

PACE-IIT & MEDICAL

MUMBAI / AKOLA / DELHI / KOLKATA / GHAZIABAD / NASHIK / GOA / BOKARO / PUNE

IIT – JEE: 2025

TW TEST (MAIN)

DATE: 23/12/23

TOPIC: CENTRE OF MASS & COLLISION

Solutions

1. (D)
The final orientation of the system will be such that the centre of mass will be vertically below the point of suspension.

2. (A)
 $MgL = (M+m)gh$

3. (D)
By the conservation of linear momentum, Velocity of the hoop is

$$V_D = \frac{2V_0}{3}$$

By conservation of Energy

$$\frac{1}{2}mV_0^2 + \frac{1}{2}mV_0^2 = \frac{1}{2}m\left(\frac{2V_0}{3}\right)^2 + \frac{1}{2}mX^2 + \frac{1}{2}mX^2$$

4. (C)
 $P_{avg} = \frac{W_{total}}{T} = \frac{W_1 + W_2}{T} \quad \dots(1)$

For W_1 ,

$$dw = Fdx = \lambda x g dx$$

$$W_1 = \int dw = \int_0^L \lambda x g dx = \frac{\lambda g L^2}{2} \quad \left(\because \lambda = \frac{m}{L} \right)$$

From work energy theorem

$$W = \Delta K.E = \frac{1}{2}Mv^2 - 0 = \frac{1}{2}Mv^2$$

Put the value of W_1 and W_2 in equation (1)

$$P_{avg} = \frac{\frac{\lambda g L^2}{2} + \frac{1}{2}\lambda L v^2}{\frac{L}{v}} \left(T = \frac{L}{v} \right)$$
$$= \frac{\lambda v g L}{2} + \frac{1}{2}\lambda v^3$$

5. (A)
Line $3x - 4y = 0$, $3x - 4y = 0$ has slope

$$\tan \theta_1 = \frac{3}{4}$$

Line $12y + 5x = 0$, $12y + 5x = 0$ has slope

$$\tan \theta_2 = -\frac{5}{12}$$

For A, $y_A = (10 \sin \theta_1)t$

For B, $y_B = (v \sin \theta_2)t$

Now according to question,

$$y_{\text{cm}} = \frac{m_A y_A + m_B y_B}{m_A + m_B} = 0$$

$$\Rightarrow (1)(10 \sin \theta_1)t - (2)(v \sin \theta_2)t = 0$$

$$\Rightarrow v = \frac{10 \sin \theta_1}{2 \sin \theta_2} = (5) \left(\frac{3}{5} \times \frac{13}{5} \right) = \frac{39}{5}$$

6. (C)

Position of Center of Gravity of first slab is at a height $= \frac{b}{2}$ from the bottom edge of the slab.

Weight of each slab = volume \times density $\times g = b^3 \rho g$

Now, for the column of N no. of slabs, the CG will be at a height

Center of gravity of column of slabs $= \frac{\text{Total height of } N \text{ slabs}}{2} = \frac{Nb}{2}$ from the ground.

So, $PE_{\text{final}} = \text{Total weight of } N \text{ slabs} \times \text{height of center of mass} = N \times b^2 \rho g \times \frac{Nb}{2} = \frac{N^2 b^4 \rho g}{2}$

And, PE of all the slabs staying on ground, before arranging those $= PE_{\text{initial}} = Nb^3 \rho g \times \frac{b}{2} = \frac{Nb^4 \rho g}{2}$

So, change in potential energy,

$$\Delta PE = PE_{\text{final}} - PE_{\text{initial}} = \text{Work done} = \frac{N^2 b^4 \rho g}{2} - \frac{N^2 b^4 \rho g}{2} = \frac{1}{2} (N^2 - N) b^4 \rho g$$

7. (C)

Given :

Mass of bullet = a

Velocity of bullet = b

Mass of wood = c

The final velocity = ?

Initial momentum = ab

Final momentum = $(a + c)v$ [v is the final velocity, and it combines with mass $(a + c)$]

Equating this, we get

$$v = \frac{ab}{a + c}$$

8. (A)

$$\therefore x_{\text{CM}} = \frac{mL + 2m \times 2L + \dots + nm \times nL}{m + 2m + \dots + nm}$$

$$\begin{aligned}
&= \frac{L(1^2 + 2^2 + \dots + n^2)}{(1 + 2 + \dots + n)} \\
&= L \frac{\sum n^2}{\sum n} = L \frac{\frac{n(n+1)(2n+1)}{6}}{\frac{n(n+1)}{2}} \\
&= \frac{(2n+1)L}{3}
\end{aligned}$$

9. (C)

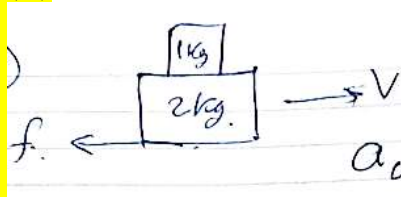
$$r = \frac{-m \left(\frac{L}{2} \hat{i} + \frac{L}{2} \hat{j} + \frac{L}{2} \hat{k} \right)}{7M} = \frac{L\sqrt{3}}{14}$$

10. (B)

CM will be at rest, $m(50 - x) = 99m(x)$

$$\Rightarrow x = \frac{1}{2}m$$

11. (D)



$$F_{\text{ext}} = f = 0.2 \times 30 = 6 \text{ N}$$

$$a_{\text{cm}} = \frac{F_{\text{ext}}}{M} = \frac{6}{3} = 2 \text{ m/s}^2$$

12. (C)

$$\vec{R}_{\text{cm}} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2} = -2\hat{i} - \hat{j} + \hat{k}$$

13. (D)

$$V_{\text{cm}} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

14. (D)

Velocity of CoM will remain unchanged as explosion was an internal force.

Ratio of velocities will be inverse of ratio of masses i.e., Speed of smaller fragment will be twice that of largest fragment.

K.E. will increase as both fragments have their K.E. where as initial mass was on rest.

Fragments have equal magnitude of momentum in ground frame and centre of mass frame as CoM is also not moving.

15. (A)

$$v = u \ln \left(\frac{m_i}{m_f} \right) - gt = 500 \ln \left(\frac{50000}{40000} \right) - 100 = 111.5 - 100 = 11.5$$

16. (B)

We are given that the mass of the big particle is $4m$ and velocity of two fragments are mutually perpendicular to each other. If p_1 and p_2 are the momentum of the fragments then momentum of third fragment is

$$p_3 = \sqrt{p_1^2 + p_2^2} = \sqrt{(mv)^2 + (mv)^2} \\ = \sqrt{2} mv$$

$$\text{Velocity of third fragment} = \frac{v}{\sqrt{2}}$$

$$\text{Total energy released in the explosion} = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2} \times 2m \left(\frac{v}{\sqrt{2}} \right)^2 = \frac{3}{2}mv^2$$

Thus, the total energy released in the explosion is $\frac{3}{2}mv^2$

17. (C)

Component of initial momentum along the wall is

$$p_{iy} = mv_0 \sin 37^\circ \hat{j}$$

Component of final momentum along the wall is

$$p_{fy} = \frac{mv_0}{2} \sin 53^\circ \hat{j}$$

So, component of impulse along the wall is $J_y = p_{fy} - p_{iy}$

$$= \frac{mv_0}{2} \times \frac{4}{5} - \frac{mv_0}{1} \times \frac{3}{5} \hat{j} = -\frac{mv_0}{5} \hat{j}$$

18. (A)

Velocity of light ball, when it is descended by height $2l$

$$v = \sqrt{2g \times 2l} = 2\sqrt{gl}$$

From conservation of linear momentum.

$$m \times 2\sqrt{gl} = 2mv' + mv'$$

$$2m\sqrt{gl} = 3mv'$$

$$\frac{2}{3}\sqrt{gl} = v'$$

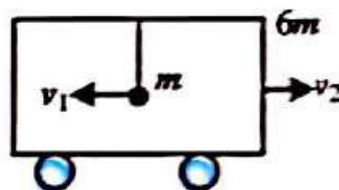
19. (C)

When the rod becomes vertical

$$mv_1 = 6mv_2 \Rightarrow v_1 = 6v_2 \quad \dots(i)$$

$$\frac{1}{2}mv_1^2 + \frac{1}{2}6mv_2^2 = mgl \quad \dots(ii)$$

Solve (i) and (ii) to get



$$v_{\text{rel}} = v_1 + v_2 = \sqrt{\frac{7}{3}} gl$$

20. (B)

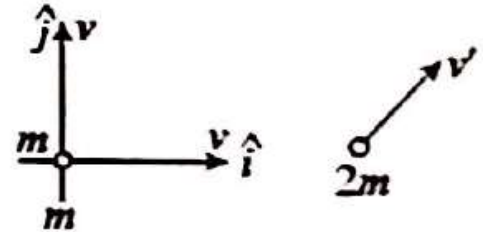
Conservation of linear momentum yields

$$mv\hat{i} + mv\hat{j} = (2m)\vec{v}'$$

$$,mv^2 \Rightarrow \vec{v}' = \frac{v}{2}(i + \hat{j}) \Rightarrow v' = \frac{v}{\sqrt{2}}$$

$$\eta = 1 - \frac{KF_f}{KE_i} = 1 - \frac{\left(\frac{1}{2}\right)(2m)v'^2}{\left(\frac{1}{2}\right)mv^2 + \left(\frac{1}{2}\right)mv^2}$$

$$\Rightarrow \eta = 1 - \left(\frac{v'}{v}\right)^2 = 1 - \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$$



21. (2)

Find the area under the graph from 0 to 2.

22. (4)

$$s = 4t = 2t + 5t^2$$

$$5t^2 = 2t$$

$$t = 0.4$$

$$v = 2 + gt = 6 \text{ m/s}$$

$$4 \times 4 + 4 \times 6 = 8v$$

$$v = 5$$

$$J = 4(v - 4) = 4$$

23. (5)

Given: $P_1 = 20 \text{ Ns}$

Let the minimum momentum of other part be P_2

Applying conservation of momentum in y direction :

$$P_1 \sin 30 = P_2 \sin \theta$$

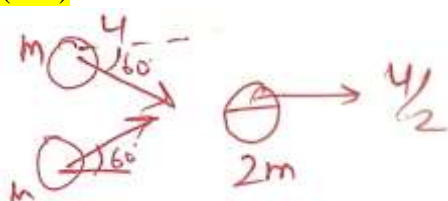
$$20 \times \frac{1}{2} = P_2 \sin \theta$$

$$\Rightarrow P_2 \sin \theta = 10 \text{ Ns}$$

Now for P_2 to be minimum, $\sin \theta$ must be maximum i.e. $\sin \theta = 1$

$$\Rightarrow P_2 = 10 \text{ Ns}$$

24. (120)



Angle between velocity is 120° .

25. (3)
 Lets take a small element at a distance x of lengths dx .
 The velocity with which element will strike the floor,

$$v = \sqrt{2gx}$$

Momentum transferred to floor is,

$$dP = dm v = \left[\frac{M}{L} x dx \sqrt{2gx} \right]$$

$$F_1 = \frac{dp}{dt} \text{ and } v = \frac{dx}{dt} = \sqrt{2gx}$$

$$F_1 = \frac{M \sqrt{2gx} \times \sqrt{2gx}}{L}$$

$$F_1 = \frac{2mgx}{L}$$

Now, the force entered by length x due to its own weight is,

$$w = \frac{mgx}{L}$$

Therefore,

$$\text{Total force} = \frac{mgx}{L} + \frac{2mgx}{L}$$

$$F = \frac{3mgx}{L}$$

26. (3)
 Apply momentum conservation.

27. (8)
 Thrust is a reaction force. When a system accelerates a mass in a particular direction, the accelerated mass will cause a force of equal magnitude and opposite direction on that system. Thrust is defined as,

$$\begin{aligned} F_{\text{thrust}} &= u \frac{dm}{dt} \\ &= (5 \times 10^4)(40) \\ &= 2 \times 10^6 \text{ N} \end{aligned}$$

28. (2)
 Horizontal Distance moved by the block wrt wedge = 12m.
 For COM to not move Wedge must move 2m in the opposite direction.

29. (5)
 It's velocity is 60 m/s downwards and 80 m/s forward. Apply momentum conservation to get this.

30. (4)

$$S_{avg} = \frac{\int_0^{\sqrt{\frac{2h}{g}}} \frac{1}{2} gt^2 dt}{\int_0^{\sqrt{\frac{2h}{g}}} dt} = \frac{g}{6} \cdot \frac{2h}{g} = \frac{h}{3}$$

$$H = h - \frac{h}{3} = \frac{2h}{3}$$

SOLUTIONS

31. (D)

Equilibrium constant is a function of temperature only.

32. (A)

$$K_p = \frac{1 \cdot P_{\text{CO}_2}}{1} = P_{\text{CO}_2}$$

33. (D)

K_{eq} is independent on concentration.

34. (B)

$$K_p = K_c \cdot (RT)^{\Delta n_g} \Rightarrow \frac{K_c}{K_p} = (RT)^{-\Delta n_g} = (RT)^{-\left(\frac{1}{2}\right)}$$
$$= \sqrt{RT}$$

35. (C)

$$K_{\text{eq}} = \frac{K_f}{K_b} = \frac{0.16}{4 \times 10^4} = 4 \times 10^{-6}$$

36. (C)

For $r_f > r_b$ means the net reaction in forward direction, the reaction quotient Q should be less than K_{eq} .

37. (A)

$$K = \frac{[\text{C}_6\text{H}_6]}{[\text{C}_2\text{H}_2]^3} \Rightarrow 4 = \frac{[\text{C}_6\text{H}_6]}{(0.5)^3} \Rightarrow [\text{C}_6\text{H}_6] = 0.5\text{M}$$

38. (D)

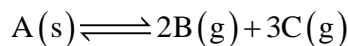
Solubility of gases in liquids : When a gas dissolves in liquid, there is decrease in volume. Thus, increase of pressure will favour the dissolution of gas in liquid.

39. (B)



For endothermic reactions, K_{eq} increases with the increase in temperature.

40. (D)



$$\therefore K_c = [B]^2 [C]^3 = x^2 (y)^3 = x^2 y^3$$

If concentration of C is two time to that of y at equilibrium *i.e.* $2y$ then if concentration of B is [B]

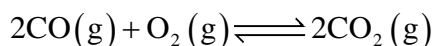
$$K_c = [B]^2 [2y]^3$$

$$\text{Or } [x]^2 [y]^3 = 8[B]^2 [y]^3$$

$$\therefore [B] = \frac{x}{\sqrt{8}} = \frac{x}{2\sqrt{2}}$$

41. (A)

From (i) and (ii)

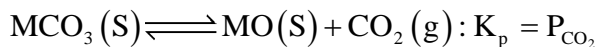


$$K_{2000} = \frac{1}{(4.4)^2 \times 5.31 \times 10^{-10}} = 9.73 \times 10^7$$

$$K_{1000} = 2.24 \times 10^{22}$$

On increasing temperature, K_{eq} is decreased and hence, $\Delta H^\circ = -ve$.

42. (A)



$$\text{Now, } \ln \frac{K_2}{K_1} = \frac{\Delta H^\circ}{R} \left(\frac{1}{T_1} - \frac{1}{T_2} \right)$$

$$\text{Or, } 2.303 \times \log 100 = \frac{\Delta H^\circ}{2} \left(\frac{1}{400} - \frac{1}{500} \right)$$

$$\Rightarrow \Delta H^\circ = 18424 \text{ cal}$$

43. (D)

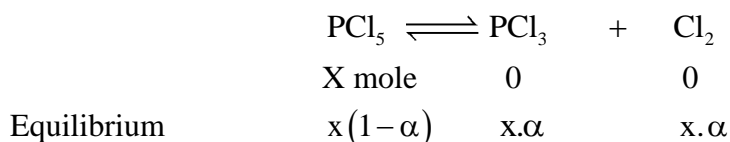
$$\Delta G^\circ = -2.303 RT \cdot \log K_{eq}$$

$$\text{Or, } -4.606 = -2.303 \times \frac{2}{1000} \times 1000 \times \log K_{eq}$$

$$\Rightarrow K_{eq} = 10$$

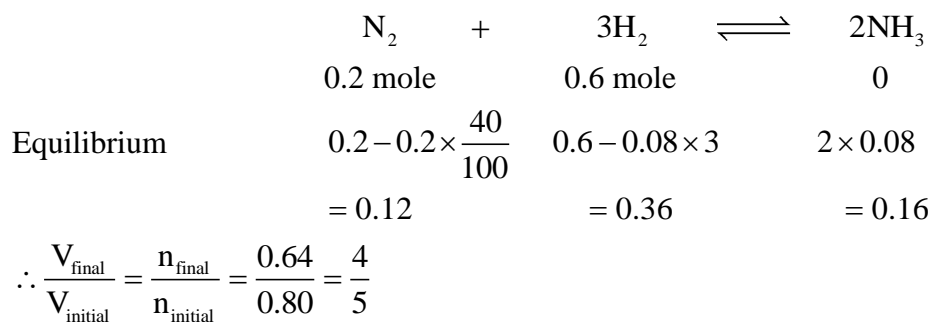
$$\text{For reverse reaction, } K'_{eq} = \frac{1}{10} = 0.1$$

44. (C)

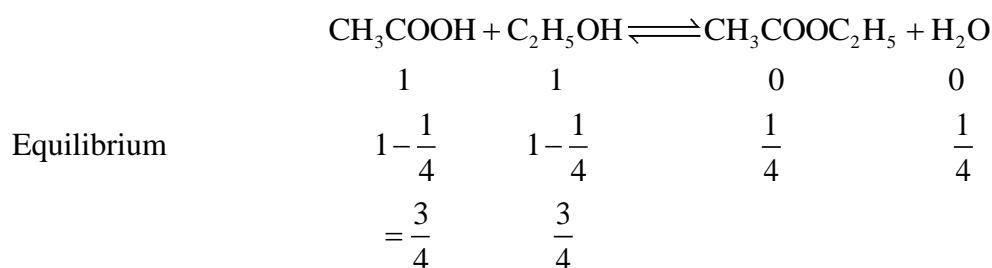


$$E \quad P_{\text{PCl}_3} = \frac{x \cdot \alpha}{x(1+\alpha)} \times P \Rightarrow P_{\text{PCl}_3} \cdot P^{-1} = \frac{\alpha}{1+\alpha}$$

45. (A)



46. (A)



$$K = \frac{\frac{1}{4} \times \frac{1}{4}}{\frac{3}{4} \times \frac{3}{4}} = \frac{1}{9}$$

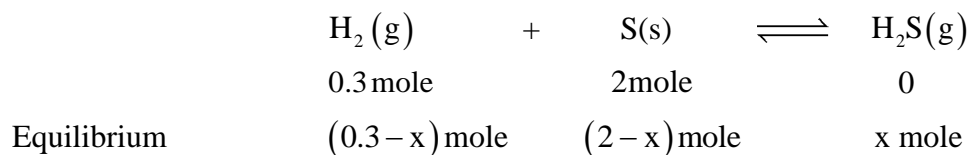
47. (A)

At equilibrium, $[\text{A}] = [\text{B}]$ and $[\text{C}] = [\text{D}]$ and hence

$$K = \frac{[\text{C}][\text{D}]}{[\text{A}][\text{B}]} = \frac{[\text{C}]^2}{[\text{A}]^2} \Rightarrow \frac{[\text{C}]}{[\text{A}]} = \sqrt{2.25}$$

$$= 1.5 \Rightarrow \frac{[\text{A}]}{[\text{C}]} = \frac{2}{3}$$

48. (A)



$$K_c = \frac{\frac{x}{2}}{\frac{(0.3-x)^2}{2}} = 0.08 \Rightarrow 0.022$$

$$\text{Now, } P_{\text{H}_2\text{S}} = \frac{x \times 0.08 \times 360}{2} = 0.32 \text{ atm}$$

49. (C)
 K_{eq} is a function of temperature only.
50. (D)
 For forward reaction (exothermic), temperature should be low. As mole of gas is decreasing, the pressure should be high.

51. (1)

$$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$$

$$= -30 - 300 \times (-0.1)$$

$$= -30 + 30 = 0$$

$$\therefore \Delta G^\circ = 2.303RT \log K_p$$

$$\therefore K_p = 1$$

52. (4)

$$2A(s) + nB(g) \rightleftharpoons 3C(g)$$

$$\frac{K_p}{K_c} = (RT)^{\Delta x}$$

$$\left(\frac{0.0105}{0.45} \right) = (0.0821 \times 523)^{\Delta x}$$

$$\therefore \Delta x = -1$$

$$\therefore \Delta x = 3 - (n) = -1$$

$$\therefore n = 4$$

53. (4)

$$\frac{K_f}{K_b} = 4$$

$$\therefore K_c = \frac{K_f}{K_b} = 4$$

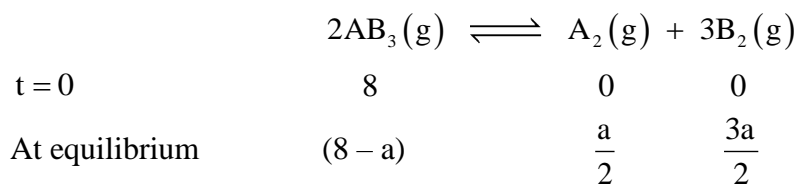
54. (1)

$$\begin{array}{ccccc}
 AB_2(g) & \rightleftharpoons & AB(g) & + & B(g) \\
 P & & 0 & & 0 \\
 P - P' & & P' & & P'
 \end{array}$$

Given $P = 6 \text{ atm}$
 $P + P' = 8 \text{ atm}$
 $\therefore P' = 2 \text{ atm}$

$$K_p = \frac{2 \times 2}{4} = 1$$

55. (27)



$$\text{Thus, } K_c = \frac{[\text{A}_2][\text{B}_2]^3}{[\text{AB}_3]^2}; \text{ Also } \frac{a}{2} = 2 \quad \therefore a = 4$$

$$\therefore [\text{AB}_3] = \frac{4}{1}; [\text{A}_2] = \frac{2}{1}; [\text{B}_2] = \frac{6}{1}$$

$$\text{Thus, } K_c = \frac{2 \times 6^3}{4^2} = 27 \text{ mol}^2\text{L}^{-2}$$

56. (3)

57. (8)

$$K_p = P'_{\text{NH}_3} \times P'_{\text{HCl}} = 16$$

$$K_p = P \times P = 16$$

$$\therefore P = 4$$

$$\therefore \text{Total pressure at eq.} = P + P = 4 + 4 = 8$$

58. (4)

K_{eq} is independent on catalyst.

59. (1)

K_c does not depend on concentration of reactants or products.

$$K_c = \frac{[\text{D}][\text{C}]}{[\text{A}][\text{B}]} = \frac{1.5 \times 1.5}{1.5 \times 1.5} = 1$$

60. (23)

$$K_p = (P_{\text{H}_2\text{O}}) = 23 \text{ mm}$$

Solutions

61. (B)

$$f(x) = \tan\left(\frac{\pi}{4} \sin^2 x\right)$$

$$\sin x \in [-1, 1]$$

$$\therefore \sin^2 x \in [0, 1]$$

$$\Rightarrow \frac{\pi}{4} \sin^2 x \in \left[0, \frac{\pi}{4}\right]$$

$$\therefore f(x) \in \tan\left[0, \frac{\pi}{4}\right]$$

$$f(x) \in \left[\tan 0, \tan \frac{\pi}{4}\right]$$

$$\therefore f(x) \in [0, 1]$$

62. (A)

Range of $f(x)$ is $(1, 2) \cup [5, \infty)$

Which is not equal to its codomain.

But $f(x)$ is monotonic increasing function

So, $f(x)$ is one-one but not onto

63. (B)

All bijective function of $f: A \rightarrow A$ are

$$[(1,1), (2,2), (3,3)], [(1,1), (2,3), (3,2)], [(1,2), (2,1), (3,3)]$$

$$[(1,2), (2,3), (3,1)], [(1,3), (2,1), (3,2)], [(1,3), (2,2), (3,1)]$$

But $f(1) \neq 3$, $f(2) \neq 1$ and $f(3) \neq 2$

\therefore Number of bijective functions are $[(1,1), (2,2), (3,3)]$ and $[(1,2), (2,3), (3,1)]$

$\therefore 2$

64. (B)

$$f: R \rightarrow R \quad f(x) = ax^3 + bx^2 + cx + d \quad (a \neq 0)$$

For f to be one-one

$$f(x_1) = f(x_2)$$

i.e., $x_1 = x_2$

for f to be onto :

range = co-domain

$f(x)$ is a (i) strictly increasing function, i.e. $f'(x) > 0$

or (ii) strictly decreasing function, i.e. $f'(x) < 0$

$$f'(x) = 3ax^2 + 2bx + c$$

$$\therefore 3ax^2 + 2bx + c > 0 \quad \dots(\text{i})$$

$$\text{or } 3ax^2 + 2bx + c < 0 \quad \dots(\text{ii})$$

$$\therefore D < 0$$

$$(2b)^2 - 4(3a)(c) < 0$$

$$\text{or } b^2 - 3ac < 0$$

65. (A)

$$x^{\sqrt{x}} = (\sqrt{x})^x$$

Take log both sides

$$\Rightarrow \sqrt{x} \log x = x \log \sqrt{x}$$

$$\Rightarrow \sqrt{x} \log x = \frac{x}{2} \log x$$

$$\Rightarrow \left(\sqrt{x} - \frac{x}{2} \right) \log x = 0$$

$$\therefore \log x = 0 \text{ or } \sqrt{x} = \frac{x}{2}$$

$$\therefore x = 1 \text{ or } 4x = x^2$$

$$x = 0 \text{ (not possible) or } x = 4$$

$$\therefore x = 1 \text{ or } x = 4.$$

Hence, one of p & q is composite number.

66. (D)

$$\text{Let } y = \sqrt{(ab) + (a-b) \sqrt{ab + (a-b) \sqrt{ab + (a-b) \sqrt{ab + \dots}}}}$$

$$\therefore y = \sqrt{ab + (a-b)y}$$

Squaring both sides

$$y^2 = ab + (a-b)y$$

$$y^2 - (a-b)y - ab = 0$$

$$\therefore y = \frac{(a-b) \pm \sqrt{(a-b)^2 + 4ab}}{2}$$

$$y = \frac{(a-b) \pm (a+b)}{2}$$

$$\therefore y = \frac{(a-b)+(a+b)}{2} \text{ or } y = \frac{(a-b)-(a+b)}{2}$$

$$y = a \text{ or } y = -b$$

\therefore independent of none of a & b .

67. (D)

$$f(x) = \frac{e^{|x|} - e^{-x}}{e^x + e^{-x}}$$

f is not one-one as $f(x) = 0$ and $f(-1) = 0$

f is also not onto as for $y = 1$, there is no $x \in R$

Such that $f(x) = 1$. If there is such an $x \in R$, then $e^{|x|} - e^{-x} = e^x + e^{-x}$. Clearly $x \neq 0$.

For $x > 0$, the equation gives $e^{-x} = 0$ (not possible) and for $x < 0$, $\frac{e^{2x} + 1}{e^x} = 0$ which is also not possible.

$\therefore f$ is neither one-one nor onto.

68. (C)

$$f(x) = 3x - 4$$

$$\Rightarrow f^{-1}(x) = \frac{x+4}{3}$$

$$g(x) = 2 + 3x$$

$$\Rightarrow g^{-1}(x) = \frac{x-2}{3}$$

$$f^{-1}(5) = \frac{5+4}{3} = 3$$

$$(g^{-1} \circ f^{-1})(5) = g^{-1}[f^{-1}(5)] = g^{-1}(3) = \frac{3-2}{3} = \frac{1}{3}$$

69. (D)

$f'(x) = \frac{e^x - e^{-x}}{2}$ changes its sign at $x = 0$. $f(x)$ is many one.

Hence inverse doesn't exist.

70. (D)

Substituting the values, we get

$$\Rightarrow \cos\left(\ln \frac{1}{x}\right) \cdot \cos\left(\ln \frac{1}{y}\right) - \frac{1}{2} \left[\cos\left(\ln \left(\frac{x}{y}\right)\right) + \cos(\ln(xy)) \right]$$

$$\Rightarrow \cos(\ln x^{-1}) \cos(\ln y^{-1}) - \frac{1}{2} [\cos(\ln x - \ln y) + \cos(\ln x + \ln y)]$$

$$\Rightarrow -\cos(\ln x) \cdot -\cos(\ln y) - \frac{1}{2} [2 \cos(\ln x) \cos(\ln y)]$$

$$\Rightarrow \cos(\ln x) \cos(\ln y) - \cos(\ln x) \cos(\ln y)$$

$$= 0$$

71. (B)

$\frac{|\sin x|}{\cos x}$ period of N' is π and D' is 2π

\therefore period of $\frac{|\sin x|}{\cos x}$ is LCM of π and $2\pi \Rightarrow$ i.e. 2π

Similarly $\frac{\sin x}{|\cos x|}$ period of N' is 2π and D' is π .

\therefore period of $\frac{\sin x}{|\cos x|}$ is LCM of π and $2\pi \Rightarrow$ i.e. 2π

Hence the period of the given function is LCM of 2π & 2π . i.e. 2π .

72. (A)

For (B) $f(x) = \sin \frac{1}{x}$

$$= (goh)x, \text{ where } g(x) = \sin x, h(x) = \frac{1}{x}$$

$\therefore f(x)$ is not periodic.

For (C) $f(x) = x \cos x = g(x) \cdot h(x)$, where $g(x) = x, h(x) = \cos x$

$\therefore f(x)$ is not periodic.

For (D) $f(x) = \sin \sqrt{x} = (goh)x$, where $g(x) = \sin x, h(x) = \sqrt{x}$

$\therefore f$ is not periodic.

For (A) $f(x) = x - [x]$

$$= \{x\}$$

$\therefore \{ \}$ is a periodic functions with period 1.

$\therefore f$ is periodic.

73. (B)

$$f(x) = \ln \left(\frac{x^2 + e}{x^2 + 1} \right) = \ln \left(\frac{x^2 + 1 - 1 + e}{x^2 + 1} \right) = \ln \left(1 + \frac{e-1}{x^2 + 1} \right)$$

\therefore clearly range is $(0, 1]$

74. (A)

$$f(x) = \frac{\sin(\pi[x])}{x^2 + 1}$$

$\therefore [x]$ always gives integer values and $\sin n\pi = 0$

$\therefore f(x) = 0$

75. (B)

$\sin(\sqrt{x})$ is non-periodic.

76. (B)

Put $x = y = 0$

$$\therefore f(0) + f(0) = 2f^2(0)$$

$$\because f(0) \neq 0$$

$$\therefore f(0) = 1$$

Put $x = 2$ and $y = 1$

$$\therefore f(3) + f(1) = 2f(2)f(1)$$

$$\therefore f(3) - 2f(1)f(2) = -f(1)$$

$$\because f(1) = K$$

$$\therefore f(3) - 2Kf(2) = -K$$

77. (A)

Put $y = 0$

$$\therefore f(x) = f(0)$$

$$\because f(2005) = 2005, \therefore f(0) = 2005$$

$$\therefore f(-2005) = f(0) = 2005$$

78. (D)

Since $f(x)$ is an odd function

$$\left[\frac{x^2}{a} \right] = 0 \text{ for all } x \in [-10, 10] \Rightarrow 0 \leq \frac{x^2}{a} < 1 \forall x \in [-10, 10]$$

$$\Rightarrow a > 100$$

79. (B)

$$f(x) = \ln\left(\frac{1+x}{1-x}\right), g(x) = \frac{3x+x^3}{1+3x^3}$$

$$\therefore f(g(x)) = \ln\left[\frac{1 + \frac{3x+x^3}{1+3x^3}}{1 - \frac{3x+x^3}{1+3x^3}} \right]$$

$$\Rightarrow f(g(x)) = \ln\left[\frac{(1+x)^3}{(1-x)^3} \right]$$

$$= 3\ln\left(\frac{1+x}{1-x}\right)$$

$$= 3f(x)$$

80. (D)

$$(hogof)(x) = h(g(f(x)))$$

$$\begin{aligned}
&= h(g(x)) \quad \because f(x) = x \\
&= h(2x^2 + 1) \\
&= 2x^2 + 1 + 1 \\
&= 2(x^2 + 1)
\end{aligned}$$

$$\therefore (hogof)(x) = 2(x^2 + 1)$$

81. (2)

$$f(1) + 2f(2) + 3f(3) + \dots + nf(n) = n(n+1)f(n)$$

$$\Rightarrow f(1) + 2f(2) + 3f(3) + \dots + nf(n) + (n+1)f(n+1) = (n+1)(n+2)f(n+1)$$

$$\Rightarrow n(n+1)f(n) = (n+1)^2 f(n+1)$$

$$\Rightarrow nf(n) = (n+1)f(n+1)$$

$$\therefore 2f(2) = 3f(3) = \dots = nf(n)$$

$$\therefore f(n) = \frac{1}{n}$$

$$\therefore 2126f(1063) = 2126 \times \frac{1}{1063} = 2$$

82. (9)

$$f(x) = x^3 + \frac{1}{x^3} - 4\left(x^2 + \frac{1}{x^2}\right) + 13$$

We know that

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right)$$

$$x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$$

$$\text{For } x = 2 + \sqrt{3}, \quad x + \frac{1}{x} = 2 + \sqrt{3} + \frac{1}{2 + \sqrt{3}}$$

$$= 2 + \sqrt{3} + \frac{2 - \sqrt{3}}{1}$$

$$= 4$$

$$\therefore f(x) = \left(x + \frac{1}{x}\right)^3 - 3\left(x + \frac{1}{x}\right) - 4\left[\left(x + \frac{1}{x}\right)^2 - 2\right] + 13$$

$$f(2 + \sqrt{3}) = 4^3 - 3 \times 4 - 4[(4)^2 - 2] + 13$$

$$= 64 - 12 - 56 + 13$$

$$= 9$$

83. (7)

$$f(2a - x) = f(x)$$

$$\Rightarrow f(2a - x) = -f(x)$$

$$\therefore f \text{ is odd} \Rightarrow f(x+4a) = f(x)$$

$$\Rightarrow f \text{ is periodic with period } 4a$$

$$\Rightarrow f(1+4r) = f(1)$$

$$\text{Now, } \sum_{r=0}^{\infty} [f(1)]^r = 8 \Rightarrow \frac{1}{1-f(1)} = 8$$

$$\Rightarrow f(1) = \frac{7}{8}$$

$$\therefore 8f(1) = 7$$

84. (2)

$$\text{Given } f(x-f(y)) = f(f(y)) + xf(y) + f(x) - 1$$

$$\text{Putting } x = f(y) = 0, \text{ then } f(0) = f(0) + 0 + f(0) - 1$$

$$\therefore f(0) = 1$$

Now, Put $x = f(y) = \lambda$ in equation, then

$$f(0) = f(\lambda) + \lambda^2 + f(\lambda) - 1$$

$$\Rightarrow 1 = 2f(\lambda) + \lambda^2 - 1$$

$$\therefore f(\lambda) = 1 - \frac{\lambda^2}{2}$$

$$\therefore f(x) = 1 - \frac{x^2}{2}$$

$$\begin{aligned} \therefore |(f(16))| - 125 &= \left| 1 - \frac{16^2}{2} \right| - 125 \\ &= 127 - 125 = 2 \end{aligned}$$

85. (1)

$$\text{If } f^3(x) - 3f^2(x) + 3f(x) - 1 = x^6$$

$$\Rightarrow (f(x) - 1)^3 = x^6$$

$$\Rightarrow f(x) - 1 = x^2$$

$$\Rightarrow f(x) = x^2 + 1$$

$$\therefore f(0) = 1$$

86. (2)

$$f(x) = \frac{1}{x}, g(x) = \sqrt{x}$$

$$4(g \circ \sqrt{f})(16) = 4g(\sqrt{f(16)})$$

$$= 4g\left(\sqrt{\frac{1}{16}}\right)$$

$$= 4g\left(\frac{1}{4}\right)$$

$$= 4 \times \sqrt{\frac{1}{4}}$$

$$= 2$$

87. (2)

Whenever, $f(x) + f\left(\frac{1}{x}\right) = f(x) \cdot f\left(\frac{1}{x}\right)$, then $f(x) = 1 + x^n$ or $f(x) = 1 - x^n$ ($n > 0$)

Here, $f(2) = -7$

$$\therefore f(x) = 1 - x^n \quad f(2) = 1 - 2^n$$

$$\Rightarrow 2^n = 8$$

$$\Rightarrow x = 3$$

$$\therefore f(x) = 1 - x^3$$

$$\therefore \frac{f(3) + 30}{2} = \frac{1 - 3^3 + 30}{2} = 2$$

88. (1)

$$G(n) = n - (-1)^{n-1}(n-1)$$

$$GoG(n) = G\left(n - (-1)^{n-1}(n-1)\right)$$

$$= n - (-1)^{n-1}(n-1) - (-1)^{n-1}\left((n-1) - (-1)^{n-1}(n-1)\right)$$

$$= n - (n-1)$$

$$= 1$$

89. (4)

$$g(f(f(2))) = K$$

$$g(f(f(2))) = g[f(f(2))]$$

$$= g[f(5)] \quad \because f(x) = 2x + 1$$

$$= g[11] \quad \because f(x) = 2x + 1$$

$$= 11^2 + 1 \quad \because g(x) = x^2 + 1$$

$$\therefore K = 122$$

No. of divisors of 122 is 4 (1, 2, 61 and 122)

90. (1)

$$f(x) = (100^5 - x^{10})^{\frac{1}{10}}$$

$$\Rightarrow (f(x))^{10} = 100^5 - x^{10}$$

$$f(f(x)) = (100^5 - f(x)^{10})^{\frac{1}{10}}$$

$$\begin{aligned} &= \left[100^5 - (100^5 - x^{10}) \right]^{\frac{1}{10}} \\ &= x \\ \therefore \frac{1}{2^{10}} f(f(1024)) &= \frac{1}{2^{10}} \times 1024 = 1 \end{aligned}$$