Inverse Trigonometric Functions

EXERCISE - 1 [A]

1. (A)

Since $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

And $\sin^{-1}\left(\frac{1}{x}\right) + \cos^{-1}\left(\frac{1}{x}\right) = \frac{\pi}{2}$

So $\sin^{-1} x + \cos^{-1} x + \sin^{-1}\left(\frac{1}{x}\right) + \cos^{-1}\left(\frac{1}{x}\right) = \pi$

2. (D)

Since domain of $\sin^{-1} x$ & $\cos^{-1} x$ is [-1, 1] but since x > 0

so 2π + x > 1 so the given terms is not defined

3. (D)

Given $\cos^{-1}(\frac{\pi}{3} + \sec^{-1}(-2)) = \cos^{-1}(\frac{\pi}{3} + \cos^{-1}\left(\frac{1}{-2}\right)) = \cos^{-1}(\frac{\pi}{3} + \frac{2\pi}{3}) = \cos^{-1}(\pi) = -1$

4. (A)

Given, $\sin^{-1}\left(\sin\left(\frac{23\pi}{7}\right)\right) + \cos^{-1}\left(\cos\left(\frac{39\pi}{7}\right)\right)$

Given $\sin^{-1}\left(\sin\left(\frac{23\pi}{7}\right)\right) = \sin^{-1}\left(\sin\left(2\pi + \frac{9\pi}{7}\right)\right) = \sin^{-1}\left(\sin\left(\frac{9\pi}{7}\right)\right) = -\frac{9\pi}{7} + \pi$

So $\cos^{-1}\left(\cos\left(\frac{39\pi}{7}\right)\right) = \cos^{-1}\left(\cos\left(4\pi + \frac{11\pi}{7}\right)\right) = \cos^{-1}\left(\cos\left(\frac{11\pi}{7}\right)\right) = -\frac{11\pi}{7} + 2\pi$

So $\sin^{-1}\left(\sin\left(\frac{23\pi}{7}\right)\right) + \cos^{-1}\left(\cos\left(\frac{39\pi}{7}\right)\right) = -\frac{9\pi}{7} + \pi - \frac{11\pi}{7} + 2\pi = \frac{\pi}{7}$

5. (D)

$\cos^{-1}\left(\cos\left(-\frac{17}{15}\pi\right)\right) = \cos^{-1}\left(\cos\left(\frac{17}{15}\pi\right)\right) = -\frac{17}{15}\pi + 2\pi = \frac{13\pi}{15}$

6. (C)

$sin\left(\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) = sin\left(\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right) = sin\left(\frac{\pi}{2} + \frac{\pi}{3}\right) = \frac{1}{2}$

7. (A)

$\sin\left(\frac{\pi}{6} + sin^{-1}\left(-\frac{1}{2}\right)\right) = \sin\left(\frac{\pi}{6} - \frac{\pi}{6}\right) = 0$

8. (C)

$\tan\left(90^{o} - \cot^{-1}\left(\frac{1}{3}\right)\right) = \cot\left(\cot^{-1}\left(\frac{1}{3}\right)\right) = \frac{1}{3}$

9. (A)

$\sin\left(\cos^{-1}\frac{12}{13}\right)$ let $\cos^{-1}\left(\frac{12}{13}\right) = \theta, \cos\theta = \frac{12}{13}, so \sin\theta = \frac{5}{13}$

10. (D)

$\text{Sin}^{-1}\left(\cos\frac{33\pi}{5}\right) = \sin^{-1}\left(\cos\left(6\pi + \frac{3\pi}{5}\right)\right) = \sin^{-1}\left(\cos\frac{3\pi}{5}\right) = \sin^{-1}\left(\cos(\frac{\pi}{2} + \frac{\pi}{10}\right)$

$= \sin^{-1}\left(\sin\left(\frac{\pi}{10}\right)\right) = \frac{\pi}{10}$

11. (B)

Given $\sin^{-1}x + \sin^{-1}y = \frac{2\pi}{3}$

So $\left(\frac{\pi}{2} - \cos^{-1}x\right) + \left(\frac{\pi}{2} - \cos^{-1}y\right) = \frac{2\pi}{3}$

Or $\cos^{-1}x + \cos^{-1}y = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$

12. (B)

13. (C)

Here θ = 10 rad doesn't lie between $-\pi^2$ and π^2

But, 3π − θ lies between −π2 andπ2

Also, sin (3π − 10) = sin10

$\Rightarrow \sin^{-1}(\sin 10) = \sin^{-1}(\sin(3\pi - 10)) = 3\pi - 10$

14. (B)

Let $y = \cos^{-1}\left(\sqrt{\frac{1+\cos x}{2}}\right) = \cos^{-1}\left(\sqrt{\frac{2\cos^2\left(\frac{x}{2}\right)}{2}}\right) = \cos^{-1}\left[\cos\left(\frac{x}{2}\right)\right] = \frac{x}{2}$

15. (A)

Let $\frac{1}{2}\cos^{-1}\frac{\sqrt{5}}{3} = \theta \Rightarrow \cos 2\theta = \frac{\sqrt{5}}{3}$

But $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta} \Rightarrow \frac{\sqrt{5}}{3} = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$\Rightarrow \sqrt{5} + \sqrt{5}\tan^2\theta = 3 - 3\tan^2\theta$

$\Rightarrow \left(\sqrt{5} + 3\right)\tan^2\theta = 3 - \sqrt{5} \Rightarrow \tan^2\theta = \frac{3-\sqrt{5}}{3+\sqrt{5}}$

$\Rightarrow \tan^2\theta = \frac{\left(3-\sqrt{5}\right)^2}{4} \Rightarrow \tan\theta = \frac{3-\sqrt{5}}{2}$

On rationalizing $\Rightarrow \tan\theta = \frac{3-\sqrt{5}}{2} \times \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{2}{3+\sqrt{5}}$

16. (B)

Given expression is: $\tan^{-1}\left(\frac{a-b}{1+ab}\right)+\tan^{-1}\left(\frac{b-c}{1+bc}\right)$

Let, $y=\tan^{-1}\left(\frac{a-b}{1+ab}\right)+\tan^{-1}\left(\frac{b-c}{1+bc}\right)$ ...

We know that: $\tan^{-1}a-\tan^{-1}b=\tan^{-1}\left(\frac{a-b}{1+ab}\right)$ ...

So, applying equation (2) in equation (1) we get:

$$\begin{aligned}y&=(\tan^{-1}a-\tan^{-1}b)+(\tan^{-1}b-\tan^{-1}c)\\&=\tan^{-1}a-\tan^{-1}b+\tan^{-1}b-\tan^{-1}c\\&=\tan^{-1}a-\tan^{-1}c\end{aligned}$$

Therefore, the expression reduces to $\tan^{-1}a-\tan^{-1}\mathbf{c}$.

17. (C)

Given that, $\sin^{-1}\frac{1}{3}+\sin^{-1}\frac{2}{3}=\sin^{-1}x$

Now using the identity, $\sin^{-1}a+\sin^{-1}b=\sin^{-1}\left[a\sqrt{(1-b^2)}+b\sqrt{((-a^2)}\right]$.

Here, $a=\frac{1}{3},b=\frac{2}{3}$

Substituting values we get:

$$\sin^{-1}\frac{1}{3}+\sin^{-1}\frac{2}{3}=\sin^{-1}\left[\frac{1}{3}\sqrt{\left(1-\frac{4}{9}\right)}+\frac{2}{3}\sqrt{\left(1-\frac{1}{9}\right)}\right]$$

$$=\sin^{-1}\left[\frac{1}{3}\sqrt{\frac{5}{9}}+\frac{2}{3}\sqrt{\frac{8}{9}}\right]=\sin^{-1}\left[\frac{1}{9}\sqrt{5}+\frac{4}{9}\sqrt{2}\right]=\sin^{-1}\left[\frac{\sqrt{5}+4\sqrt{2}}{9}\right]$$

18. (B)

Given $\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$

$\Rightarrow \tan^{-1}\frac{2x+3x}{1-2x\times 3x}=\frac{\pi}{4}$

$\Rightarrow \frac{5x}{1-6x^2}=1$

$\Rightarrow 6x^2+5x-1=0$

$\Rightarrow (6x-1)(x+1)=0$

$\Rightarrow x=\frac{1}{6},-1$

19. (A)

L.H.S

$= \cot^{-1}\left(\frac{xy+1}{x-y}\right) + \cot^{-1}\left(\frac{yz+1}{y-z}\right) + \cot^{-1}\left(\frac{zx+1}{z-x}\right)$

$= \tan^{-1}\left(\frac{x-y}{xy+1}\right) + \tan^{-1}\left(\frac{y-z}{yz+1}\right) + \tan^{-1}\left(\frac{z-x}{zx+1}\right)$

$= [\tan^{-1}x - \tan^{-1}y] + [\tan^{-1}y - \tan^{-1}z] + [\tan^{-1}z - \tan^{-1}x]$

(since $0 < xy, yz, zx < 1$)

$= 0$

$=$ RHS

20. (B)

$y = \cot^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$

$y = \frac{\pi}{2} - \tan^{-1}(\sqrt{\cos x}) - \tan^{-1}(\sqrt{\cos x})$

$y = \frac{\pi}{2} - 2\tan^{-1}(\sqrt{\cos x})$

$\frac{\pi}{2} - y = 2\tan^{-1}(\sqrt{\cos x})$

$\Rightarrow \cos\left(\frac{\pi}{2} - y\right) = \cos\left(2\tan^{-1}(\sqrt{\cos x})\right)$

Now apply, $\cos 2A = \frac{1-\tan^2 A}{1+\tan^2 A}$ on R.H.S.

$$\sin y = \frac{1 - \tan^2\left[\tan^{-1}(\sqrt{\cos x})\right]}{1 + \tan^2\left[\tan^{-1}(\sqrt{\cos x})\right]}$$

$$\sin y = \frac{1 - \cos x}{1 + \cos x}$$

$$\sin y = \frac{2\sin^2 \frac{x}{2}}{2\cos^2 \frac{x}{2}}$$

$$\Rightarrow \sin y = \tan^2 \frac{x}{2}$$

1. (C)
The above expression is true for
$\alpha = 1, \beta = 1$ and $\gamma = 1$
Since $\frac{-\pi}{2} \leq \sin^{-1}(x) \leq \frac{\pi}{2}$
Hence
$\alpha\beta + \beta\gamma + \gamma\alpha$
$= (1) + (1) + (1)$
$= 3$

2. (B)
$\frac{-2\pi}{5}$
$= -\sin^{-1}\left(\sin\left(\frac{2\pi}{5}\right)\right)$
$= \sin^{-1}\left(-\sin\left(\frac{2\pi}{5}\right)\right)$
$= \sin^{-1}\left(\sin\left(\pi + \frac{2\pi}{5}\right)\right)$
$= \sin^{-1}\left(\sin\left(\frac{7\pi}{5}\right)\right)$

3. (C)
$cos^{-1}\left(-sin\frac{7\pi}{6}\right) = cos^{-1}\left(\cos\frac{\pi}{3}\right) = \frac{\pi}{3}$

4. (C)
$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$
$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6}$
$= \frac{3\pi + 8\pi - 2\pi}{12} = \frac{9\pi}{12} = \frac{3\pi}{4}$

5. (A)
Since $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$
$\tan^{-1}x + \cot^{-1}x + \sin^{-1}x = \frac{\pi}{2} + \sin^{-1}x$
As $-\frac{\pi}{2} \leq \sin^{-1}\text{x} \leq \frac{\pi}{2} \Rightarrow 0 \leq \frac{\pi}{2} + \sin^{-1}\text{x} \leq \pi$
$\Rightarrow 0 \leq \tan^{-1}x + \cot^{-1}x + \sin^{-1}x \leq \pi$
$\therefore \text{a} = \text{o}, \text{b} = \pi$

6. (A)
If $sin^{-1}x + tan^{-1}x = y(-1 < x < 1)$ then $y = \frac{3\pi}{2}$

7. (B)

We have

$\sin^{-1}x - \cos^{-1}x = \pi/6$

$\Rightarrow \sin^{-1}x + \cos^{-1}x - 2 \cdot \cos^{-1}x = \pi/6$

$\Rightarrow \pi/2 - 2 \cdot \cos^{-1}x = \pi/6$

$\Rightarrow -2 \cdot \cos^{-1}x = \pi/6 - \pi/2$

$\Rightarrow -2 \cdot \cos^{-1}x = \frac{\pi - 3\pi}{6}$

$\Rightarrow -2 \cdot \cos^{-1}x = \frac{\pi}{6} - \frac{3\pi}{6}$

$\Rightarrow 2 \cdot \cos^{-1}x = 2\pi/6$

$\Rightarrow \cos^{-1}x = \frac{2\pi}{6.2}$

$\Rightarrow \cos^{-1}x = \pi/6$

$\Rightarrow x = \cos\pi/6$

8. (C)

The formula

$\tan^{-1}[\tan(a)] = a$

Works for $a \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$

In our case $\frac{5\pi}{7} > \frac{\pi}{2}$ but we can use the periodicity of tan:

$\tan\left(\frac{5\pi}{7}\right) = \tan\left(\frac{5\pi}{7} - \pi\right) = \tan\left(-\frac{2\pi}{7}\right)$

$\tan^{-1}\left[\tan\left(\frac{5\pi}{7}\right)\right] = \tan^{-1}\left[\tan\left(-\frac{2\pi}{7}\right)\right] = -\frac{2\pi}{7}$

9. (D)

The number of positive integral solutions of $cos^{-1}\left(4x^2 - 8x + \frac{7}{2}\right) = \frac{\pi}{3}$ is None of the above

10. (D)

Given $a\sin^{-1}\mathrm{x} - \mathrm{b}\cos^{-1}\mathrm{x} = \mathrm{c}$

$\Rightarrow a\sin^{-1}x - b\left(\frac{\pi}{2} - \sin^{-1}x\right) = c$

$\Rightarrow (a+b)\sin^{-1}x = c + \frac{b\pi}{2}$

$\Rightarrow \sin^{-1}x = \frac{2c + b\pi}{2(a+b)}$

$a\sin^{-1}x + b\cos^{-1}x = a\sin^{-1}x + b\left(\frac{\pi}{2} - \sin^{-1}x\right)$

$= (a-b)\sin^{-1}x + b\frac{\pi}{2}$

$= \frac{(a-b)(2c+b\pi)}{2(a+b)} + \frac{b\pi}{2}$

$= \frac{2c(a-b) + b\pi(a-b+a+b)}{2(a+b)}$

$= \frac{c(a-b) + ab\pi}{(a+b)}$

11. (B)

$$\cos^{-1}\left(\frac{1+x^2}{2x}\right)=\frac{\pi}{2}+\left(\sin^{-1}x+\cos^{-1}x\right)$$

$$\Rightarrow \quad \cos^{-1}\left(\frac{1+x^2}{2x}\right)=\pi$$

$$\Rightarrow \quad \left(\frac{1+x^2}{2x}\right)=\cos\pi=-1$$

$$\Rightarrow \quad x^2+1+2x=0$$

$$\Rightarrow \quad x=-1$$

12. (C)

We have $\cos^{-1}x+\cos^{-1}(2x)=-\pi$, which is not possible as $\cos^{-1}x$ and $\cos^{-1}2x$ never take negative values

13. (B)

The given equation is $ax^2+\sin^{-1}\left((x-1)^2+1\right)+\cos^{-1}\left((x-1)^2+1\right)=0$.

Now, $-1\le(x-1)^2+1\le1 \Rightarrow x=1$

So, we have $a+\frac{\pi}{2}=0 \Rightarrow a=-\frac{\pi}{2}$

14. (C)

Put $\sin^{-1}\frac{5}{x}=A\Rightarrow\frac{5}{x}=\sin A$

$\sin^{-1}\frac{12}{x}=B\Rightarrow\frac{12}{x}=\sin B \qquad \Rightarrow A+B=\frac{\pi}{2}$

$$\Rightarrow \quad \sin A=\sin\left(\frac{\pi}{2}-B\right)=\cos B=\sqrt{1-\sin^2 B}$$

$$\Rightarrow \quad \frac{5}{x}=\sqrt{1-\frac{144}{x^2}} \quad \Rightarrow \frac{169}{x^2}=1$$

$\Rightarrow \quad x^2=169 \qquad \Rightarrow \quad x=13 \qquad$ [$\because$ $x=-13$ does not satisfy the given equation]

15. (D)

$$\sin\left(2\sin^{-1}(0.8)\right)=\sin\left(\sin^{-1}\left(2\times0.8\sqrt{1-(0.8)^2}\right)\right)=\sin\left(\sin^{-1}0.96\right)=0.96$$

16. (B)

Let $x=\sin\theta$ where $-\frac{1}{2}\le x\le1 \Rightarrow -\frac{\pi}{6}\le\theta\le\frac{\pi}{2}$

Then $f(x)=\sin^{-1}\left(\frac{\sqrt{3}}{2}x-\frac{1}{2}\sqrt{1-x^2}\right)$

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta\right)$$

$$= \sin^{-1}\left(\sin\left(\theta - \frac{\pi}{6}\right)\right)$$

$$= \theta - \frac{\pi}{6} = \sin^{-1}x - \frac{\pi}{6} \qquad \left[\because\ \theta - \frac{\pi}{6} \in \left(\frac{-\pi}{3}, \frac{\pi}{3}\right)\right]$$

17. (C)

$\sin^{-1}x = 2\sin^{-1}a$

Now $-\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}$

$\Rightarrow\ -\frac{\pi}{2} \le 2\sin^{-1}a \le \frac{\pi}{2}$

$\Rightarrow\ -\frac{\pi}{4} \le \sin^{-1}a \le \frac{\pi}{4}$

$\Rightarrow\ \frac{1}{\sqrt{2}} \le a \le \frac{1}{\sqrt{2}}$

$\Rightarrow\ |a| \le \frac{1}{\sqrt{2}}$

18. (C)

$$\sin^{-1}\left(\frac{\sqrt{r}-\sqrt{r-1}}{\sqrt{r(r+1)}}\right) = \tan^{-1}\left(\frac{\sqrt{r}-\sqrt{r+1}}{1+\sqrt{r(r-1)}}\right)$$

$$\Rightarrow\ \sum_{r=1}^{n}\sin^{-1}\left(\frac{\sqrt{r}-\sqrt{r-1}}{\sqrt{r(r+1)}}\right) = \sum_{r=1}^{n}\left(\tan^{-1}\sqrt{r} - \tan^{-1}\sqrt{r-1}\right) = \tan^{-1}\sqrt{n}$$

19. (B)

$x = \sin(\theta+\beta)$ and $y = \sin(\theta-\beta)$

$\Rightarrow\ 1 + xy = 1 + \sin(\theta+\beta)\sin(\theta-\beta) = 1 + \sin^2\theta - \sin^2\beta = \sin^2\theta + \cos^2\beta$

20. (B)

$$\tan\left[\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right] + \tan\left[\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right]$$

Let $\frac{1}{2}\cos^{-1}\frac{a}{b} = \theta \Rightarrow \cos 2\theta = \frac{a}{b}$

$$\tan\left[\frac{\pi}{4}+\theta\right] + \tan\left[\frac{\pi}{4}-\theta\right] = \frac{1+\tan\theta}{1-\tan\theta} + \frac{1-\tan\theta}{1+\tan\theta}$$

$$= \frac{1+\tan^2\theta + 2\tan\theta + 1 + \tan^2\theta - 2\tan\theta}{\left(1-\tan^2\theta\right)} = \frac{2}{\cos 2\theta} = \frac{2b}{a}$$

21. (D)

$$\tan^{-1}\left(\frac{\tan x}{4}\right)+\tan^{-1}\left(\frac{3\sin 2x}{5+3\cos 2x}\right)=\tan^{-1}\left(\frac{\tan x}{4}\right)+\tan^{-1}\left(\frac{\frac{6\tan x}{1+\tan^2 x}}{5+\frac{3\left(1-\tan^2 x\right)}{1+\tan^2 x}}\right)$$

$$=\tan^{-1}\left(\frac{\tan x}{4}\right)+\tan^{-1}\left(\frac{6\tan x}{8+2\tan^2 x}\right)$$

$$=\tan^{-1}\left(\frac{\tan x}{4}\right)+\tan^{-1}\left(\frac{3\tan x}{4+\tan^2 x}\right)$$

$$=\tan^{-1}\left(\frac{\frac{\tan x}{4}+\frac{3\tan x}{4+\tan^2 x}}{1-\frac{3\tan^2 x}{4\left(4+\tan^2 x\right)}}\right)$$

$$=\tan^{-1}\left(\frac{16\tan x+\tan^3 x}{16+\tan^2 x}\right)$$

$$=\tan^{-1}\left(\tan x\right)=x$$

1. (0)

Let $y = \sin^{-1}\left(\frac{-1}{2}\right)$

We know that

$\sin^{-1}(-x) = -\sin^{-1}x$

$y = -\sin^{-1}\left(\frac{1}{2}\right)$ Since $\sin\frac{\pi}{6} = \frac{1}{2}$

$y = -\frac{\pi}{6}$ $\frac{\pi}{6} = \sin^{-1}\left(\frac{1}{2}\right)$

$\sin\left(\frac{\pi}{6} - \frac{\pi}{6}\right) = 0$

2. (9)

$9\,cot\left(cot^{-1}\frac{1}{3}\right)$ = 9x 1/3 = 3

3. (18)

$sin\left(cos^{-1}\frac{12}{13}\right) = sin\left(sin^{-1}\frac{5}{13}\right) = \frac{5}{13}$

5 + 13 = 18

4. (0)

Given, $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$

This will happen only when $\sin^{-1}x = \sin^{-1}y = \sin^{-1}z = \frac{\pi}{2}$

Since $\sin^{-1}\text{x} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

then $= \sin\frac{\pi}{2} = 1 \Rightarrow x = y = z = 1$

Hence desired value is $= 1 + 1 + 1 + -\frac{9}{1+1+1} = 3 - \frac{9}{3} = 0$

5. (15)

$\sec^2(\tan^{-1}2) + \csc^2(\cot^{-1}3)$

$= 1 + \tan^2(\tan^{-1}2) + 1 + \cot^2(\cot^{-1}3)$

$= 1 + [\tan(\tan^{-1}2)]^2 + 1 + [\cot(\cot^{-1}3)]^2 = 1 + 2^2 + 1 + 3^2 = 15$

6. (9)

$\sin^{-1}\sin 15 + \cos^{-1}\cos 20 + \tan^{-1}\tan 25 = 30 - 9\pi$

So k = 9

7. (1)

$\cos^{-1}\left(\frac{3}{5}\right) - \sin^{-1}\left(\frac{4}{5}\right) = \cos^{-1}\text{x}$

or, $\frac{\pi}{2} - \sin^{-1}\left(\frac{3}{5}\right) - \sin^{-1}\left(\frac{4}{5}\right) = \cos^{-1}\text{x}$

or, $\frac{\pi}{2} - \sin^{-1}\left(\frac{3}{5}\cdot\frac{3}{5} + \frac{4}{5}\cdot\frac{4}{5}\right) = \cos^{-1}\text{x}$

or, $\frac{\pi}{2} - \sin^{-1}1 = \cos^{-1}\text{x}$

or, $\cos^{-1}\text{x} = 0$

or, $\text{x} = 1$.

8. (0)

$\theta = \tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ac}} + \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$

$a = \tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}}, \beta = \tan^{-1}\sqrt{\frac{b(a+b+c)}{ac}}, Y = \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$

$\tan\alpha = \sqrt{\frac{a(a+b+c)}{bc}}, \tan\beta = \sqrt{\frac{b(a+b+a)}{ac}}, \ \tan\gamma = \sqrt{\frac{c(a+b+c)}{ab}}$

$\tan\alpha + \tan\beta + \tan\gamma = \sqrt{\frac{a(a+b+c)}{bc}} + \sqrt{\frac{b(a+b+c)}{ac}} + \sqrt{\frac{c(a+b+c)}{ab}}$

$= \frac{(a+b-c)^{\frac{1}{2}}}{\sqrt{3bc}}$

$= \tan\alpha\tan\beta\tan\gamma$

$= \tan\theta = \left[\frac{(\tan\alpha+\tan\beta+\tan\gamma)-\tan\alpha\tan\beta\tan\gamma}{1-\tan\alpha\tan\beta-\tan\beta\tan\gamma-\tan\gamma\tan\alpha}\right]$

$= \tan\theta = 0$

9. (2)

$3\sqrt{5}\,tan\left\{\left(cos^{-1}\left(-\frac{2}{7}\right)-\frac{\pi}{2}\right)\right\} = -3\sqrt{5}\,tan\left\{\left(sin^{-1}\left(\frac{2}{7}\right)\right)\right\} = -3\sqrt{5}\,tan\left\{\left(tan^{-1}\left(\frac{2}{3\sqrt{5}}\right)\right)\right\} =$

= 2

10. (7)

$sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

$tan^{-1}(1) = \frac{\pi}{4}$

$cos^{-1}\left(\cos(-\frac{\pi}{2})\right) = \frac{\pi}{2}$

$sin^{-1}\left(-\frac{1}{2}\right) + tan^{-1}(1) + cos^{-1}\left(\cos(-\frac{\pi}{2})\right) = \frac{7\pi}{12}$

k = 7

11. (2)

$sin\left[cot^{-1}\left(cot\frac{17\pi}{3}\right)\right] = \frac{\sqrt{3}}{2}, k = 2$

12. (20)

$sin^{-1}x_i = \frac{\pi}{2}\ so\ x_i = 1$

So $\sum_{i=1}^{20} x_i = 20$

13. (2)

Here, $\tan^{-1}x + \cos^{-1}\frac{y}{\sqrt{1-y^2}} = \sin^{-1}\frac{3}{\sqrt{10}}$

or $\tan^{-1}x + \tan^{-1}\left(\frac{1}{y}\right) = \tan^{-1}(3)$

or $\tan^{-1}\left(\frac{1}{y}\right) = \tan^{-1}3 - \tan^{-1}(x)$

or $\tan^{-1}\left(\frac{1}{y}\right) = \tan^{-1}\left(\frac{3-x}{1+3x}\right)$

or $y=\frac{1+3x}{3-x}$

14. (0)

We have, $\sin^{-1}\left\{\cot\left(\sin^{-1}\sqrt{\frac{2-\sqrt{3}}{4}}+\cos^{-1}\frac{\sqrt{12}}{4}+\sec^{-1}\sqrt{2}\right)\right\}$

$=\sin^{-1}\left\{\cot\left(\sin^{-1}\frac{\sqrt{3}-1}{2\sqrt{2}}+\cos^{-1}\frac{\sqrt{3}}{2}+\sec^{-1}\sqrt{2}\right)\right\}$

$=\sin^{-1}\left\{\cot\left(\frac{\pi}{12}+\frac{\pi}{6}+\frac{\pi}{4}\right)\right\}$

$=\sin^{-1}\left(\cot\frac{\pi}{2}\right)$

$=\sin^{-1}0=0$

15. (2)

We know that $\cos^{-1}x\in[0,\pi]$

$\therefore\cos^{-1}(a)+\cos^{-1}(b)+\cos^{-1}(c)=3\pi$ is possible iff $a=b=c=-1$

Now, $f(1)=2$ and $f(x+y)=f(x)\cdot f(y)$

Put $x=y=1$, we get

$f(2)=f(1)\cdot f(1)=4$

Put $x=2,y=1$, we get

$f(3)=f(2)\cdot f(1)=4\times 2=8$

$\therefore a^{2f(1)}+b^{2f(2)}+c^{2f(3)}+\frac{a+b+c}{a^{2f(1)}+b^{2f(2)}+c^{2f(3)}}$

$=1+1+1-\frac{3}{1+1+1}$

$=2$

1. (A)

Since, x, y, z are in A.P.

Also, we have

$$2\tan^{-1} y = \tan^{-1} x + \tan^{-1}(z)$$

$$\Rightarrow \tan^{-1}\left(\frac{2y}{1-y^2}\right) = \tan^{-1}\left(\frac{x+z}{1-xz}\right)$$

$$\Rightarrow \frac{x+z}{1-y^2} = \frac{x+z}{1-xz} \quad (\because 2y = x+z)$$

$$\Rightarrow y^2 = xz \text{ or } x+z=0$$

$$\Rightarrow x = y = z = 0$$

2. (C)

Consider, $\tan^{-1}\left[\cot\frac{43\pi}{4}\right] = \tan^{-1}\left[\cot\left(10\pi + \frac{3\pi}{4}\right)\right]$

$$= \tan^{-1}\left[\cot\frac{3\pi}{4}\right] \quad [\because \cot(2n\pi + \theta) = \cot\theta]$$

$$= \tan^{-1}\left[\tan\left(\frac{\pi}{2} - \frac{3\pi}{4}\right)\right] = \frac{\pi}{2} - \frac{3\pi}{4}$$

$$= \frac{2\pi - 3\pi}{4} = \frac{-\pi}{4}$$

3. (C)

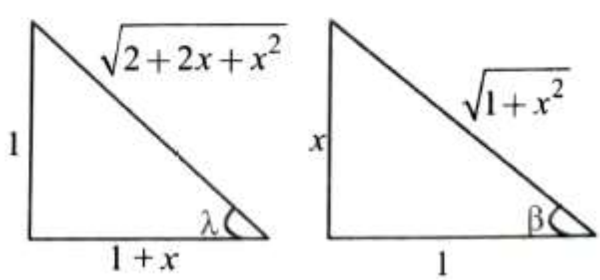
Given that, $\tan^{-1} y = \tan^{-1} x + \tan^{-1}\left[\frac{2x}{1-x^2}\right]$

$$= \tan^{-1} x + 2\tan^{-1} x = 3\tan^{-1} x$$

$$\tan^{-1} y = \tan^{-1}\left[\frac{3x - x^3}{1-3x^2}\right] \Rightarrow y = \frac{3x - x^3}{1-3x^2}$$

4. (A)

$$\sin\left[\cot^{-1}(1+x)\right] = \cos\left(\tan^{-1} x\right)$$



Let $\cot\lambda = 1 + x, \tan\beta = x$

$$\Rightarrow \sin\lambda = \cos\beta$$

$$\Rightarrow \frac{1}{\sqrt{x^2+2x+2}} = \frac{1}{\sqrt{1+x^2}}$$

$$\Rightarrow x^2+2x+2 = x^2+1 \Rightarrow x = -\frac{1}{2}$$

5. (A)

Let $x^2 = \cos 2\theta \Rightarrow \theta = \frac{1}{2}\cos^{-1} x^2$

$$\Rightarrow \tan^{-1}\left[\frac{\sqrt{1+x^2}}{\sqrt{1+x^2}} - \frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}\right] = \left[\frac{\sqrt{1+\cos 2\theta}}{\sqrt{1+\cos 2\theta}} - \frac{\sqrt{1-\cos 2\theta}}{\sqrt{1-\cos 2\theta}}\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{1+\tan\theta}{1-\tan\theta}\right] = \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\theta\right)\right]$$

$$= \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2$$

6. (D)

$\because \cos\alpha = \frac{3}{5}$, then $\sin\alpha = \frac{4}{5}$

$\Rightarrow \tan\alpha = \frac{4}{3}$ and $\tan\beta = \frac{1}{3}$

$$\because \tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\cdot\tan\beta}$$

$$= \frac{\frac{4}{3}-\frac{1}{3}}{1+\frac{4}{9}} = \frac{1}{\frac{13}{9}} = \frac{9}{13}$$

$$\therefore \alpha-\beta = \tan^{-1}\left(\frac{9}{13}\right) = \sin^{-1}\left(\frac{9}{5\sqrt{10}}\right) = \cos^{-1}\left(\frac{13}{5\sqrt{10}}\right)$$

7. (D)

$x = \sin^{-1}(\sin 10)$

$\Rightarrow x = 3\pi - 10$ $\quad \begin{cases} 3\pi - \frac{\pi}{2} < 10 < 3\pi + \frac{\pi}{2} \\ \Rightarrow 3\pi - x = 10 \end{cases}$

and $y = \cos^{-1}(\cos 10)$ $\quad \begin{cases} 3\pi < 10 < 4\pi \\ \Rightarrow 4\pi - x = 10 \end{cases}$

$\Rightarrow y = 4\pi - 10$

$\therefore y - x = (4\pi-10)-(3\pi-10) = \pi$

8. (C)

$\therefore\ f(x) = \sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$

$\therefore\ -1 \le \frac{|x|+5}{x^2+1} \le 1$

$\Rightarrow |x|+5 \le x^2+1 \qquad \left[\because\ x^2+1 \ne 0\right]$

$\Rightarrow x^2 - |x| - 4 \ge 0$

$\Rightarrow \left(|x| - \frac{1-\sqrt{17}}{2}\right)\left(|x| - \frac{1+\sqrt{17}}{2}\right) \ge 0$

$\Rightarrow x \in \left(-\infty, -\frac{1+\sqrt{17}}{2}\right] \cup \left[\frac{1+\sqrt{17}}{2}, \infty\right)$

$\therefore\ a = \frac{1+\sqrt{17}}{2}$

9. (C)

$-1 \le \frac{x-1}{x+1} \le 1 \Rightarrow \frac{x-1}{x+1} \le 1$

$\Rightarrow \frac{2}{x+1} \ge 0 \Rightarrow x \in (-1, \infty)$ ….. (i)

and $\frac{x-1}{x+1} \ge -1 \Rightarrow \frac{2x}{x+1} \ge 0 \Rightarrow x \in (-\infty, -1) \cup [0, \infty)$ ….. (ii)

from (i) and (ii), $x \in [0, \infty)$ ….. (iii)

Now, $-1 \le \frac{3x^2+x-1}{(x-1)^2} \le 1 \Rightarrow \frac{3x^2+x-1}{(x-1)^2} \le 1$

$\Rightarrow \frac{2x^2+3x-2}{(x-1)^2} \le 0$

$\Rightarrow \frac{(2x-1)(x+2)}{(x-1)^2} \le 0 \Rightarrow x \in \left[-2, \frac{1}{2}\right]$ ….. (iv)

and $\frac{3x^2+x-1}{(x-1)^2} \ge -1 \Rightarrow \frac{x(4x-1)}{(x-1)^2} \ge 0$

$\Rightarrow x \in (-\infty, 0] \cup \left[\frac{1}{4}, \infty\right)$ ….. (v)

From (iv) and (v); $x \in [-2, 0] \cup \left[\frac{1}{4}, \frac{1}{2}\right]$ ….. (vi)

From (iii) and (vi); $x \in \{0\} \cup \left[\frac{1}{4}, \frac{1}{2}\right]$

10. (A)

Let $\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}=\theta \Rightarrow \sin 4\theta=\frac{\sqrt{63}}{8}$

or $\cos 4\theta=\frac{1}{8}$ $\left[\because \cos\theta=\sqrt{1-\sin^2\theta}\right]$

$\Rightarrow 2\cos^2 2\theta-1=\frac{1}{8}$ $\left[\because \cos 2\theta=2\cos^2\theta-1\right]$

$\Rightarrow \cos^2 2\theta=\frac{9}{16} \Rightarrow \cos 2\theta=\frac{3}{4}$

$\Rightarrow 2\cos^2\theta-1=\frac{3}{4} \Rightarrow \cos^2\theta=\frac{7}{8} \Rightarrow \cos\theta=\frac{\sqrt{7}}{2\sqrt{2}}$

$\therefore \tan\theta=\frac{1}{\sqrt{7}}$ $[\because \sin\theta=\sqrt{\sec^2\theta}-1]$

11. (A)

$0\le x^2-x+1\le 1$

$\Rightarrow x^2-x\le 0 \Rightarrow x\in[0,1]$

Also, $0<\sin^{-1}\left(\frac{2x-1}{2}\right)\le\frac{\pi}{2}$

$\Rightarrow 0<\frac{2x-1}{2}\le 1 \Rightarrow 0<2x-1\le 2$

$1<2x\le 3 \Rightarrow \frac{1}{2}<x\le\frac{3}{2}$

From (i) and (ii), we get

$x\in\left(\frac{1}{2},1\right] \Rightarrow \alpha=\frac{1}{2}, \beta=1$

So, $\alpha+\beta=\frac{3}{2}$

12. (C)

$f(x)=\sin^{-1}\left(\frac{x^2-3x+2}{x^2+2x+7}\right)$

Now, take domain of $\sin^{-1}$

$\frac{x^2-3x+2}{x^2+2x+7}\ge -1$ and $\frac{x^2-3x+2}{x^2+2x+7}\le 1$

$2x^2-x+9\ge 0$ and $5x\ge -5 \Rightarrow x\ge -1$

$x\in R$

Hence, Domain $x\in[-1,\infty)$.

13. (D)

$$-1 \le \frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi} \le 1$$

$$-\frac{\pi}{2} \le \sin^{-1}\frac{1}{4x^2-1} \le \frac{\pi}{2}$$

On solving inequalities we get

Always $-1 \le \frac{1}{4x^2-1} \le 1$

$$x \in \left(-\infty, \frac{-1}{\sqrt{2}}\right) \cup \left[\frac{1}{\sqrt{2}}, \infty\right) \cup \{0\}$$

14. (B)

$$\left|\frac{x^2-4x+2}{x^2+3}\right| \le 1$$

$$\Rightarrow \left(x^2-4x+2\right)^2 \le \left(x^2+x^3\right)^2$$

$$\Rightarrow \left(x^2-4x+2\right)^2 - \left(x^2+3\right)^2 \le 0$$

$$\Rightarrow \left(2x^2-4x+5\right)(-4x-1) \le 0$$

$$-4x-1 \le 0 \;\Rightarrow\; x \ge \frac{-1}{4}$$

15. (A)

We are given that

$$\cos^{-1}x - 2\sin^{-1}x = \cos^{-1}2x \qquad \text{...A}$$

$$\Rightarrow \cos^{-1}x - 2\left(\frac{\pi}{2} - \cos^{-1}x\right) = \cos^{-1}2x$$

$$\Rightarrow \cos^{-1}x - \pi + 2\cos^{-1}x = \cos^{-1}2x$$

$$\Rightarrow 3\cos^{-1}x = \pi + \cos^{-1}2x \qquad \text{... (i)}$$

$$\Rightarrow \cos\left(3\cos^{-1}x\right) = \cos\left(\pi + \cos^{-1}2x\right) \qquad \left[\because\ 3\cos^{-1}x = \cos^{-1}\left(4x^3-3x\right)\right]$$

$$\Rightarrow 4x^3 - 3x = -2x$$

$$\Rightarrow 4x^3 = x \;\Rightarrow\; x = 0, \pm\frac{1}{2}$$

Here all values of x satisfy the eqn. (A)

$\therefore$ Sum of all the solutions of the eqn. $= -\frac{1}{2} + \frac{1}{2} + 0 = 0$

16. (C)

Given functions is

$$f(x) = \sin^{-1}\left[2x^3-3\right] + \log_2\left(\log_{\frac{1}{2}}\left(x^2-5x+5\right)\right)$$

Take angle of $\sin^{-1}$ as T_1. Which lies between –1 and 1

$T_1 : -1 \le \left[2x^2 - 3\right] < 1$

$\Rightarrow\ -1 \le 2x^2 - 3 < 2 \ \Rightarrow\ 2 < 2x^2 < 5$

$\Rightarrow\ 1 < x^2 < \frac{5}{2} \ \Rightarrow\ T_1 : x \in \left(-\frac{5}{2}, -1\right) \cup \left(1, \frac{5}{2}\right)$

Similarly,

$T_2 : x^2 - 5x + 5 > 0$

$\Rightarrow\ \left(x - \left(\frac{5-\sqrt{5}}{2}\right)\right)\left(x - \left(\frac{5+\sqrt{5}}{2}\right)\right) > 0$

$T_3 : \log_{\frac{1}{2}}\left(x^2 - 5x + 5\right) > 0$

$\Rightarrow\ x^2 - 5x - 5 < 1$

$\Rightarrow\ x^2 - 5x + 4 < 0$

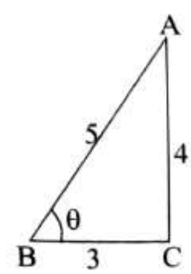
$\Rightarrow\ T_3 : x \in (1, 4)$

Now, take intersection of T_1, T_2, &T_3,

$T_1 \cap T_2 \cap T_3 = \left(1, \frac{5-\sqrt{5}}{2}\right)$

17. (C)

Let $\tan^{-1}\frac{4}{3} = \theta \ \Rightarrow\ \tan\theta = \frac{4}{3}$



$\Rightarrow\ \cos^{-1}\left(\frac{3}{10}\cos\theta + \frac{2}{5}\sin\theta\right)$

$= \cos^{-1}\left(\frac{3}{10} \times \frac{3}{5} + \frac{2}{5} \cdot \frac{4}{5}\right)$

$= \cos^{-1}\left(\frac{9}{50} + \frac{8}{25}\right) = \cos^{-1}\left(\frac{25}{50}\right) = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

18. (D)

Given function domain is [–1, 1].

$-1 \le \frac{x^2 - 5x + 6}{x^2 - 9} \le 1$

Take maximum value and subtract l both sides.

$\frac{x^2 - 5x + 6}{x^2 - 9} - 1 \le 0,\ \frac{1}{x+3} \ge 0$

$x \in (-3, \infty)$ ….. (i)

Take minimum value and add 1 both sids.

$\frac{x^2 - 5x + 6}{x^2 - 9} + 1 \ge 0,\ \frac{2x+1}{x+3} \ge 0$

$x \in (-\infty, -3) \cup \left[-\frac{1}{2}, \infty\right)$ ….. (ii)

Now, take the intersection of two equations (i) and (ii)

$x \in \left[-\frac{1}{2}, \infty\right)$

Now, take $x^2 - 3x + 2 > 0,\ x \in (-\infty, 1) \cup (2, \infty)$

$x^2 - 3x + 2 \neq 1,\ x \neq \frac{3 \pm \sqrt{5}}{2}$

Take intersection of all the solutions.

$\left[-\frac{1}{2}, 1\right) \cup (2, \infty) - \left\{\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}\right\}$

19. (B)

Given line is $\frac{\sin^{-1} x}{\alpha} = \frac{\cos^{-1} x}{\beta} = k$

$\sin^{-1} x = k\alpha \Rightarrow \cos^{-1} x = k\beta$

$k = \frac{\pi}{2(\alpha+\beta)}$

$\sin\left(\frac{2\pi\alpha}{\alpha+\beta}\right) = \sin\left(4\sin^{-1} x\right)$

$2\sin\left(2\sin^{-1} x\right)\cos\left(2\sin^{-1} x\right)$

$= 4x\sqrt{1-x^2}\left(1-2x^2\right)$

20. (A)

We have $f(x) = \sin^{-1} 2x + \sin 2x + \cos^{-1} 2x + \cos 2x$

$= \sin(2x) + \cos(2x) + \frac{\pi}{2}$

Now, $f(0) = 1 + \frac{\pi}{2}(m)$ and $f\left(\frac{\pi}{8}\right) = \sqrt{2} + \frac{\pi}{2}(M)$

Now, $m + M = 1 + \sqrt{2} + \pi$

21. (A)

Given equation is

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right) + \tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \pi - \frac{2\pi}{3} = \frac{\pi}{3}$

$\cos^{-1}\left(\cos\frac{7\pi}{6}\right) = 2\pi - \frac{7\pi}{3} = \frac{5\pi}{6}$

$tab^{-1}\left(\tan\frac{3\pi}{4}\right) = \tan^{-1}(-1) = \tan^{-1}\left(\tan\left(-\frac{\pi}{4}\right)\right) = -\frac{\pi}{4}$

$\Rightarrow \sin^{-1}\left(\sin\frac{2\pi}{3}\right) + \cos^{-1}\left(\cos\frac{7\pi}{6}\right) + \tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

$$=\frac{\pi}{3}+\frac{5\pi}{6}-\frac{\pi}{4}=\frac{4\pi+10\pi-3\pi}{12}=\frac{11\pi}{12}$$

22. (A)

Take function $\tan^{-1}\frac{1}{1+n+n^2}=\tan^{-1}\left(\frac{(n+1)-n}{1+n(n+1)}\right)$

$=\tan^{-1}(n+1)-\tan^{-1}n$

So, $\sum_{n=1}^{50}\left(\tan^{-1}(n+1)-\tan^{-1}n\right)$

$=\tan^{-1}51-\tan^{-1}1$

Take cot both sides,

$$\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)=\cot\left(\tan^{-1}51+\tan^{-1}1\right)$$

$$=\frac{1}{\tan\left(\tan^{-1}51-\tan^{-1}1\right)}=\frac{1+51\times1}{51-1}=\frac{52}{50}=\frac{26}{25}$$

23. (D)

$$\lim_{x\to\frac{1}{\sqrt{2}}}\frac{\sin\left(\cos^{-1}x\right)-x}{1-\tan\left(\cos^{-1}x\right)}$$

Let $x=\cos y$

$\Rightarrow\ \cos^{-1}(x)=\sin^{-1}\sqrt{1-x^2}$

$$\lim_{x\to\frac{1}{\sqrt{2}}}\frac{\sin\left(\sin^{-1}\sqrt{1-x^2}\right)-x}{1-\tan\left(\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)\right)}$$

$$\lim_{x\to\frac{1}{\sqrt{2}}}\frac{\sqrt{1-x^2}-x}{1-\left(\frac{\sqrt{1-x^2}}{x}\right)}$$

$$\lim_{x\to\frac{1}{\sqrt{2}}}(-x)=-\frac{1}{\sqrt{2}}$$

24. (B)

Given equation is $\tan^{-1}\left[\dfrac{\cos\left(\dfrac{15\pi}{4}\right)-1}{\sin\dfrac{\pi}{4}}\right]$

$\tan^{-1}\left[\dfrac{\cos\left(4\pi-\dfrac{\pi}{4}\right)-1}{\sin\dfrac{\pi}{4}}\right] \quad \Rightarrow \quad \tan^{-1}\left(\dfrac{\cos\dfrac{\pi}{4}-1}{\sin\dfrac{\pi}{4}}\right)$

$\tan^{-1}\left(\dfrac{1-\sqrt{2}}{1}\right)=-\dfrac{\pi}{8}$

25. (130)

Given curves are

$x=\sin\left(2\tan^{-1}\alpha\right)=\dfrac{2\alpha}{1+\alpha^2}$ and $y=\sin\left(\dfrac{1}{2}\tan^{-1}\dfrac{4}{3}\right)=\sin\left(\sin^{-1}\dfrac{1}{\sqrt{5}}\right)=\dfrac{1}{\sqrt{5}}$

We have set with relation $y^2=1-x$

$\dfrac{1}{5}=1-\dfrac{2\alpha}{1+\alpha^2}...$ {from value of x and y)

$\Rightarrow 1+\alpha^2=5+5\alpha^2-10\alpha \Rightarrow 2\alpha^2-5\alpha+2=0$

So, $\alpha=2\cdot\dfrac{1}{2}$

Take, $\sum_{a\in S}16\alpha^3=16\times 2^3+16\times\dfrac{1}{2^3}=130$

26. (29)

$50\tan\left(3\tan^{-1}\left(\dfrac{1}{2}\right)+2\cos^{-1}\left(\dfrac{1}{\sqrt{5}}\right)\right)+4\sqrt{2}\tan\left(\dfrac{1}{2}\tan^{-1}\left(2\sqrt{2}\right)\right)$

$=50\tan\left(\tan^{-1}\left(\dfrac{1}{2}\right)+2\left(\tan^{-1}\left(\dfrac{1}{2}\right)+\tan^{-1}(2)\right)\right)+4\sqrt{2}\tan\left(\dfrac{1}{2}\tan^{-1}\left(2\sqrt{2}\right)\right)$

$=50\tan\left(\tan^{-1}\left(\dfrac{1}{2}\right)+2\cdot\dfrac{\pi}{2}\right)+4\sqrt{2}\times\dfrac{1}{\sqrt{2}}$

$=50\left(\tan\left(\tan^{-1}\left(\dfrac{1}{2}\right)\right)\right)+4=25+4=29$

EXERCISE - 2 [A]

1. (B)

$\tan^{-1}A + \tan^{-1}B + \tan^{-1}C = \tan^{-1}\left(\frac{A+B+C-ABC}{1-AB-BC-AC}\right)$

$\tan^{-1}\left(\frac{xy}{zr}\right) + \tan^{-1}\left(\frac{yz}{xr}\right) + \tan^{-1}\left(\frac{xz}{yr}\right)$

$= \tan^{-1}\left(\frac{-\frac{xyz}{r^3}+\frac{1}{rxyz}(x^2y^2+y^2z^2+x^2z^2)}{1-\frac{1}{r^2}(y^2+z^2+x^2)}\right)$

As $r^2 = x^2 + y^2 + z^2$

As doniminator $\to 0$

$\tan^{-1}(\infty) = \frac{\pi}{2}$

2. (B)

$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z$

$= \tan^{-1}\left(\frac{x+y}{1-xy}\right) + \tan^{-1}z$

$= \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)$

$= \tan^{-1}\infty$[Since $xy + yz + zx = 1$]

$= \frac{\pi}{2}$

3. (A)

$\cos^{-1}\left(\cos\frac{2\pi}{3}\right) = \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

$\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \frac{2\pi}{3} + \frac{\pi}{3} = \pi$

4. (C)

$3\cos^{-1}\left(x^2 - 7x + \frac{25}{2}\right) = \pi$

$\cos^{-1}\left(x^2 - 7x + \frac{25}{2}\right) = \frac{\pi}{3}$

$x^2 - 7x + \frac{25}{2} = \cos\frac{\pi}{3}$

$\Rightarrow x^2 - 7x + \frac{25}{2} = \frac{1}{2}$

$\Rightarrow x^2 - 7x + \frac{24}{2} = 0 \Rightarrow x^2 - 7x + 12 = 0$

$\Rightarrow (x-3)(x-4) = 0$

$\Rightarrow\ x = 3$ or 4

5. (C)

Given,

$\tan(x+y) = 33$

$x = \tan^{-1}3$

$\Rightarrow \tan x = 3$

Formula,

$\tan(x+y) = \frac{\tan x+\tan y}{1-\tan x\tan y}$

$33 = \frac{3+\tan y}{1-(3)\tan y}$

$33(1-3\tan y) = 3+\tan y$

$33-99\tan y = 3+\tan y$

$30 = 100\tan y$

$\tan y = \frac{30}{100} = 0.3$

$\therefore y = \tan^{-1}0.3$

6. (C)

Let $\frac{1}{2}\cos^{-1}(\mathrm{x}) = \mathrm{A}$

Therefore simplifying the above expression, we get

$\frac{1-\tan A}{1+\tan A} + \frac{1+\tan A}{1-\tan A}$

$= \frac{2(1+\tan^2 A)}{1-\tan^2 A}$

$= \frac{2}{\cos 2A}$

$= \frac{2}{\cos(\cos^{-1}(x))}$

$= \frac{2}{x}$

7. (D)

$3\,tan^{-1}\left(\frac{1}{2}\right) + 2\,tan^{-1}\left(\frac{1}{5}\right) = \; tan^{-1}\left(\frac{11}{2}\right) + tan^{-1}\left(\frac{5}{12}\right) = \; \pi + \tan^{-1}\left(-\frac{142}{31}\right)$

8. (B)

$sin^2\left(cos^{-1}\frac{1}{2}\right) + cos^2\left(sin^{-1}\frac{1}{3}\right) = 1 - \; cos^2\left(cos^{-1}\frac{1}{2}\right) + 1 - sin^2\left(sin^{-1}\frac{1}{3}\right)$

$1-\frac{1}{4}+1-\frac{1}{9} = \frac{59}{36}$

9. (B)

We have $\tan(\cos^{-1}\mathrm{x}) = \sin\left(\cot^{-1}\frac{1}{2}\right)$

$\Rightarrow \tan\left[\tan^{-1}\left(\frac{\sqrt{1-\mathrm{x}^2}}{\mathrm{x}}\right)\right] = \sin(\tan^{-1}2)$

$\Rightarrow \tan\left[\tan^{-1}\left(\frac{\sqrt{1-\mathrm{x}^2}}{\mathrm{x}}\right)\right] = \sin\left(\sin^{-1}\frac{2}{\sqrt{1+4}}\right)$

$\Rightarrow \tan\left[\tan^{-1}\left(\frac{\sqrt{1-\mathrm{x}^2}}{\mathrm{x}}\right)\right] = \sin\left(\sin^{-1}\frac{2}{\sqrt{5}}\right)$

$\Rightarrow \left(\frac{\sqrt{1-\mathrm{x}^2}}{\mathrm{x}}\right) = \frac{2}{\sqrt{5}} \Rightarrow 5(1-\mathrm{x}^2) = 4\mathrm{x}^2$

$\Rightarrow 9\mathrm{x}^2 = 5 \Rightarrow \mathrm{x} = \pm\frac{\sqrt{5}}{3}$

10. (C)

Let $\cot^{-1}x = y \Rightarrow x = \cot y \Rightarrow \sin y = \frac{1}{\sqrt{1+x^2}}$

Let $\tan^{-1}\frac{1}{\sqrt{x^2+1}} = z \Rightarrow \tan z = \frac{1}{\sqrt{1+x^2}}$

So $\cos z = \frac{1}{\sqrt{1+\left(\frac{1}{\sqrt{x^2+1}}\right)^2}} = \frac{1}{\sqrt{1+\frac{1}{x^2+1}}} = \frac{1}{\sqrt{\frac{x^2+2}{x^2+1}}} = \sqrt{\frac{x^2+1}{x^2+2}}$

11. (D)

We have given $\cos^{-1}x > \sin^{-1}x$, and we know that,

$\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x$

But $\frac{\pi}{2} - \sin^{-1}x > \sin^{-1}x$

$\Rightarrow \frac{\pi}{2} > 2\sin^{-1}x$

$\Rightarrow \frac{\pi}{4} > \sin^{-1}x$

Also $-\frac{\pi}{2} \leq \sin^{-1}x \leq \frac{\pi}{2}$

Step 2. From equation (1) & (2), we get

$-\frac{\pi}{2} \leq \sin^{-1}x < \frac{\pi}{4}$

12. (A)

$$\sin\left(-\frac{\pi}{2}\right) \leq \sin(\sin^{-1}x) < \sin\left(\frac{\pi}{4}\right)$$

$$\therefore -1 \leq x < \frac{1}{\sqrt{2}}$$

$$(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$$

$$(\tan^{-1}x)^2 + \left(\frac{\pi}{2} - \tan^{-1}x\right)^2 = \frac{5\pi^2}{8}$$

$$2(\tan^{-1}x)^2 - \pi\tan^{-1}x + \left(\frac{\pi^2}{4}\right) - \left(\frac{5\pi^2}{8}\right) = 0$$

$$2(\tan^{-1}x)^2 - \pi\tan^{-1}x - \left(\frac{3\pi^2}{8}\right) = 0$$

$$\tan^{-1}x = \frac{\left(\pi \pm \sqrt{\pi^2+3\pi^2}\right)}{4}$$

$$\tan^{-1}x = \frac{3\pi}{4}, \frac{-\pi}{4}$$

$$x = -1$$

13. (B)

$$\sum_{r=1}^{n} \tan^{-1}\left(\frac{2^{r-1}}{1+2^{2r-1}}\right) = \sum_{r=1}^{n} \tan^{-1}\left(\frac{2^{r}-2^{r-1}}{1+2^{r}\cdot 2^{r-1}}\right)$$

$$\sum_{r=1}^{n} (\tan^{-1}2^{r} - \tan^{-1}2^{r-1}) = (\tan^{2} - \tan^{1}) + (\tan^{-1}3 - \tan^{-1}2) + \cdots\cdots (\tan^{-1}\varepsilon$$

$$= \tan^{-1}2^{n} - \tan^{-1}1 = \tan^{-1}2^{n} - \frac{\pi}{4}$$

14. (C)

$$tan^{-1}\frac{1}{a-1} = tan^{-1}\frac{1}{x} + tan^{-1}\frac{1}{a^2-x+1} = tan^{-1}\left(\frac{\frac{1}{x}+\frac{1}{a^2-x+1}}{1-\left(\frac{1}{x}\right)\left(\frac{1}{a^2-x+1}\right)}\right)$$

On solving we will get the value of x.

15. (C)

$$tan^{-1}n + cot^{-1}(n+1) = tan^{-1}n + tan^{-1}\frac{1}{(n+1)} = tan^{-1}\left(\frac{n+\frac{1}{n+1}}{1-\frac{n}{n+1}}\right) = \tan^{-1}\left(n^2+n+1\right)$$

16. (A)

Assume the value of $\text{cosec}^{-1}x = A$

Then, the value of $\text{cosec}A = x$ and hence $\sin A = \frac{1}{x}$.

Using the trigonometric identity to get the cosine value.

$$\Rightarrow \quad \cos A = \sqrt{1-\frac{1}{x^2}}$$

$$\Rightarrow \quad \cos A = \frac{\sqrt{x^2-1}}{x}$$

So, using the obtained value to find the secant value.

$$\Rightarrow \quad \sec A = \frac{x}{\sqrt{x^2-1}}$$

$$\Rightarrow \sec(\text{cosec}^{-1}x) = \frac{x}{\sqrt{x^2-1}} \qquad \left[\because \cos\mathbf{x} = \frac{1}{\text{secx}}\right]$$

$[\because \text{cosec}^{-1}x = A,$ assumed $]$

Again, assume $\sec^{-1}x = B$

Then, the value of $\sec B = x$

So, $\cos B = \frac{1}{x}$

Determining the sine value for the assumption.

Use the trigonometric identity to get the sine value.

$\sin^2 B + \cos^2 B \quad = 1$

$\Rightarrow \sin B = \sqrt{1 - \cos^2 B}$

$= \frac{\sqrt{x^2 - 1}}{x}$

$\operatorname{cosec} B = \frac{x}{\sqrt{x^2 - 1}}$

$\Rightarrow \operatorname{cosec}(\sec^{-1} x) = \frac{x}{\sqrt{x^2 - 1}}$

So, $\sec(\operatorname{cosec}^{-1} x) = \operatorname{cosec}(\sec^{-1} x)$

17. (A)

$cot^{-1} 3 + sec^{-1} \frac{\sqrt{5}}{2} = \ tan^{-1} \frac{1}{3} + tan^{-1} \frac{1}{2} = \ tan^{-1} 1 = \frac{\pi}{4}$

18. (A)

Given, $\theta = \sin^{-1}\{\sin(-600°)\}$

$= \sin^{-1}\{-\sin(360° + 240°)\}$

$= \sin^{-1}\{-\sin(240°)\}$

$= \sin^{-1}\{\sin(180° + 60°)\}$

$= \sin^{-1}\{\sin(60°)\}$

$= \frac{\pi}{3}\left[\because \sin 60° = \frac{\sqrt{3}}{2}\right]$

19. (D)

Given, $\sin\left(2\cos^{-1}\left(\frac{-3}{5}\right)\right) = \sin\left(\cos^{-1}\left(2\left(\frac{-3}{5}\right)^2 - 1\right)\right)$

$= \sin\left(\cos^{-1}\left(\frac{-7}{25}\right)\right)$

$= \sin\left(\sin^{-1}\left(\frac{24}{25}\right)\right)$

$= \frac{24}{25}$

20. (B)

From graph we will get x = $\left(0, \frac{\pi}{2}\right]$

EXERCISE - 2 [B]

One or More than One Option(s) Correct

1. (B, C)

$6x^2+11x+3=0$

$\Rightarrow\ (2x+3)(3x+1)=0$

$\Rightarrow\ x=-\frac{3}{2},-\frac{1}{3}$

For $x=-\frac{3}{2}$, $\cos^{-1}x$ is not defined as domain of $\cos^{-1}x$ is $[-1,1]$ and for $x=-\frac{1}{3}$, $\text{cosec}^{-1}x$ is not defined as domain of $\text{cosec}^{-1}x$ is $R-(-1,1)$.

2. (A, B, C)

Let $\tan^{-1}(-2)=\theta \Rightarrow \tan\theta=-2 \Rightarrow \theta=\left(-\frac{\pi}{2},0\right)$

$\Rightarrow\ 2\theta=(-\pi,0)$

$\cos(-2\theta)=\cos 2\theta=\frac{1-\tan^2\theta}{1+\tan^2\theta}=-\frac{3}{5}$

$\Rightarrow\ -2\theta=\cos^{-1}\left(\frac{-3}{5}\right)=\pi-\cos^{-1}\frac{3}{5}$

$\Rightarrow\ -2\theta=-\pi+\cos^{-1}\frac{3}{5}=-\pi+\tan^{-1}\frac{4}{3}=-\pi+\cot^{-1}\frac{3}{4}=-\pi+\frac{\pi}{2}-\tan^{-1}\frac{3}{4}=-\frac{\pi}{2}-\tan^{-1}\frac{3}{4}$

$=-\frac{\pi}{2}+\tan^{-1}\left(-\frac{3}{4}\right)$

3. (A, B, D)

$\tan^{-1}(x-1)+\tan^{-1}(x)+\tan^{-1}(x+1)=\tan^{-1}3x$

$\Rightarrow\ \tan^{-1}(x-1)+\tan^{-1}(x)=\tan^{-1}3x-\tan^{-1}(x+1)$

$\Rightarrow\ \tan^{-1}\left[\frac{(x-1)+x}{1-(x-1)(x)}\right]=\tan^{-1}\left[\frac{3x-(x+1)}{1+3x(x+1)}\right]$

$\Rightarrow\ \frac{2x-1}{1-x^2+x}=\frac{2x-1}{1+3x^2+3x}$

$\Rightarrow\ (1-x^2+x)(2x-1)=(1+3x^2+3x)(2x-1)$

$\Rightarrow\ x=0,\pm\frac{1}{2}$

4. (B)

We know that $\sin^{-1}x$ is define for $x\in[-1,1]$ and $\sec^{-1}x$ is defined for $x\in(-\infty,-1]\cup[1,\infty)$.
Hence, the given function is defined for $x\in\{-1,1\}$.
Therefore, $f(1)=\frac{\pi}{2},\ f(-1)=\frac{\pi}{2}$.

5. (A, B, C, D)

Since, $\left|\tan^{-1}x\right| = \begin{cases} \tan^{-1}x, \text{ if } x \ge 0 \\ -\tan^{-1}x, \text{ if } x < 0 \end{cases}$

$\Rightarrow \quad \left|\tan^{-1}x\right| = \tan^{-1}|x| \; \forall \; x \in R$

$\Rightarrow \quad \tan\left|\tan^{-1}x\right| = \tan\tan^{-1}|x| = |x|$

Also $\left|\cot^{-1}x\right| = \cot^{-1}x; \forall \; x \in R$

$\Rightarrow \quad \cot\left|\cot^{-1}x\right| = x, \forall \; x \in R$

$\tan^{-1}|\tan x| = \begin{cases} x, & \text{if} \quad \tan x > 0 \\ -x, & \text{if} \quad \tan x < 0 \end{cases}$

$\sin\left|\sin^{-1}x\right| = \begin{cases} x, & x \in [0, 1] \\ -x, & x \in [-1, 0) \end{cases}$

6. (A, C)

Domain of $f(x) = \log_e \cos^{-1}x$ is $x \in [-1, 1)$

$\therefore \quad [\alpha] = -1$ or 0

7. (C, D)

$xy < 0 \Rightarrow x + \frac{1}{x} \ge 2, \; y + \frac{1}{y} \le -2$

or $\quad x + \frac{1}{x} \le -2, \; y + \frac{1}{y} \ge 2$

$x + \frac{1}{x} \ge 2 \Rightarrow \sec^{-1}\left(x + \frac{1}{x}\right) \in \left[\frac{\pi}{3}, \frac{\pi}{2}\right)$

$y + \frac{1}{y} \le -2 \Rightarrow \sec^{-1}\left(y + \frac{1}{y}\right) \in \left(\frac{\pi}{2}, \frac{2\pi}{3}\right]$

$\Rightarrow \quad z \in \left(\frac{5\pi}{6}, \frac{7\pi}{6}\right)$

8. (A, D)

Let $f(x) = \left(\sin^{-1}x\right)^2 + \left(\cos^{-1}x\right)^2$

$$= \left(\sin^{-1}x + \cos^{-1}x\right)^2 - 2\sin^{-1}x\cos^{-1}x$$

$$= \frac{\pi^2}{4} - 2\sin^{-1}x\left[\frac{\pi}{2} - \sin^{-1}x\right]$$

$$= \frac{\pi^2}{4} - \pi\sin^{-1}x + 2\left(\sin^{-1}x\right)^2$$

$$= 2\left[\left(\sin^{-1}x\right)^2 - \frac{\pi}{2}\sin^{-1}x + \frac{\pi^2}{8}\right]$$

$$= 2\left(\sin^{-1}x - \frac{\pi}{4}\right)^2 + 2\left[\frac{\pi^2}{16}\right]$$

Now, $-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$

$\Rightarrow \quad -\frac{3\pi}{4} \le \sin^{-1} x - \frac{\pi}{4} \le \frac{\pi}{4}$

$\Rightarrow \quad 0 \le \left(\sin^{-1} x - \frac{\pi}{4}\right)^2 \le \frac{9\pi^2}{16}$

$\Rightarrow \quad 0 \le 2\left(\sin^{-1} x - \frac{\pi}{4}\right)^2 \le \frac{9\pi^2}{8}$

$\Rightarrow \quad \frac{\pi^2}{8} \le \left(\sin^{-1} x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{8} \le \frac{5\pi^2}{4}$

9. (A, C)

We have, $\cot^{-1}\left(\frac{n^2 - 10n + 21.6}{\pi}\right) < \frac{\pi}{6}$

$\Rightarrow \quad \frac{n^2 - 10n + 21.6}{\pi} < \cot\frac{\pi}{6}$

$\Rightarrow \quad n^2 - 10b + 21.6 < \pi\sqrt{3}$

$\Rightarrow \quad n^2 - 10n + 25 + 21.6 - 25 < \pi\sqrt{3}$

$\Rightarrow \quad (n-5)^2 < \pi\sqrt{3} + 3.4$

$\Rightarrow \quad -\sqrt{\sqrt{3}\pi + 3.4} < n - 5 < \sqrt{\sqrt{3}\pi + 3.4}$

$\Rightarrow \quad 5 - \sqrt{\sqrt{3}\pi + 3.4} < n < 5 + \sqrt{\sqrt{3}\pi + 3.4}$

Since, $\sqrt{3}\,\pi = 5.5$ nearly, $\sqrt{\sqrt{3}\pi + 3.4} - \sqrt{8.9} \sim 2.9$

$\Rightarrow \quad 2.1 < n < 7.9$

$\therefore \quad n = 3, 4, 5, 6, 7$

10. (B)

$f(x) = \sin^{-1}|\sin kx| + \cos^{-1}(\cos kx)$

Let $g(x) = \sin^{-1}|\sin x| + \cos^{-1}(\cos x)$

$$g(x) = \begin{cases} 2x, & 0 \le x \le \frac{\pi}{2} \\ \pi, & \frac{\pi}{2} < x \le \frac{3\pi}{2} \\ 4\pi - 2x, & \frac{3\pi}{2} < x \le 2\pi \end{cases}$$

$g(x)$ is periodic with period 2π and is constant in the continuous interval $\left[2n\pi + \frac{\pi}{2}, 2n\pi + \frac{3\pi}{2}\right]$ (where $n \in I$) and $f(x) = g(kx)$.

So, $f(x)$ is constant in the interval $\left[\frac{2n\pi}{k} + \frac{\pi}{2k}, \frac{2n\pi}{k} + \frac{3\pi}{2k}\right]$

$\Rightarrow \quad \frac{\pi}{4} = \frac{3\pi}{2k} - \frac{\pi}{2k} \Rightarrow \frac{\pi}{k} = \frac{\pi}{4} \Rightarrow k = 4$

11. (A, C)

The given relation is possible when $a-\frac{a^2}{3}+\frac{a^3}{9}+...=1+b+b^2+...$

Also, $-1\le a-\frac{a^2}{3}+\frac{a^3}{9}+....\le 1$ and $-1\le 1+b+b^2+...\le 1$

$\Rightarrow |b|<1 \Rightarrow |a|<3$ and $\frac{a}{1+\frac{a}{3}}=\frac{1}{1-b}$

12. (A, B)

We know that

If $|x|\le 1$, then $2\tan^{-1}x=\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

If $x>1, 2\tan^{-1}x=\pi-\sin^{-1}\frac{2x}{1+x^2}$

If $x<-1, 2\tan^{-1}x=-\pi-\sin^{-1}\frac{2x}{1+x^2}$

Hence, the required values are $x<-1$ or $x>1$.

13. (A, B, C)

(A) $\cos\left(\tan^{-1}\left(\tan\left(4-\pi\right)\right)\right)=\cos\left(4-\pi\right)=\cos\left(4-\pi\right)=-\cos 4>0$

(B) $\sin\left(\cot^{-1}\left(\cot\left(4-\pi\right)\right)\right)=\sin\left(4-\pi\right)=-\sin 4>0$ (as $\sin 4<0$)

(C) $\tan\left(\cos^{-1}\left(\cos\left(2\pi-5\right)\right)\right)=\tan\left(2\pi-5\right)=-\tan 5>0$ (as $\tan 5<0$)

(D) $\cot\left(\sin^{-1}\left(\sin\left(\pi-4\right)\right)\right)=\cot\left(\pi-4\right)=-\cot 4<0$

14. (B, C, D)

$cos\left(-\frac{14\pi}{5}\right)=cos\frac{14\pi}{5}=cos\frac{4\pi}{5}$

Hence, $cos\frac{1}{2}cos^{-1}\left(cos\frac{4\pi}{5}\right)=cos\frac{2\pi}{5}$

Paragraph Based Questions

1. (D)

The value of cos $[\tan^{-1}\tan 2]$ is $-\cos 2$

2. (D)

If $\pi\le x\le 2\pi$, then $\cos^{-1}\cos x$ is equal to $2\pi-x$

3. (B)

If $x+\frac{1}{x}=2$, the principal value of $\sin^{-1}x$ is $\frac{\pi}{2}$

4. (D)

The trigonometric equation $\sin^{-1}x=2\sin^{-1}a$ has a solution for $|a|\le\frac{1}{\sqrt{2}}$

5. (A)

The value of $\sin\left[\frac{\pi}{6}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$ is equal to $\frac{\sqrt{3}}{2}$

6. (C)

If $\sin^{-1}(\sin x)=\pi-x$, then x belongs to $\left[\frac{\pi}{2},\frac{3\pi}{2}\right]$

Matrix-Match Type

1. (I) $\rightarrow$ Q, R, S; (II) $\rightarrow$ S; (III) $\rightarrow$ R, S; (IV) $\rightarrow$ P

(I) $\left(\sin^{-1}x\right)^2+\left(\sin^{-1}y\right)^2=\frac{\pi^2}{2}$

$\Rightarrow \left(\sin^{-1}x\right)^2=\left(\sin^{-1}y\right)^2=\frac{\pi^2}{4}$

$\Rightarrow \sin^{-1}x=\pm\frac{\pi}{2}, \sin^{-1}y=\pm\frac{\pi}{2}$

$\Rightarrow x=\pm1$ and $y=\pm1$

$\Rightarrow x^3+y^3=-2, 0, 2$

(II) $\left(\cos^{-1}x\right)^2+\left(\cos^{-1}y\right)^2=2\pi^2$

$\Rightarrow \left(\cos^{-1}x\right)^2=\left(\cos^{-1}y\right)^2=\pi$

$\Rightarrow x=y=-1$

(III) $\left(\sin^{-1}x\right)^2\left(\cos^{-1}y\right)^2=\frac{\pi^4}{4}$

$\Rightarrow \left(\sin^{-1}x\right)^2=\frac{\pi^2}{4}$ and $\left(\cos^{-1}y\right)^2=\pi$

$\Rightarrow \left(\sin^{-1}x\right)=\pm\frac{\pi}{2}$ and $\left(\cos^{-1}y\right)=\pi$

$\Rightarrow x=\pm1$ and $y=-1$

$\Rightarrow -|x-y|=0, 2$

(IV) $\left|\sin^{-1}x-\sin^{-1}y\right|=\pi$

$\Rightarrow \sin^{-1}x=-\frac{\pi}{2}$ and $\sin^{-1}y=\frac{\pi}{2}$

or $\sin^{-1}x=\frac{\pi}{2}$ and $\sin^{-1}y=-\frac{\pi}{2}$

$\Rightarrow x^y=1^{(-1)}$ or $(-1)^1=1$ or -1

2. (I) $\rightarrow$ P, Q; (II) $\rightarrow$ Q; (III) $\rightarrow$ Q, R, S; (IV) $\rightarrow$ P, R

1. (3)

We must have $x(x+3) \geq 0$

$\Rightarrow \quad x \geq 0$ or $x \leq -3$

Also, $-1 \leq x^2 + 3x + 1 \leq 1$

$\Rightarrow \quad x(x+3) \leq 0$

$\Rightarrow \quad -3 \leq x \leq 0$

From Eqs. (i) and (ii), we get $x = \{0, -3\}$

Hence, required sum is 3.

2. (6)

$$T_n = \tan^{-1}\left(\frac{n+1-1}{1+(n+1)1}\right)$$

$$= \tan^{-1}(n+1) - \tan^{-1}(n)$$

Hence, $S_n = \tan^{-1}(n+1) - \tan^{-1}1$

$$= \tan^{-1}\left(\frac{n+1-1}{1+(n+1)1}\right) = \left(\tan^{-1}\frac{n}{n+2}\right) = \frac{1}{2}\cos^{-1}\left(\frac{24}{145}\right)$$

$\Rightarrow \quad 2\left(\tan^{-1}\frac{n}{n+2}\right) = \cos^{-1}\left(\frac{24}{145}\right)$

$\Rightarrow \quad \cos^{-1}\left(\frac{2(n+1)}{n^2+2n+2}\right) = \cos^{-1}\left(\frac{24}{145}\right)$

$\Rightarrow \quad 12(n+1)^2 - 145(n+1) + 12 = 0$

$\Rightarrow \quad ((n+1)-12)(12(n+1)-1) = 0$

$\Rightarrow \quad n+1 = 12$

$\Rightarrow \quad n = 11$

3. (1)

Given expression is defined only for $x = 1$ and -1

$\therefore \quad f(1) = 1$ and $f(-1) = (1+\pi)(1+\pi) = (1+\pi)^2$

Hence, the least value is 1.

4. (1)

$\tan^{-1}(3x) + \tan^{-1}(5x) = \tan^{-1}(7x) + \tan^{-1}(2x)$

$\Rightarrow \quad \tan^{-1}(3x) - \tan^{-1}(2x) = \tan^{-1}(7x) - \tan^{-1}(5x)$

$\Rightarrow \quad \tan^{-1}\left(\frac{3x-2x}{1+6x^2}\right) = \tan^{-1}\left(\frac{7x-5x}{1+35x^2}\right)$

$\Rightarrow \quad \frac{x}{1+6x^2} = \frac{2x}{1+35x^2}$

$\Rightarrow \quad x = 0$ or $1 + 35x^2 = 2 + 12x^2$

$\Rightarrow\ x=0$ or $x=\dfrac{1}{\sqrt{23}}$ or $-\dfrac{1}{\sqrt{23}}$

5. (5)

$\left(\cot^{-1}x\right)\left(\tan^{-1}x\right)+\left(2-\dfrac{\pi}{2}\right)\cos^{-1}x-3\tan^{-1}x-3\left(2-\dfrac{\pi}{2}\right)>0$

$\Rightarrow\ \left(\cot^{-1}x\right)>0$

$\Rightarrow\ \left(\cot^{-1}x-3\right)\left(2-\cot^{-1}x\right)>0$

$\Rightarrow\ \left(\cot^{-1}x-3\right)\left(\cos^{-1}x-2\right)<0$

$\Rightarrow\ 2<\cot^{-1}x<3$

$\Rightarrow\ \cot 3<x<\cot 2$ [as $\cot^{-1}x$ is a decreasing function]

$\Rightarrow$ Hence, $x\in\left(\cot 3,\cot 2\right)$

$\Rightarrow\ \cot^{-1}a+\cot^{-1}b=\cot^{-1}\left(\cot 3\right)+\cot^{-1}\left(\cot 2\right)=5$

6. (3)

$\sin^{-1}\left(x^2-\dfrac{x^4}{3}+\dfrac{x^6}{9}-...\right)+\cos^{-1}\left(x^4-\dfrac{x^8}{3}+\dfrac{x^{12}}{9}...\right)=\dfrac{\pi}{2}$

$\Rightarrow\ \left(x^2-\dfrac{x^4}{3}+\dfrac{x^6}{9}.....\right)=\left(x^4-\dfrac{x^8}{3}+\dfrac{x^{12}}{9}.....\right)$

$\Rightarrow\ \dfrac{x^2}{1+\dfrac{x^2}{3}}=\dfrac{x^4}{1+\dfrac{x^4}{3}}$

$\Rightarrow\ \dfrac{3}{3+x^2}=\dfrac{3x^2}{3+x^4}$ or $x=0$

$\Rightarrow\ 9+3x^4=9x^2+3x^4$ or $x=0$

$\Rightarrow\ x^2=1\ \Rightarrow\ x=0,1$ or -1

Therefore, the number of values is equal to 3.

7. (2)

Since $\sin^{-1}$ is defined for [–1, 1]

$\therefore\ a=0$

$\therefore\ x=\sin^{-1}1+\cos^{-1}1-\tan^{-1}1=\dfrac{\pi}{4}$

$\Rightarrow\ \sec^2 x=2$

8. (4)

$f\left(x\right)=\sin^{-1}x+2\tan^{-1}x+\left(x+2\right)^2-3$

Domain of $f\left(x\right)$ is [–1, 1].

Also $f\left(x\right)$ is an increasing function in the domain

$\therefore\ p = f_{\min.}(x) = f(-1) = -\frac{\pi}{2} + 2\left(\frac{-\pi}{4}\right) + 1 - 3 = -\pi - 2$

and $q = f_{\max.}(x) = f(1) = \frac{\pi}{2} + 2\left(\frac{\pi}{4}\right) + 9 - 6 = \pi + 6$.

Therefore, the range of $f(x)$ is $[-\pi - 2, \pi + 6]$.

Hence, $(p + q) = 4$.

9. (6)

Let $\tan^{-1} u = \alpha \Rightarrow \tan\alpha = u$

$\tan^{-1} y = \beta \Rightarrow \tan\beta = v$

$\tan^{-1} w = \gamma \Rightarrow \tan\gamma = w$

$$\tan(\alpha + \beta + \gamma) = \frac{s_1 - s_3}{1 - s_2} = \frac{0 - (-11)}{1 - (-10)} = \frac{11}{11} = 1$$

$\therefore\ \alpha + \beta + \gamma = \tan^{-1}(1) = \frac{\pi}{4}$

$\Rightarrow\ 3\operatorname{cosec}^2\left(\tan^{-1} u + \tan^{-1} v + \tan^{-1} w\right) = 6$

10. (7)

$f(x) = \sqrt{3\cos^{-1}(4x) - \pi}$ is defined

If $\cos^{-1} 4x \geq \frac{\pi}{3} \Rightarrow 4x \leq \frac{1}{2} \Rightarrow x \leq \frac{1}{8}$ ...(i)

Also, $-1 \leq 4x \leq 1 \Rightarrow \frac{-1}{4} \leq x \leq \frac{1}{4}$ ...(ii)

Therefore, from Eqs. (i) and (ii), we have domain: $x \in \left[\frac{-1}{4}, \frac{1}{8}\right]$

$\Rightarrow 4a + 64b = 7$

11. (9)

$1 + \sin\left(\cos^{-1} x\right) + \sin^2\left(\cos^{-1} x\right) + \ldots\infty = 2$

$$\Rightarrow\ \frac{1}{1 - \sin\left(\cos^{-1} x\right)} = 2$$

$\Rightarrow\ \frac{1}{2} = 1 - \sin\left(\cos^{-1} x\right)$

$\Rightarrow\ \sin\left(\cos^{-1} x\right) = \frac{1}{2} \Rightarrow \cos^{-1} x = \frac{\pi}{6}$

12. (3)

$\cos^{-1}(x) + \cos^{-1}(2x) + \cos^{-1}(3x) = \pi$

$\Rightarrow\ \cos^{-1}(2x) + \cos^{-1}(3x) = \pi - \cos^{-1}(x) = \cos^{-1}(-x)$

$\Rightarrow\ \cos^{-1}\left[(2x)(3x) - \sqrt{1 - 4x^2}\sqrt{1 - 9x^2}\right] = \cos^{-1}(-x)$

$\Rightarrow \; 6x^2 - \sqrt{1-4x^2}\sqrt{1-9x^2} = -x$

$\Rightarrow \; \left(6x^2 + x\right)^2 = \left(1-4x^2\right)\left(1-9x^2\right)$

$\Rightarrow \; x^2 + 12x^3 = 1 - 13x^2$

$\Rightarrow \; 12x^3 + 14x^2 - 1 = 0$

$\Rightarrow \; a = 12; \; b = 14; \; c = 0$

$\Rightarrow \; b - a - c = 14 - 12 + 1 = 3$

13. (9)

$$\tan^{-1}\left(x+\frac{3}{x}\right) - \tan^{-1}\left(x-\frac{3}{x}\right) = \tan^{-1}\frac{6}{x}$$

$$\Rightarrow \; \tan^{-1}\left(\frac{\left(x+\frac{3}{x}\right)-\left(x-\frac{3}{x}\right)}{1+\left(x+\frac{3}{x}\right)\left(x-\frac{3}{x}\right)}\right) = \tan^{-1}\frac{6}{x}$$

$$\Rightarrow \; x^2 - \frac{9}{x^2} = 0 \; \Rightarrow \; x^4 = 9$$

Only One Option Correct

1. (D)

The principal value of $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

$$=\sin^{-1}\left(\sin\left(\pi-\frac{\pi}{3}\right)\right)=\sin^{-1}\left(\sin\frac{\pi}{3}\right)=\frac{\pi}{3}$$

2. (B)

$$\sin^{-1}\left(x-\frac{x^2}{2}+\frac{x^3}{4}...\right)+\cos^{-1}\left(x^2-\frac{x^4}{2}+\frac{x^6}{4}...\right)=\frac{\pi}{2}$$

$$\Rightarrow \cos^{-1}\left(x^2-\frac{x^4}{2}+\frac{x^6}{4}...\right)=\frac{\pi}{2}-\sin^{-1}\left(x-\frac{x^2}{2}+\frac{x^3}{4}...\right)$$

$$\Rightarrow \cos^{-1}\left(x^2-\frac{x^4}{2}+\frac{x^6}{4}...\right)=\cos^{-1}\left(x-\frac{x^2}{2}+\frac{x^3}{4}...\right)$$

$$\Rightarrow x^2-\frac{x^4}{2}+\frac{x^6}{4}...=x-\frac{x^2}{2}+\frac{x^3}{4}...$$

On both sides we have G.P. of infinite terms.

$$\therefore \frac{x^2}{1-\left(\frac{-x^2}{2}\right)}=\frac{x}{1-\left(\frac{-x}{2}\right)} \Rightarrow \frac{2x^2}{2+x}=\frac{2x}{2+x}$$

$$\Rightarrow 2x+x^3=2x^2+x^3 \Rightarrow x(x-1)=0$$

$$\Rightarrow x=0,1 \text{ but } 0<|x|<\sqrt{2} \Rightarrow x=1.$$

3. (D)

$$\sin\left[\cot^{-1}(1+x)\right]=\cos\left(\tan^{-1}x\right)$$

$$\Rightarrow \sin\left[\sin^{-1}\left(\frac{1}{\sqrt{1+(1+x)^2}}\right)\right]=\cos\left[\cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right]$$

$$\Rightarrow \frac{1}{\sqrt{1+(1+x)^2}}=\frac{1}{\sqrt{1+x^2}}$$

$$\Rightarrow 1+1+2x+x^2=1+x^2$$

$$\Rightarrow 2x+1=0$$

$$\Rightarrow x=-\frac{1}{2}$$

4. (C)

$$\sqrt{1+x^2}\left[\left\{x\cos\left(\cot^{-1}x\right)+\sin\left(\cot^{-1}x\right)\right\}^2-1\right]^{\frac{1}{2}}$$

$$=\sqrt{1+x^2}\left[\left\{x\cos\left(\cos^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right)+\sin\left(\sin^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right)\right\}^2-1\right]^{\frac{1}{2}}$$

$$=\sqrt{1+x^2}\left[\left\{x\cdot\frac{x}{\sqrt{1+x^2}}+\frac{1}{\sqrt{1+x^2}}\right\}^2-1\right]^{\frac{1}{2}}$$

$$=\sqrt{1+x^2}\left[\left(\sqrt{1+x^2}\right)^2-1\right]^{\frac{1}{2}}=x\sqrt{1+x^2}$$

5. (B)

$$\cot^{-1}\left[1+\sum_{k=1}^{n}2k\right]=\cot^{-1}\left[1+n(n+1)\right]$$

$$=\tan^{-1}\left[\frac{(n+1)-n}{1+(n+1)n}\right]=\tan^{-1}(n+1)-\tan^{-1}n$$

$$\therefore\ \sum_{n=1}^{23}\left[\tan^{-1}(n+1)-\tan^{-1}n\right]=\tan^{-1}24-\tan^{-1}1=\tan^{-1}\frac{23}{25}$$

$$\therefore\ \cot\left[\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{n=1}^{n}2k\right)\right]=\cot\left[\tan^{-1}\frac{23}{25}\right]=\frac{25}{23}$$

One or More than One Correct Answer

1. (B, C, D)

$$\alpha=3\sin^{-1}\frac{6}{11}>3\sin^{-1}\frac{1}{2}=\frac{\pi}{2}\ \Rightarrow\ \alpha>\frac{\pi}{2}$$

$\therefore\ \cos\alpha<0$

$$\beta=3\cos^{-1}\frac{4}{9}>3\cos^{-1}\frac{1}{2}=\pi\ \Rightarrow\ \beta>\pi$$

$\therefore\ \cos\beta<0$ and $\sin\beta<0$

Now, $\alpha+\beta>\frac{3\pi}{2}$, $\quad\therefore\ \cos(\alpha+\beta)>0$

2. (B, C, D)

$$f(n)=\frac{\sum_{k=0}^{n}\sin\left(\frac{k+1}{n+2}\pi\right)\sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^{n}\sin^2\left(\frac{k+1}{n+2}\pi\right)}$$, where n is non negative integer

$$=\frac{\sum_{k=0}^{n}\left[\cos\left(\frac{\pi}{n+2}\right)-\cos\frac{(2k+3)\pi}{n+2}\right]}{\sum_{k=0}^{n}\left[1-\cos\frac{2(k+1)\pi}{n+2}\right]}$$

$$=\frac{(n+1)\cos\left(\frac{\pi}{n+2}\right)-\left[\cos\frac{3\pi}{n+2}+\cos\frac{5\pi}{n+2}+.....+\cos\frac{(2n+3)\pi}{n+2}\right]}{n+1-\left[\cos\frac{2\pi}{n+2}+\cos\frac{4\pi}{n+2}+.....+\cos\frac{2(n+1)\pi}{n+2}\right]}$$

$$=\frac{(n+1)\cos\left(\frac{\pi}{n+2}\right)-\frac{\sin\frac{(n+1)\pi}{n+2}}{\sin\left(\frac{\pi}{n+2}\right)}\cdot\cos\frac{(2n+6)\pi}{2(n+2)}}{n+1-\frac{\sin\frac{(n+1)\pi}{n+2}}{\sin\left(\frac{\pi}{n+2}\right)}\cdot\cos\frac{(2n+4)\pi}{2(n+2)}}$$

$$=\frac{(n+1)\cos\frac{\pi}{n+2}+\cos\frac{\pi}{n+2}}{n+1+1}=\frac{(n+2)\cos\left(\frac{\pi}{n+2}\right)}{n+2}$$

$\therefore\quad f(n)=\cos\left(\frac{\pi}{n+2}\right)$

$\lim_{n\to\infty} f(n)=\lim_{n\to\infty}\cos\left(\frac{\pi}{n+2}\right)=1$

$\therefore$ Option (A) is incorrect.

$f(4)=\cos\left(\frac{\pi}{4+2}\right)=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$

$\therefore$ Option (B) is correct.

If $\alpha=\tan\left(\cos^{-1}f(6)\right)$

$$=\tan\left(\cos^{-1}\left(\cos\frac{\pi}{8}\right)\right)=\tan\frac{\pi}{8}$$

Now, $\tan\frac{\pi}{4}=1\Rightarrow\frac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}}=1$

$\Rightarrow\frac{2\alpha}{1-\alpha^2}=1\Rightarrow\alpha^2+2\alpha-1=0$

$\therefore$ Option (C) is correct.

$\sin\left(7\cos^{-1}f(5)\right)=\sin\left(7\cos^{-1}\left(\cos\frac{\pi}{7}\right)\right)=\sin\left(7\times\frac{\pi}{7}\right)$

$=\sin\pi=0$

$\therefore$ Option (D) is correct.

3. (A, B)

Given that $S_n(x) = \sum_{k=1}^{n} \cot^{-1}\left(\frac{1+k(k+1)x^2}{x}\right)$

$$= \sum_{k=1}^{n} \tan^{-1}\left(\frac{x}{1+kx(kx+x)}\right)$$

$$= \sum_{k=1}^{n} \tan^{-1}\left(\frac{(kx+x)-(kx)}{1+(kx+x)(kx)}\right)$$

$\Rightarrow \quad S_n(x) = \tan^{-1}(nx+x) - \tan^{-1} x$

$$= \tan^{-1}\left(\frac{nx}{1+(n+1)x^2}\right)$$

(A) $S_{10}(x) = \tan^{-1}\frac{10x}{1+11x^2} = \frac{\pi}{2} - \tan^{-1}\left(\frac{1+11x^2}{10x}\right)(x>0)$

(Option (A) is correct)

(B) $\lim_{n\to\infty} \cot(S_n(x)) = \lim_{n\to\infty} \cot\left(\cot^{-1}\left(\frac{1+(n+1)x^2}{nx}\right)\right)$

$$= \lim_{n\to\infty} \frac{\frac{1}{n} + \left(1+\frac{1}{n}\right)x^2}{x} = x(x>0)$$

(Option (B) is correct)

(C) $S_3(x) = \tan^{-1}\frac{3x}{1+4x^2} = \frac{\pi}{4}$

$\Rightarrow \quad 4x^2 - 3x + 1 = 0 \quad \Rightarrow \quad x \notin \mathbb{R}$ [$\because$ D is negative]

(Option (C) is incorrect)

(D) For $x = 1$

$\tan(S_n(x)) = \frac{n}{n+2} \geq \frac{1}{2}$

For $n \geq 3$.

(Option (D) is incorrect).

Matrix – Match Type :

1. (A) $\to$ P; (B) $\to$ R; (C) $\to$ Q

(A) $t = \sum_{i=1}^{\infty} \tan^{-1}\left(\frac{1}{2i^2}\right) = \sum_{i=1}^{\infty} \tan^{-1}\left[\frac{(2i+1)-(2i-1)}{1+4i^2-1}\right]$

$$= \sum_{i=1}^{\infty}\left[\tan^{-1}(2i+1) - \tan^{-1}(2i-1)\right]$$

$$= \tan^{-1}3 - \tan^{-1}1 + \tan^{-1}5 - \tan^{-1}3 + ... + \tan^{-1}(2n+1) - \tan^{-1}(2n-1) + ...\infty$$

$$= \lim_{n\to\infty}\left[\tan^{-1}(2n+1) - \tan^{-1}1\right]$$

$$= \lim_{n\to\infty} \tan^{-1}\left[\frac{2n}{1+(2n+1)}\right] = \lim_{n\to\infty} \tan^{-1}\left[\frac{1}{1+\frac{1}{n}}\right]$$

$$= \tan^{-1}(1) = \frac{\pi}{4} \Rightarrow \tan t = 1 \qquad (A) \to P$$

(B) $\because a, b, c$ are in AP $\Rightarrow 2b = a + c$

Now, $\cos\theta_1 = \frac{a}{b+c} \Rightarrow \frac{1-\tan^2\theta_1/2}{1+\tan^2\theta_1/2} = \frac{a}{b+c}$

$\Rightarrow \tan^2\frac{\theta_1}{2} = \frac{b+c-a}{b+c+a}$

Similarly, $\tan^2\frac{\theta_3}{2} = \frac{a+b-c}{a+b+c}$

$\Rightarrow \tan^2\frac{\theta_1}{2} + \tan^2\frac{\theta_3}{2} = \frac{2b}{3b} = \frac{2}{3}$ (B) $\to$ R

(C) Equation of line through (0, 1, 0) and perpendicular to $x+2y+2z=0$ is $\frac{x}{1} = \frac{y-1}{2} = \frac{z}{2} = \lambda$

For some value of λ, the foot of perpendicular from origin to line is $(\lambda, 2\lambda+1, 2\lambda)$

For some value of λ, the foot of perpendicular from origin to line is $(\lambda, 2\lambda+1, 2\lambda)$

Direction ratios of this $\perp$ from origin are $\lambda, 2\lambda+1, 2\lambda$

$\therefore 1.\lambda + 2(2\lambda+1) + 2.2\lambda = 0 \Rightarrow \lambda = -\frac{2}{9}$

$\therefore$ Foot of perpendicular is $\left(-\frac{2}{9}, \frac{5}{9}, -\frac{4}{9}\right)$

Hence required distance

$= \sqrt{\frac{4}{81}+\frac{25}{81}+\frac{16}{81}} = \sqrt{\frac{45}{81}} = \frac{\sqrt{5}}{3}$ (C) $\to$ Q

2. (A) $\to$ P; (B) – Q; (C) $\to$ P; (D) $\to$ S

$\sin^{-1}(ax) + \cos^{-1} y + \cos^{-1}(bxy) = \frac{\pi}{2}$

$\Rightarrow \cos^{-1} y + \cos^{-1}(bxy) = \frac{\pi}{2} - \sin^{-1}(ax) = \cos^{-1}(ax)$

Let $\cos^{-1} y = \alpha, \cos^{-1}(bxy) = \beta, \cos^{-1}(ax) = \gamma$, then $y = \cos\alpha, bxy = \cos\beta, ax = \cos\gamma$

$\therefore$ We get $\alpha + \beta = \gamma$ and $\cos\beta = bxy$

$\Rightarrow \cos(\gamma - \alpha) = \cos\beta = bxy$

$\Rightarrow \cos\gamma\cos\alpha + \sin\gamma\sin\alpha = bxy$

$\Rightarrow axy + \sin\gamma\sin\alpha = bxy \Rightarrow (a-b)xy = -\sin\alpha\sin y$

$\Rightarrow (a-b)^2 x^2 y^2 = \sin^2\alpha\sin^2\gamma$

$= (1-\cos^2\alpha)(1-\cos^2\gamma)$

$\Rightarrow (a-b)^2 x^2 y^2 = (1-y^2)(1-a^2x^2)$ ...(i)

(A) For $a=1, b=0$, equation (i) reduces to

$x^2y^2=(1-x^2)(1-y^2) \Rightarrow x^2+y^2=1$

(B) for $a=1, b=1$ equation (i) becomes

$(1-x^2)(1-y^2)=0 \Rightarrow (x^2-1)(y^2-1)=0$

(C) For $a=1, b=2$ equation (i) reduces to

$x^2y^2=(1-x^2)(1-y^2) \Rightarrow x^2+y^2=1$

(D) For $a=2, b=2$ equation (i) reduced to

$0=(1-4x^2)(1-y^2) \Rightarrow (4x^2-1)(y^2-1)=0$

3. (B)

(P) $\left[\frac{1}{y^2}\left(\frac{\cos(\tan^{-1}y)+y\sin(\tan^{-1}y)}{\cot(\sin^{-1}y)+\tan(\sin^{-1}y)}\right)^2+y^4\right]^{\frac{1}{2}}$

$$=\left[\frac{1}{y^2}\left(\frac{\cos\left(\cos^{-1}\frac{1}{\sqrt{1+y^2}}\right)+y\sin\left(\sin^{-1}\frac{y}{\sqrt{1+y^2}}\right)}{\cot\left(\cot^{-1}\frac{\sqrt{1-y^2}}{y}\right)+\tan\left(\tan^{-1}\frac{y}{\sqrt{1-y^2}}\right)}\right)+y^4\right]^{\frac{1}{2}}$$

$$=\left[\frac{1}{y^2}\left(\frac{\frac{\sqrt{1+y^2}}{1}}{\frac{1}{y\left(\sqrt{1-y^2}\right)}}\right)+y^4\right]^{\frac{1}{2}}$$

$=(1-y^4+y^4)^{\frac{1}{2}}=1$

$\therefore$ (P) $\to$ (4)

(Q) $\cos x+\cos y=-\cos z$ ….. (i)

and $\sin x+\sin y=-\sin z$ ….. (ii)

On squaring (i) and (ii) and then adding, we get

$(\cos x+\cos y)^2+(\sin x+\sin y)^2=\cos^2 z+\sin^2 z$

$\Rightarrow 2+2\cos(x-y)=1$

$\Rightarrow 4\cos^2\frac{x-y}{2}=1 \Rightarrow \cos\frac{x-y}{2}=\pm\frac{1}{2}$

$\therefore$ Q $\to$ (3)

(R) $\cos\left(\frac{\pi}{4}-x\right)\cos 2x+\sin x\sin 2x\sec x$

$=\cos x\sin 2x\sec x+\cos\left(\frac{\pi}{4}+x\right)\cos 2x$

$$\Rightarrow \cos 2x\left[\cos\left(\frac{\pi}{4}-x\right)-\cos\left(\frac{\pi}{4}+x\right)\right]$$

$$=\sin 2x \sec x(\cos x-\sin x)$$

$$\Rightarrow 2\sin x\left[\frac{1}{\sqrt{2}}\left(\cos^2 x-\sin^2 x\right)-(\cos x-\sin x)\right]=0$$

$$\Rightarrow 2\sin x(\cos x-\sin x)\left(\frac{\cos x+\sin x}{\sqrt{2}}-1\right)=0$$

$$\Rightarrow \sin x=0 \text{ or } \tan x=1 \text{ or } \cos\left(x-\frac{\pi}{4}\right)=1$$

$$\Rightarrow x=0 \text{ or } \frac{\pi}{4} \Rightarrow \sec x=1 \text{ or } \sqrt{2}$$

$\therefore$ (R) $\to$ (2, 4)

(S) $\cot\left(\sin^{-1}\sqrt{1-x^2}\right)=\sin\left(\tan^{-1}x\sqrt{6}\right)$

$$\Rightarrow \frac{x}{\sqrt{1-x^2}}=\frac{x\sqrt{6}}{\sqrt{1+6x^2}} \Rightarrow x=\pm\frac{1}{2}\sqrt{\frac{5}{3}}$$

$\therefore$ (S) $\to$ (1)

Hence, (P) $\to$ (4), (Q) $\to$ (3), (R) $\to$ (2, 4), (S) $\to$ (1)

Integer Value Answer/ Non-Negative Integer

1. (2)

$$\sin^{-1}\left(\sum_{i=1}^{\infty}x^{i+1}-x\sum_{i=1}^{\infty}\left(\frac{x}{2}\right)^i\right)$$

$$=\frac{\pi}{2}-\cos^{-1}\left(\sum_{i=1}^{\infty}\left(-\frac{x}{2}\right)^i-\sum_{i=1}^{\infty}(-x)^i\right)$$

$$\sin^{-1}\left(\frac{x^2}{1-x}-x\cdot\frac{\frac{x}{2}}{1-\frac{x}{2}}\right)=\sin^{-1}\left(\frac{-\frac{\pi}{2}}{1+\frac{x}{2}}-\frac{-x}{1+x}\right)$$ [$\because$ sum of infinite terms of a G.P. $=\frac{a}{1-r}$, if $|r|<1$]

$$\Rightarrow \frac{x^2}{1-x}-\frac{x^2}{2-x}=\frac{-x}{2+x}+\frac{x}{1+x}$$

$$\Rightarrow \frac{x^2}{1-x}-\frac{x}{1+x}+\frac{x}{2+x}-\frac{x^2}{2-x}=0$$

$$\Rightarrow \frac{x\left(x^2+2x-1\right)}{1-x^2}+\frac{x\left(2-3x-x^2\right)}{4-x^2}=0$$

$$\Rightarrow x\left[x^3+2x^2+5x-2\right]=0$$

$$\Rightarrow x=0 \text{ or } x^3+2x^2+5x-2=0 = p(x)(\text{say})$$

We observe that $p(0)<0$ and $p\left(\frac{1}{2}\right)>0$

$\therefore$ One root of $p(x)=0$ lies in $\left(0, \frac{1}{2}\right)$.

Thus two solutions lie between $-\frac{1}{2}$ and $\frac{1}{2}$.

2. (0)

$$\sec^{-1}\left[\frac{1}{4}\sum_{k=0}^{10}\sec\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\sec\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)\right]$$

$$=\sec^{-1}\left[\frac{1}{2}\sum_{k=0}^{10}\frac{1}{2\cos\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\cos\left(\frac{7\pi}{12}+\frac{k\pi}{2}+\frac{\pi}{2}\right)}\right]$$

$$=\sec^{-1}\left[\frac{1}{2}\sum_{k=0}^{10}\frac{-1}{\sin\left(\frac{7\pi}{6}+k\pi\right)}\right]$$

$$=\sec^{-1}\left[\frac{1}{2}\sum_{k=0}^{10}\frac{-1}{\sin(k+1)\pi+\frac{\pi}{6}}\right]$$

If k is an even integer, then

$$\sin\left((k+1)\pi+\frac{\pi}{6}\right)=-\sin\frac{\pi}{6}=-\frac{1}{2}$$

If k is an odd integer, then $\sin\left((k+1)\pi+\frac{\pi}{6}\right)$

$$=\sin\frac{\pi}{6}=\frac{1}{2}$$

$$\therefore \sum_{k=0}^{9}\frac{1}{\sin\left((k+1)\pi+\frac{\pi}{6}\right)}=0$$

Hence, $\sec^{-1}\left[\frac{1}{2}\sum_{k=0}^{10}\frac{-1}{\sin\left((k+1)\pi+\frac{\pi}{6}\right)}\right]$

$$=\sec^{-1}\left[\frac{1}{2}\left(\frac{-1}{-\frac{1}{2}}\right)\right]=\sec^{-1}(1)=0$$

3. (2.36)

$$\left[\text{Let, } \cos^{-1}\sqrt{\frac{2}{2+\pi^2}} = t = \tan^{-1}\frac{\pi}{\sqrt{2}}\right] \qquad \left\{\text{similarly for } \sin^{-1}\frac{2\sqrt{2}\pi}{2+\pi^2}\right\}$$

Now, we have

$$\frac{3}{2}\tan^{-1}\frac{\pi}{\sqrt{2}} + \frac{1}{4}\tan^{-1}\left(\frac{2\sqrt{2}\pi}{\pi^2-2}\right) + \tan^{-1}\frac{\sqrt{2}}{\pi}$$

$$= \frac{\pi}{2} + \frac{1}{2}\tan^{-1}\frac{\pi}{\sqrt{2}} - \frac{1}{4}\tan^{-1}\left(\frac{2\sqrt{2}\pi}{2-\pi^2}\right)$$

$$= \frac{\pi}{2} + \frac{1}{2}\tan^{-1}\left(\frac{\pi}{\sqrt{2}}\right) - \frac{1}{4}\tan^{-1}\left(\frac{2\cdot\left(\frac{\pi}{\sqrt{2}}\right)}{1-\left(\frac{\pi}{\sqrt{2}}\right)^2}\right)$$

$$= \frac{\pi}{2} + \frac{1}{2}\tan^{-1}\left(\frac{\pi}{\sqrt{2}}\right) - \frac{1}{4}\left(-\pi + 2\tan^{-1}\left(\frac{\pi}{\sqrt{2}}\right)\right)$$

$$= \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4} \simeq 2.36$$