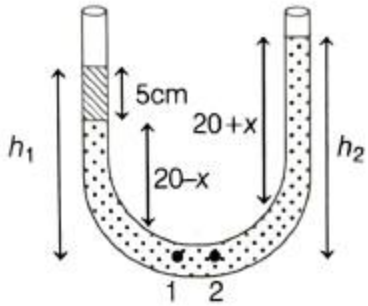
1. (C)



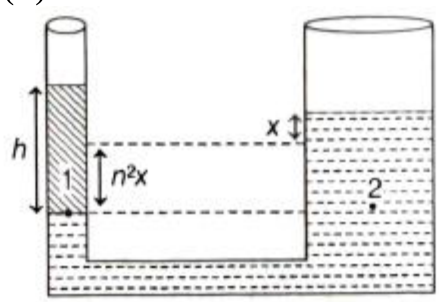
$p_1 = p_2$

$\Rightarrow \quad p_0 + 4g(5) + 1g(20 - x) = p_0 + 1g(20 + x)$

$\Rightarrow \quad x = 10\,\text{cm}$

$\frac{h_2}{h_1} = \frac{20 + x}{(20 - x) + 5} = \frac{30}{15} = 2$

2. (B)



$p_1 = p_2 \Rightarrow p_0 + \rho_w gh = p_0 + \sigma\rho_w(n^2x + x + x)$

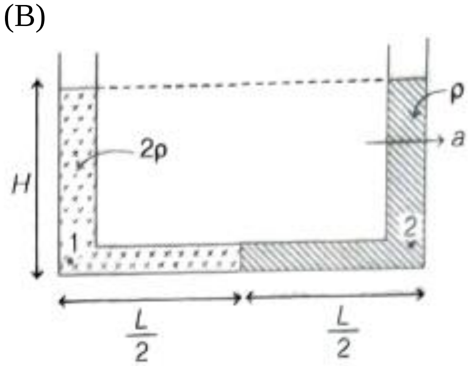
$\Rightarrow \quad x = \frac{h}{(n^2 + 1)\sigma}$

3. (C)

$p_1 = p_2 \Rightarrow p_0 + 1000\,gh = p_0 + \frac{12g}{800 \times 10^{-4}}$

$\Rightarrow h = 15\,\text{cm}$

4. (B)



Along vertical,

$$p_1 = p_0 + (2\rho)gh$$

$$p_2 = p_0 + \rho gh$$

Along horizontal,

$$p_1 = p_2 + \rho a\left(\frac{L}{2}\right) + 2\rho a\left(\frac{L}{2}\right)$$

$$\Rightarrow \; \Rightarrow \; \left(p_0 + 2\rho gh\right) = \left(p_0 + \rho gH\right) + \frac{3\rho aL}{2}$$

$$\Rightarrow \; H = \frac{3aL}{2g}$$

5. (A)

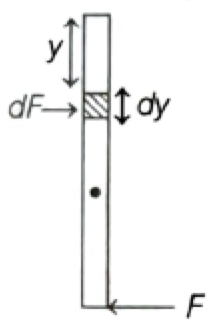
$$\int_{p_1}^{p_2} dp = \int_{r_1}^{r_2} \rho\left(\omega^2 r\right) dr$$

$$\Rightarrow \; p_2 - p_1 = \rho\omega^2\left(\frac{r_2^2 - r_1^2}{2}\right)$$

$$\Rightarrow \; \omega = \sqrt{\frac{2\left(p_2 - p_1\right)}{\rho\left(r_2^2 - r_1^2\right)}}$$

6. (C)

The net force acting on the gate element of width dy at a depth y from the surface of the fluid.



$$dF = \left(p_0 + \rho gy - P_0\right)\left(1dy\right)$$

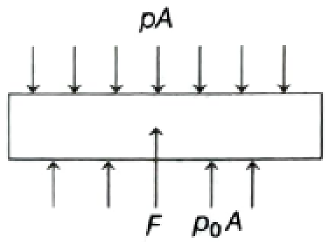
Torque about the hinge $d\tau = \rho gy\, dy\left(\frac{1}{2} - y\right)$

Net torque experienced by the gate.

$$\int d\tau + F\left(\frac{1}{2}\right) = 0$$

$$\Rightarrow \; \int_0^1 \rho gy\, dy\left(\frac{1}{2} - y\right) + \frac{F}{2} = 0 \; \Rightarrow \; F = \frac{\rho g}{6}$$

7. (B)



For equilibrium of piston,

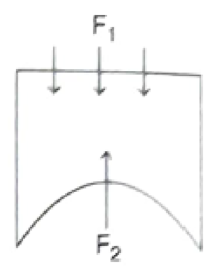
$$pA = p_0A + F$$

$\Rightarrow$ $(p_0 + \rho gH)A = p_0A + F$

$\Rightarrow$ $F = \rho gHA$

8. (D)

Let force on the bottom of cylinder by liquid be F_2.



$F_1 = pA = (\rho gh)\pi R^2$

Buoyant Force = Net hydrostatic force

$\Rightarrow$ $\rho Vg = F_2 - F_1$

$\Rightarrow$ $\rho Vg = F_2 - (\rho gh)\pi R^2$

$\Rightarrow$ $F_2 = \rho g\left(V + \pi R^2 h\right)$

9. (B)

Weight removed = Decrease in buoyant force

$\Rightarrow$ $200g = 1\left(l^2 \times 2\right)g$

$\Rightarrow$ l = 10 cm

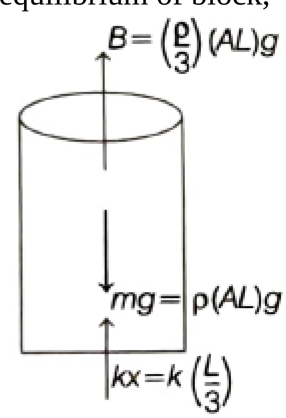
10. (C)

Block experiences a buoyant force due to liquid in upward direction. So, reading of spring balance will be less than 2 kg.

Liquid will experience buoyant force due to block in downward direction. So, reading of balance B will be more than 5 kg.

11. (B)

For equilibrium of block,



$\sum F = 0$

$\Rightarrow \quad B + kx = mg$

$\Rightarrow \quad \frac{\rho}{3} ALg + k\left(\frac{L}{3}\right) = \rho ALg$

$\Rightarrow \quad K = 2\rho Ag$

12. (A)

Fraction of volume immersed $\frac{V_d}{V} = \frac{\rho_S}{\rho_L}$

It depends upon the densities of solid and liquid only. It is independent of acceleration of system. So, no change in fraction of volume immersed.

13. (A)

For equilibrium of cylinder,

$$B_1 + B_2 = mg$$

$\Rightarrow \quad (2d)\left(A\frac{L}{4}\right)g + dA\left(\frac{3L}{4}\right)g = D(AL)g$

$\Rightarrow \quad D = \frac{5d}{4}$

14. (A)

In steady horizontal flow,

$p + \frac{1}{2}\rho v^2 = \text{constant}$

So, pressure is greatest where the speed is least.

15. (A)

Applying Bernoulli's equation for the horizontal pipe,

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$\Rightarrow \quad \rho_{Hg} g(1\text{ cm}) + \frac{1}{2}\rho_w (35\text{ cm/s})^2$

$= \rho_{Hg} gh + \frac{1}{2}\rho_w (65\text{ cm/s})^2$

$\Rightarrow \quad 13.6 \times 980 \times 1 + \frac{1}{2} \times 1 \times (35)^2$

$= 13.6 \times 980h + \frac{1}{2} \times 1 \times (65)^2$

$\Rightarrow \quad h = 0.89\text{ cm of Hg}$

16. (C)

Applying Bernoulli's equation,

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$\Rightarrow \quad v_2^2 = v_1^2 + 2g(h_1 - h_2) \qquad \because \{p_1 = p_2\}$

$\Rightarrow \quad v_2^2 = (1)^2 + 2 \times 10 \times (0.15)$

$\Rightarrow \quad v_2 = 2 \text{ m/s}$

Applying equation of continuity,

$$A_1 v_1 = A_2 v_2$$

$\Rightarrow \quad 10^{-4} \times 1 = A_2(2)$

$\Rightarrow \quad A_2 = 5 \times 10^{-5} \text{ m}^2$

17. (B)

If a ball is moving from left to right and also spinning about a horizontal axis in anti-clockwise direction of motion, then relative to the ball air will be moving from right to left. The resultant velocity of air above and below the ball will be $(v + \omega r)$ and $(v - \omega r)$, respectively.

So, according to Bernoulli's principle, due to this differences of pressure, an upward force will act on the ball and hence the ball will get maximum flight.

18. (A)

$$v^2 = \frac{2gh}{1 - \left(\frac{a}{A}\right)^2}$$

$$= \frac{2 \times 10 \times (3 - 0.525)}{1 - (0.1)^2} = 50 \text{ m}^2/\text{s}^2$$

19. (A)

Volume flow rate $= \frac{dV}{dt} = Av$

Volume flow rate is same for both holes.

$\Rightarrow \quad A_1 v_1 = A_2 v_2$

$\Rightarrow \quad (L^2)\sqrt{2gy} = (\pi R^2)\left(\sqrt{2g(4y)}\right)$

$\Rightarrow \quad R = \frac{L}{\sqrt{2\pi}}$

20. (D)

Applying Bernoulli's equation,

$$p_1 + \frac{1}{2}\rho v_1^2 = \rho_2 + \frac{1}{2}\rho v_2^2$$

$\Rightarrow \quad \left(p_0 + 10^3 \times 10 \times 10\right) + 0 = p_0 + \frac{1}{2} \times 10^3 v_2^2$

$\Rightarrow \quad v_2 = \sqrt{200} \text{ m/s}$

So horizontal range, $R = v_2 T$

$$2R = v'T$$

$\Rightarrow \quad v_2' = 2v_2 = 2\sqrt{200} \text{ m/s}$

Applying Bernoulli's equation,

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow \left(p_0 + 10^3 \times 10 \times 10 + p_{\text{extra}}\right) + 0$$

$$= p_0 + \frac{1}{2} \times 10^3 \times \left(2\sqrt{200}\right)^2$$

$$\Rightarrow p_{\text{extra}} = 3 \times 10^5 \text{ Pa} = 3 \text{ atm}$$

21. (C)

Let height of p above ground be y.

$v_p = \sqrt{2g(H-y)}$ and $v_Q = \sqrt{2g(H-h)}$

$R_1 = R_2$

$$\Rightarrow \sqrt{2g(H-y)}\sqrt{\frac{2y}{g}} = \sqrt{2g(H-h)}\sqrt{\frac{2h}{g}}$$

$$\Rightarrow y = H - h$$

22. (A)

Where tube is narrower, liquid is faster and hence according to Bernoulli's principle pressure will be lesser. So, height of liquid in the vertical tubes will be lesser where the tube is narrower.

23. (A)

When air is blown over the tank, atmospheric pressure over the tank decreases. Due to lesser pressure difference at the hole, velocity of efflux decreases.

24. (D)

Initial momentum of liquid $= d\,mv\,\hat{\mathbf{j}}$

Final momentum of liquid $= d\,mv\,\hat{\mathbf{i}}$

Change in momentum, $\mathbf{dp} = dmv\left(\hat{\mathbf{i}} - \hat{\mathbf{j}}\right)$

$$\Rightarrow dp = \sqrt{2}\,dmv$$

$$\Rightarrow \frac{dp}{dt} = \sqrt{2}\frac{dm}{dt}v = \sqrt{2}\frac{d(\rho v)}{dt}v$$

$$\Rightarrow F = \sqrt{2}\rho LV$$

25. (A)

$F = \rho a v^2$

$$\Rightarrow m\alpha = \rho a\left(\sqrt{2gh}\right)^2$$

$$(\rho A h)\alpha = \rho a(2gh)$$

$$\Rightarrow \alpha = \frac{2ga}{A}$$

So, acceleration at any instant is independent of h.

Velocity acquired by container,

$$v = u + at \Rightarrow v = \frac{2gat}{A}$$

Hence, v depends on h as t depends on h.

26. (1.9)

Weight of liquid added = Increase in buoyant force

$$\Rightarrow \quad (\rho\rho_w)\left(\frac{1}{3}\pi\left(\frac{r}{3}\right)^2\left(\frac{h}{3}\right)\right)g = (0.8\rho_w)$$

$$\left(\frac{1}{3}\pi\left(\frac{r}{2}\right)^2\left(\frac{r}{2}\right) - \frac{1}{3}\pi\left(\frac{r}{3}\right)^2\left(\frac{h}{3}\right)\right)g$$

$$\Rightarrow \quad \rho = 1.9$$

27. (100)

As water level decreases, buoyant force on the block decreases and tension in the wire increases.

Let water level comes down by y, then wire breaks.

$$T = mg - B$$

$$\Rightarrow \quad 7\times10^6\times10^{-6} = 1500\times(0.1)^3 g - 1000\times(0.1)^2\frac{(10-y)}{100}g$$

$$y = 2\text{ cm}$$

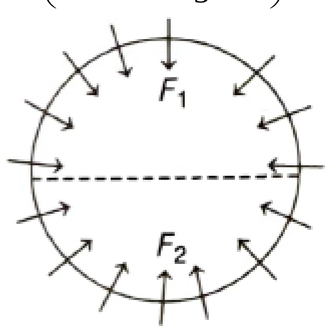
Let the level descends by cm in time it.

$$2t = (200-100)2 \Rightarrow t = 100\text{ s}$$

28. (3)

$F_1 =$ Hydrostatic force on the upper half of sphere weight of the liquid column above the upper half of sphere

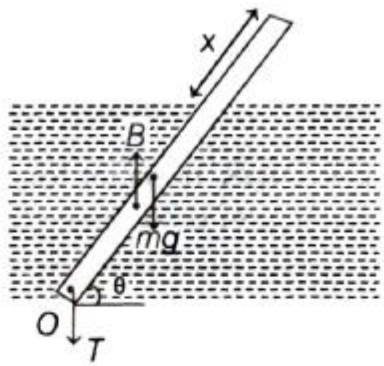
$$= \rho\left(\pi R^2(R) - \frac{2}{3}\pi R^3\right)g = \frac{1}{3}\rho\pi R^3 g \text{ (downward)}$$



$$F_2 - F_1 = B \Rightarrow F_2 - \frac{1}{3}\rho\pi R^3 g = \rho\left(\frac{4}{3}\pi R^3\right)g$$

$$\Rightarrow \quad F_2 = \frac{5}{3}\rho\pi R^3 g$$

29. (2)



Balancing torque about O,

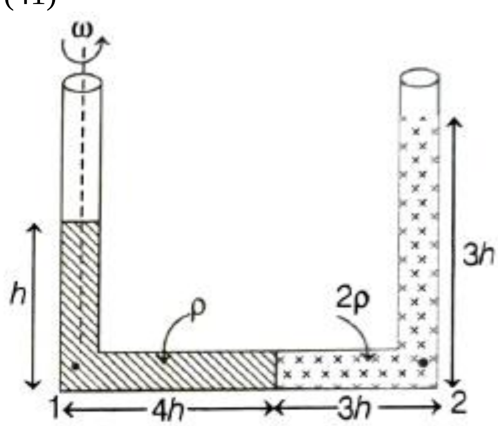
$$\left(\frac{25}{36}\rho_w\right)(12A)\,g\,(6\cos\theta)-\rho_w$$

$$\left[(12-x)A\right]g\left(\frac{12-x}{2}\right)\cos\theta=0$$

$\Rightarrow \quad (12-x)^2=100$

$\Rightarrow \quad x=2\text{ m}$

30. (41)



Along vertical, $p_1=p_0+\rho gh$

$$p_2=p_0+(2\rho)\,g\,(3h)=p_0+6\rho gh$$

Along horizontal,

$$\int dp=\int \rho\omega^2 x\,dx$$

$$\int_{p_1}^{p_2} dP=\int_0^{4h}\rho\omega^2 x\,dx+\int_{4h}^{7h}(2\rho)\,\omega^2 x\,dx$$

$$\Rightarrow \quad p_2-p_1=\rho\omega^2\left(\frac{16h^2}{2}\right)+2\rho\omega^2\left(\frac{33h^2}{2}\right)$$

$$\Rightarrow \quad 5g\rho h=41\rho\omega^2h^2 \;\Rightarrow\; \omega=\sqrt{\frac{5g}{41h}}$$

31. (2)

Equation of continuity, $A_1v_1=A_2v_2$

$\Rightarrow \quad \frac{\pi(8)^2}{4} \times 0.25 = \frac{\pi(2)^2}{4} v_2$

$\Rightarrow \quad v_2 = 4\ \text{m/s}$

Horizontal range $= v_2 T = v_2 \sqrt{\frac{2h}{g}} = 4\sqrt{\frac{2(1.25)}{10}} = 2\ \text{m}$

32. (3.2)

Let point B at the nozzle.

Applying equation of continuity at A and B,

$A_1 v_1 = A_2 v_2$

$\Rightarrow \quad (4A) v_A = A v_B$

$\Rightarrow \quad v_A = \frac{v_B}{4}$

Applying Bernoulli's equation between A and B,

$p_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$

$\Rightarrow \quad p_A + \frac{1}{2}\rho v_A^2 + 0 = p_B + \frac{1}{2}\rho v_B^2 + \rho g l$

$\Rightarrow \quad \left(p_0 + 41 \times 10^3\right) + \frac{1}{2} \times 10^3 \times \left(\frac{V_B}{4}\right)^2$

$= p_0 + \frac{1}{2} \times 10^3 \times v_B^2 + 10^3 \times 10 \times 1.1$

$\Rightarrow \quad v_B = 8\ \text{m/s}$

$\Rightarrow \quad h = \frac{v_B^2}{2g}$

$\Rightarrow \quad h = \frac{v_B^2}{2g}$

$\Rightarrow \quad h = \frac{(g)^2}{2(10)} = 3.2\ \text{m}$

33. (80)

In steady state, $A_1 v_1 = A_2 v_2$

$\Rightarrow \quad \pi(0.02)^2 \times 1 = \pi(0.01)^2 \times \sqrt{2 \times 10 \times h}$

$h = 0.8\ \text{m} = 80\ \text{cm}$

34. (2)

Applying Bernoulli's equation,

$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$

$\Rightarrow \quad \left(p_0 + \rho g h + 2\rho g\left(\frac{h}{2}\right)\right) + 0 = p_0 + \frac{1}{2}(2\rho) v_2^2$

$\Rightarrow \quad v_2 = \sqrt{2gh}$

Speed just before hitting the ground,

$$v = \sqrt{v_2^2 + \left(2g\left(\frac{h}{2}\right)\right)} = \sqrt{3gh}$$

Applying equation of continuity,

$$A_1 v_1 = A_2 v_2$$

$\Rightarrow \quad \sqrt{6} \times \sqrt{2gh} = A\sqrt{3gh}$

$\Rightarrow \quad A = 2\ \text{cm}^2$

35. (2.78)

$$R = v\sqrt{\frac{2h}{g}}$$

$\Rightarrow \quad 80v\sqrt{\frac{2(20)}{10}}$

$\Rightarrow \quad v = 40\ \text{m/s}$

Applying Bernoulli's equation,

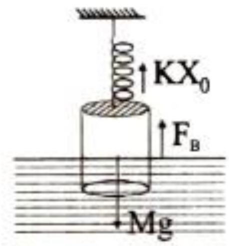
$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow (p + 660 \times 10 \times 53) + 0 = 10^5 + \frac{1}{2} \times 660 \times (40)^2$$

$$p = 2.78 \times 10^5\ \text{Pa}$$

1. (C)

(c) From figure, $kx_0 + F_B = Mg$



$$kx_0 + \sigma \frac{L}{2} Ag = Mg$$

[$\because$ mass = density × volume]

$$\Rightarrow kx_0 = Mg - \sigma \frac{L}{2} Ag$$

$$\Rightarrow x_0 = \frac{Mg - \frac{\sigma LAg}{2}}{k} = \frac{Mg}{k}\left(1 - \frac{LA\sigma}{2M}\right)$$

Hence, extension of the spring when it is in equilibrium is,

$$x_0 = \frac{Mg}{k}\left(1 - \frac{LA\sigma}{2M}\right)$$

2. (C)

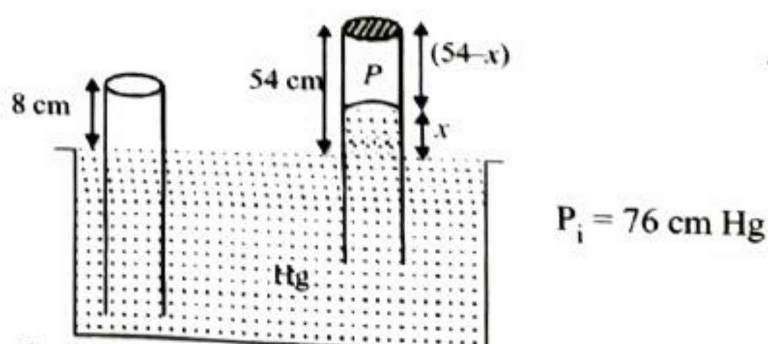
(c) Pressure difference

$$P_2 - P_1 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) = \frac{1}{2} \times 1.2\left((150)^2 - (100)^2\right)$$

$$= \frac{1}{2} \times 1.2(22500 - 10000) \simeq 7500 \text{ Nm}^{-2}$$

3. (A)

(a)



Length of the air column above mercury in the tube is,

$P_f + x = P_0$

$\Rightarrow \quad P_f = (76 - x)$

As T = cons. $\Rightarrow PV$ = cons.

$\Rightarrow P_iV_i = P_fV_f \Rightarrow (8 \times A) \times 76 = (76 - x) \times A \times (54 - x)$

$\therefore \quad x = 38$

Thus, length of air column = 54 – 38 = 16 cm.

4. (B)

(b) Volume flow rate = AV

Mass flow rate = ρAV

Momentum flow rate = ρAV^2

$\therefore \quad F = \rho AV^2 = 1000 \times 10^{-2} \times 1.5^2 = 22.5$ N

5. (D)

(d) Given: Diameter of water tap = $\frac{2}{\sqrt{\pi}}$ cm

$\therefore \quad$ Radius, $r = \frac{1}{\sqrt{\pi}} \times 10^{-2}$ m

Let A_V = Volume flow rate

$$A_V = \frac{3\ell}{\min} = \frac{3 \times 10^{-3}}{60}\frac{m^3}{s} = \frac{1}{20} \times 10^{-3} \text{ m/s}$$

$A_V = VA$

$$V = \frac{\frac{1}{20} \times 10^{-3}}{\pi \times \left(\frac{1}{\sqrt{\pi}}\right)^2 \times 10^{-4}} = 0.5 \text{ m/s}$$

Reynold's number, $R_e = \frac{\rho V d}{\eta}$

$$= \frac{10^3 \times 0.5 \times \frac{2}{\sqrt{\pi}} 10^{-2}}{10^{-3}} \cong 5500$$

6. (A)

(a) According to Bernoulli's Principle,

$$\frac{1}{2}\rho v_1^2 + \rho gh = \frac{1}{2}\rho v_2^2 \Rightarrow v_1^2 + 2gh = v_2^2$$

$$\Rightarrow 2gH + 2gh = v_2^2 \qquad \text{...(i)}$$

$a_1 v_1 = a_2 v_2$

$$\pi r^2 \sqrt{2gh} = \pi x^2 v_2 \Rightarrow v_2 = \frac{r^2}{x^2}\sqrt{2gh}$$

Substituting the value of v_2 in equation (i)

$$2gH + 2gh = \frac{r^4}{x^4} 2gh \text{ or, } x = r\left[\frac{H}{H+h}\right]^{\frac{1}{4}}$$

7. (D)

The volume of liquid flowing through both the tubes i.e., rate of flow of liquid is same i.e., Q= cons.

$$Q = \frac{\Delta P \pi r^4}{8\eta L} \quad \text{[By Poiseuille equation]}$$

i.e., $\frac{\pi P_1 r_1^4}{8\eta l_1} = \frac{\pi P_2 r_2^4}{8\eta l_2} \Rightarrow \frac{P_1 r_1^4}{l_1} = \frac{P_2 r_2^4}{l_2}$

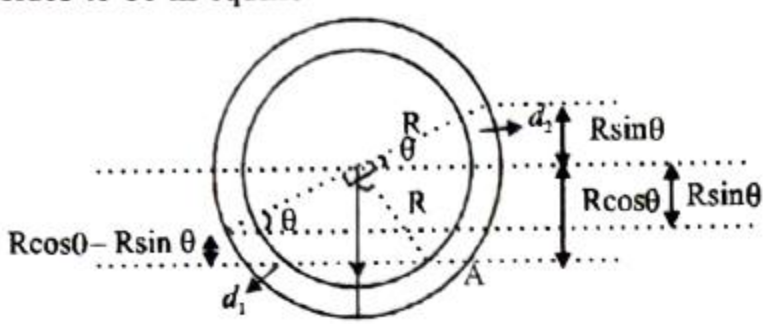
$\because \quad P_2 = 4\,P_1$ and $l_2 = l_1/4$

$\frac{P_1 r_1^4}{l_1} = \frac{4P_1 r_2^4}{l_1/4} \Rightarrow r_2^4 = \frac{r_1^4}{16}$

$r_2 = r_1/2$

8. (D)

(d) Pressure at interface A must be same from both the sides to be in equilibrium.



$\therefore \quad (R\cos\theta + R\sin\theta)\rho_2 g = (R\cos\theta - R\sin\theta)\rho_1 g$

$\Rightarrow \quad \frac{d_1}{d_2} = \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta} = \frac{1+\tan\theta}{1-\tan\theta}$

$\Rightarrow \quad \rho_1 - \rho_1 \tan\theta = \rho_2 + \rho_2 \tan\theta$

$\Rightarrow \quad (\rho_1 + \rho_2)\tan\theta = \rho_1 - \rho_2$

$\therefore \quad \theta = \tan^{-1}\left(\frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}\right)$

9. (D)

(d) Using $P_1V_1 = P_2V_2$

$$(P_1)\frac{4}{3}\pi r^3 = (P_2)\frac{4}{3}\pi\frac{125r^3}{64}$$

$$\frac{\rho g(10)+\rho g h}{\rho g(10)} = \frac{125}{64} \Rightarrow 640 + 64\,h = 1250$$

On solving we get $h = 9.5$ m

10. (A)

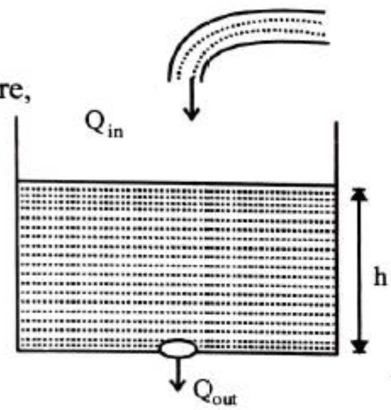
(a) Since height of water column is constant therefore, water inflow rate (Q_{in}) = water outflow rate

$Q_{in} = 10^{-4}\ m^3s^{-1}$

$Q_{out} = Au = 10^{-4} \times \sqrt{2gh}$

$\therefore 10^{-4} = 10^{-4} \times \sqrt{20 \times h}$

$\therefore h = \frac{1}{20} m = 5\ cm$



11. (B)

(b) Here, volume tric flow rate

$$= \frac{0.74}{60} = \pi r^2 v = \left(\pi \times 4 \times 10^{-4}\right) \times \sqrt{2gh}$$

$$\Rightarrow \sqrt{2gh} = \frac{74 \times 100}{240\pi} \Rightarrow \sqrt{2gh} = \frac{740}{24\pi}$$

$$\Rightarrow 2gh = \frac{740 \times 740}{24 \times 24 \times 10}\left(\because \pi^2 = 10\right) \Rightarrow h = \frac{74 \times 74}{2 \times 24 \times 24} \approx 4.8\text{m}$$

i.e., The depth of the centre of the opening from the level of water in the tank is close to 4.8 m

12. (B)

(b) When a body floats then the weight of the body = upthrust

$$\therefore (50)^3 \times \frac{30}{100} \times (1) \times g = M_{\text{cube}} g \qquad ...(i)$$

Let m mass should be placed, then

$(50)^3 \times (1) \times g = (M_{\text{cube}} + m)g \qquad ...(ii)$

Subtracting equation (i) from equation (ii), we get

$\Rightarrow mg = (50)^3 \times g\,(1 - 0.3) = 125 \times 0.7 \times 10^3 g \Rightarrow m = 87.5\text{ kg}$

13. (A)

(a) $P_1 = P_0 + rgd_1$

$P_2 = P_0 + rgd_2$

$\Delta P = P_2 - P_1 = \rho g \Delta d$

$3.03 \times 10^6 = 10^3 \times 10 \times \Delta d \Rightarrow \Delta d \simeq 300\text{ m}$

14. (C)

(c) $Mg = \left(\frac{4V}{5}\right)\rho_\omega g$ or $\left(\frac{M}{V}\right) = \frac{4\rho_\omega}{5}$ or $\rho = \frac{4\rho_\omega}{5}$

When block floats fully in water and oil, then

$Mg = F_{b_1} + F_{b_2}$

$(\rho V)g = \left(\frac{V}{2}\right)\rho_{\text{oil}}\, g + \frac{V}{2}\rho_\omega g$ or $r_{\text{oil}} = \frac{3}{5}\rho_\omega = 0.6\rho_\omega$

15. (A)

(a) Using $\frac{F}{A} = Y \cdot \frac{\Delta\ell}{\ell} \Rightarrow \Delta\ell \propto F \qquad ...(i)$

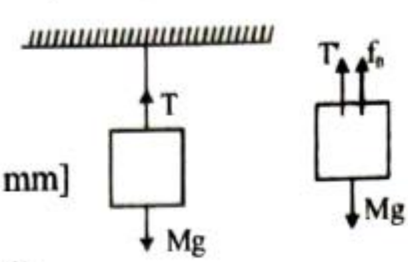
$T = Mg$

$T = Mg - f_B = Mg - \frac{M}{\rho_b} \cdot \rho_\ell \cdot g = \left(1 - \frac{\rho_\ell}{\rho_b}\right)Mg = \left(1 - \frac{2}{8}\right)Mg$

$T = \frac{3}{4}Mg$

From eqn (i)

$\frac{\Delta\ell'}{\Delta\ell} = \frac{T'}{T} = \frac{3}{4}$ [Given: $\Delta\ell = 4$ mm]



$\therefore \quad \Delta\ell' = \frac{3}{4} \cdot \Delta\ell = \frac{3}{4} \times 4 = 3\text{ mm}$

16. (C)

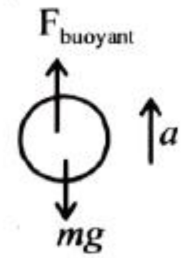
(c) Given :

Radius of air bubble = 1 cm,

Upward acceleration of bubble, $a = 9.8\ \text{cm/s}^2$,

$\rho_{water} = 1\ \text{g cm}^{-3}$

Volume $V = \frac{4\pi}{3}r^3 = \frac{4\pi}{3}\times(1)^3 = 4.19\ \text{cm}^3$



$F_{buoyant} - mg = ma \Rightarrow m = \frac{F_{buoyant}}{g+a}$

$\therefore m = \frac{(V\rho_\omega g)}{g+a} = \frac{V\rho_\omega}{1+\frac{a}{g}} = \frac{(4.19)\times 1}{1+\frac{9.8}{980}} = \frac{4.19}{1.01} = 4.15\,\text{g}$

17. (C)

(c) For minimum density of liquid, solid sphere has to float (completely immersed) in the liquid.

$mg = F_B$ (also $V_{immersed} = V_{total}$)

Now, $m = \int \rho\, dV$ and, $F_B = \frac{4}{3}\pi R^3 \rho_\ell g$

So, $\int_0^R \rho_0 4\pi\left(1-\frac{r^2}{R^2}\right).\, r^2 dr = \frac{4}{3}\pi R^3\rho_\ell$

$\Rightarrow 4\pi\rho_0\left[\frac{r^3}{3}-\frac{r^5}{5R^2}\right]_0^R = \frac{4}{3}\pi R^3\rho_\ell$

$\Rightarrow \frac{4\pi\rho_0 R^3}{3}\times\frac{2}{5} = \frac{4}{3}\pi R^3\rho_\ell \qquad \therefore \rho_\ell = \frac{2\rho_0}{5}$

18. (D)

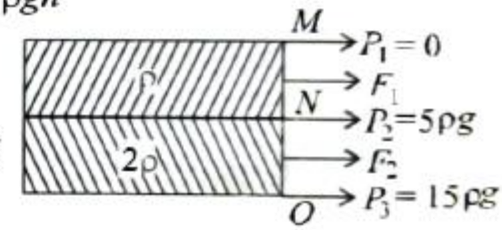
(d) Let P_1, P_2 and P_3 be the pressure at points M, N and O respectively.

Pressure is given by $P = \rho g h$

Now, $P_1 = 0$ ($\because$ $h = 0$)

$P_2 = \rho g(5)$

$P_3 = \rho g(15) = 15\,\rho g$



Force on upper part, $F_1 = \dfrac{(P_1 + P_2)}{2} A$

Force on lower part, $F_2 = \dfrac{(P_2 + P_3)}{2} A$

$\therefore \dfrac{F_1}{F_2} = \dfrac{5\rho g}{20\rho g} = \dfrac{5}{20} = \dfrac{1}{4}$

19. (D)

(d) Using Bernoulli's equation

$$P_1 + \frac{1}{2}\rho \upsilon_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho \upsilon_2^2 + \rho g h_2$$

For horizontal pipe, $h_1 = 0$ and $h_2 = 0$ and taking

$P_1 = P, P_2 = \dfrac{P}{2}$, we get $\Rightarrow P + \dfrac{1}{2}\rho\upsilon^2 = \dfrac{P}{2} + \dfrac{1}{2}\rho V^2$

$\Rightarrow \dfrac{P}{2} + \dfrac{1}{2}\rho\upsilon^2 = \dfrac{1}{2}\rho V^2 \Rightarrow V = \sqrt{\upsilon^2 + \dfrac{P}{\rho}}$

20. (A)

(a) From the equation of continuity

$A_1 v_1 = A_2 v_2$

Here, v_1 and v_2 are the velocities at two ends of pipe.

A_1 and A_2 are the area of pipe at two ends

$\Rightarrow \dfrac{v_1}{v_2} = \dfrac{A_2}{A_1} = \dfrac{\pi(4.8)^2}{\pi(6.4)^2} = \dfrac{9}{16}$

21. (D)

(d) F = Momentam transferred by water per sec

$= \rho a V \times V$

$F = \rho A v^2 = 10^3 \times 10 \times 10^{-4} \times 20 \times 20$

$F = 400$ N

22. (B)

(b) Let 'x' be the rise (or fall) in water.

Final volume of both vessel will be same

So, $16 \times 10^{-4} \times (150 - x) = 16 \times 10^{-4} \times (100 + x)$

$\Rightarrow 50 = 2x$

$\Rightarrow x = 25$ cm

Work done = Potential Energy of extra water that enters in cylindrical vessel $= m_{extra} \times g \times x$

$= \rho_w A_w x \times g \times x$

$= \rho_w A_w g \times x^2$

$= 10^3 \times 16 \times 10^{-4} \times 10 \times (25 \times 10^{-2})^2$

= 1 joule

23. (C)

(c) Apply Bernoulli's theorem between Piston and hole

$$P_A + \rho gh = P_0 + \frac{1}{2}\rho v_e^2$$

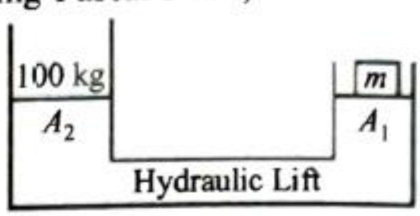
Assuming there is no atmospheric pressure on piston

$$\frac{5 \times 10^5}{\pi} + 10^3 \times 10 \times 10 = 1.01 \times 10^5 + \frac{1}{2} \times 10^3 \times v_e^2$$

$\Rightarrow v_e = 1.78$ m/s

24. (25600)

(25600) Using Pascal's law,



$$\frac{100 \times g}{A_2} = \frac{mg}{A_1} (A_1 < A_2) \quad ...(i)$$

Let m mass can lift M_0 in second case then

$$\frac{M_0 g}{16 A_2} = \frac{mg}{\frac{A_1}{16}} \quad \left(\because A = \frac{\pi d^2}{4}\right) \quad ...(ii)$$

From equation (i) and (ii), we get

$$\frac{M_0}{16 \times 100} = 16 \Rightarrow M_0 = 25600 \text{ kg}$$

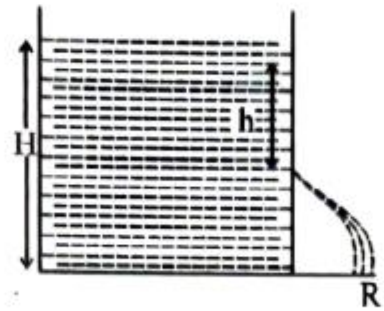
25. (6)

(6) Given,

Height of the water, H = 12 cm

Velocity of water coming out of hole, $v = \sqrt{2gh}$

Range of water, R = vt



$\Rightarrow \quad R = \sqrt{2gh} \times \sqrt{\frac{2(H-h)}{g}}$

$= 2\sqrt{h(H-h)}$

For maximum range $\frac{dR}{dh} = 0$

$\therefore \quad h = \frac{H}{2}$

Range is maximum when $h = \frac{12}{2} = 6m$

26. (24)

(24) Here, h = constant

So, $P + \frac{1}{2}\rho V^2 =$ constant

$\Rightarrow P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2 \Rightarrow P_1 - P_2 = \frac{1}{2}\rho(V_2^2 - V_1^2)$

Now, by equation of continuty.

$A_1V_1 = A_2V_2$

$\Rightarrow 2A_2V_1 = A_2V_2 \Rightarrow V_2 = 2V_1$

So, $P_1 - P_2 = \frac{1}{2}\rho 3V_1^2$

$\Rightarrow V_1^2 = \frac{2}{3}\frac{P_1 - P_2}{\rho} = \frac{2}{3} \times \frac{4500}{750} = \frac{9000}{2250} = 4$

So, $V_1 = 2$ m/s.

So, Volume flow rate $= A_1V_1 = 1.2 \times 10^{-2} \times 2$

$= 24 \times 10^{-3}\ m^3/s$

27. (300)

(300) By Bernoull's theorem

$P + \rho gh + \frac{1}{2}\rho V^2 = \text{constant}$

$\Rightarrow \left(P_0 + \frac{mg}{A}\right) + \rho gh = P_0 + \frac{1}{2}\rho V^2$

[$\because$ Speed of water in tank = 0]

$\Rightarrow \frac{25 \times 10}{0.5} + 1000 \times 10 \times 0.4 = \frac{1}{2} \times 1000 \times V^2$

$\Rightarrow 500 + 4000 = 500V^2 \Rightarrow V^2 = 9$

$\Rightarrow V = 3 \text{ m/s} = 300 \text{ cm/sec}$

28. (363)

(363) By equation of continuity,

$av_1 = \frac{a}{2}v_2 \Rightarrow v_2 = 2v_1$

From Bernoulli's theorem,

$P_1 + \rho gh_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$

$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) - \rho gh_1$

$4100 = \frac{1}{2} \times 800 \ (4v_1^2 - v_1^2) - 800 \times 10$

$4100 = 400 \ \times 3v_1^2 - 8000$

$12100 = 1200 \times v_1^2$

$v_1^2 = \frac{121}{12}$

$v_1^2 = \frac{121 \times 3}{12 \times 3} \Rightarrow v_1 = \frac{\sqrt{363}}{6}$ m/s.