EXERCISE 1(A)

INDEFINITE INTEGRATION

1 $\int \sin^2 (x/2)\, dx$ equals-

(A) $\frac{1}{2}$ (x + sin x) + c (B) $\frac{1}{2}$ (x + cos x) + c

(C) $\frac{1}{2}$ (x – sin x) + c (D) None of these

Sol. Here $I = \int \frac{1-\cos x}{2} dx = \frac{1}{2}$ (x – sin x) + c **Ans.[C]**

2 $\int \cot^2 x\, dx$ equals -

(A) – sec x + x + c (B) – cot x – x + c

(C) – sin x + x + c (D) None of these

Sol. $\int (\text{cosec}^2 x - 1)\, dx = - \cot x - x + c$ **Ans. [B]**

3 $\int \frac{5x+7}{x} dx$ equals-

(A) 5x + 7 log x (B) 7x + 5 log x +c

(C) 5x + 7 log x + c (D) None of these

Sol. $\int \frac{5x+7}{x} dx = \int \left(\frac{5x}{x} + \frac{7}{x}\right) dx$

$= \int 5\, dx + \int \frac{7}{x} = 5 \int 1 dx + 7 \int \frac{1}{x} dx = 5x + 7 \log x + c$ **Ans.[C]**

4 $\int \left(x - \frac{1}{x}\right)^3 dx$, (x > 0) equals-

(A) $\frac{x^3}{3} - \frac{3}{2}x^2 + 3 \log x + \frac{1}{2x^2} + c$ (B) $\frac{x^4}{3} - \frac{3}{2}x^2 + 3 \log x + \frac{1}{2x^2} + c$

(C) $\frac{x^4}{4} + 3 \log x + \frac{1}{2x^2} + c$ (D) None of these

Sol. $\int \left(x - \frac{1}{x}\right)^3 dx$

$= \int \left(x^3 - 3x^2.\frac{1}{x} + 3x.\frac{1}{x^2} - \frac{1}{x^3}\right) Edx$

$[\because (a-b)^3 = (a^3 - 3a^2b + 3ab^2 - b^3)]$

$= \int \left(x^3 - 3x + \frac{3}{x} - \frac{1}{x^3}\right) dx$

$= \int x^3\, dx - 3 \int x dx + 3 \int \frac{1}{x} dx - \int \frac{1}{x^3} dx = \frac{x^{3+1}}{3+1} - 3.\ \frac{x^{1+1}}{1+1} + 3\log x - \frac{x^{-3+1}}{-3+1} + c$

$= \frac{x^4}{4} - \frac{3}{2}x^2 + 3 \log x + \frac{1}{2x^2} + c$ **Ans.[B]**

5 The value of $\int\left(\frac{6}{1+x^2}+10^x\right)dx$ is -

(A) $6\tan^{-1}x + 10^x \log_e 10 + c$

(B) $6\tan^{-1}x + \frac{10^x}{\log_e 10} + c$

(C) $3\tan^{-1}x + \frac{10^x}{\log_e 10} + c$

(D) None of these

Sol. $\int\left(\frac{6}{1+x^2}+10^x\right)dx$

$= 6\int\frac{1}{1+x^2}dx + \int 10^x dx = 6\tan^{-1}x + \frac{10^x}{\log_e 10} + C$ **Ans.[B]**

6 $\int(\tan x + \cot x)^2 dx$ is equal to-

(A) $\tan x - \cot x + c$

(B) $\tan x + \cot x + c$

(C) $\cot x - \tan x + c$

(D) None of these

Sol. $I = \int(\tan^2 x + \cot^2 x + 2)dx$

$= \int(\sec^2 x + \text{cosec}^2 x)dx$

$= \tan x - \cot x + c$ **Ans. [A]**

7 $\int \sin 2x \sin 3x\, dx$ equals-

(A) $\frac{1}{2}(\sin x - \sin 5x) + c$

(B) $\frac{1}{10}(\sin x - \sin 5x) + c$

(C) $\frac{1}{10}(5\sin x - \sin 5x) + c$

(D) None of these

Sol. $I = \frac{1}{2}\int[\cos(-x) - \cos 5x]dx$

$= \frac{1}{2}\left[\sin x - \frac{\sin 5x}{5}\right] + c$

$= \frac{1}{10}[5\sin x - \sin 5x] + c$ **Ans. [C]**

8 $\int\frac{x^2}{x^2-1}dx$ equals-

(A) $x + \log\sqrt{\frac{x-1}{x+1}} + c$

(B) $x + \log\sqrt{\frac{x+1}{x-1}} + c$

(C) $x + \log\left(\frac{x-1}{x+1}\right) + c$

(D) $x + \log\left(\frac{x+1}{x-1}\right) + c$

Sol. $\int\frac{x^2-1+1}{x^2-1}dx$

$= \int\left(1+\frac{1}{x^2-1}\right)dx = x + \frac{1}{2}\log\left(\frac{x-1}{x+1}\right) + c$

$= x + \log\sqrt{\frac{x-1}{x+1}} + c$ **Ans.[A]**

9 $\int \frac{x^5}{\sqrt{1+x^3}} dx$ equals-

(A) $\frac{2}{9}(x^3-2)\sqrt{1+x^3}+c$ (B) $\frac{2}{9}(x^3+2)\sqrt{1+x^3}+c$

(C) $(x^3+2)\sqrt{1+x^3}+c$ (D) None of these

Sol. Put $1+x^3 = t^2 \Rightarrow 3x^2\,dx = 2\,t\,dt$

$\therefore I = \int \frac{x^3}{\sqrt{1+x^3}}(x^2\,dx) = \frac{2}{3}\int (t^2-1)\,dt$

$= \frac{2}{3}\left[\frac{t^3}{3} - t\right] + c$

$= \frac{2}{3}\left[\frac{1}{3}(1+x^3)^{3/2} - \sqrt{1+x^3}\right] + c$

$= \frac{2}{9}\sqrt{1+x^3}\,(1+x^3-3)+c$

$= \frac{2}{9}(x^3-2)\sqrt{1+x^3}+c$ **Ans. [A]**

10 $\int \frac{1}{x\log x} dx$ is equal to-

(A) $\log(x\log x)+c$ (B) $\log(\log x + x)+c$

(C) $\log x + c$ (D) $\log(\log x)+c$

Sol. $\int \frac{1}{x\log x}dx = \int \frac{1}{x}\cdot\frac{1}{\log x}dx$

put $\log x = t$, $\frac{1}{x}dx = dt$

$\therefore \int \frac{1}{x}\cdot\frac{1}{\log x}dx = \int \frac{1}{t}\,dt$

$\therefore \int \frac{1}{t}dt = \log t + c = \log(\log x)+c$

(putting the value of $t = \log x$) **Ans.[D]**

11 $\int \sec^2 x \cos(\tan x)\,dx$ equals-

(A) $\sin(\cos x)+c$ (B) $\sin(\tan x)+c$

(C) $\operatorname{cosec}(\tan x)+c$ (D) None of these

Sol. Let $\tan x = t$, then $\sec^2 x\,dx = dt$

$\therefore I = \int \cos t\,dt = \sin t + c$

$= \sin(\tan x)+c$ **Ans.[B]**

12 $\int \tan^n x \sec^2 x\,dx$ equals-

(A) $\frac{\tan^{n-1}x}{n-1}+c$ (B) $\frac{\tan^{n-1}x}{n+1}+c$

(C) $\tan^{n+1}x+c$ (D) None of these

Sol. $\int \tan^n x \sec^2 x\,dx$

putting $\tan x = t$, $\sec^2 x\,dx = dt$

$\int \tan^n x \sec^2 x\, dx = \int t^n\, dt = \frac{\tan^{n+1}}{n+1} + c$

$= \frac{(\tan x)^{n+1}}{n+1} + c$ **Ans.[B]**

13 $\int \frac{\sin 2x}{1+\cos^4 x} dx$ is equal to-

(A) $\cos^{-1}(\cos^2 x) + c$

(B) $\sin^{-1}(\cos^2 x) + c$

(C) $\cot^{-1}(\cos^2 x) + c$

(D) None of these

Sol. Here differential coefficient of

$\cos^2 x$ is $-\sin 2x$

Let $\cos^2 x = t$

$\therefore 2\cos x(-\sin x)\, dx = dt$

or $\sin 2x\, dx = -dt$

$\therefore \int \frac{\sin 2x}{1+\cos^4 x} dx = \int \frac{-dt}{1+t^2}$

$= \cot^{-1} t + c$

$= \cot^{-1}(\cos^2 x) + c$ **Ans.[C]**

14 $\int \frac{be^x}{\sqrt{a+be^x}} dx$ equals-

(A) $\frac{2}{b}\sqrt{a+be^x} + c$

(B) $\frac{1}{b} \cdot \sqrt{a+be^x} + c$

(C) $2\sqrt{a+be^x} + c$

(D) None of these

Sol. $\int \frac{be^x}{\sqrt{a+be^x}} dx$, putting $a + be^x = t$

$be^x\, dx = dt$

$\therefore \int \frac{be^x}{\sqrt{a+be^x}} dx = \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} + c$

$= 2\sqrt{a+be^x} + c$ **Ans.[C]**

15 $\int \sqrt{\frac{1+\cos x}{1-\cos x}} dx$ equals-

(A) $\log \cos\left(\frac{x}{2}\right) + c$

(B) $2\log \sin\left(\frac{x}{2}\right) + c$

(C) $2\log \sec\left(\frac{x}{2}\right) + c$

(D) None of these

Sol. $I = \int \sqrt{\frac{1+\cos x}{1-\cos x}}\, dx$

$= \int \sqrt{\frac{2\cos^2(x/2)}{2\sin^2(x/2)}}\, dx$

$= \int \cot\left(\frac{x}{2}\right) dx$

$= 2\log \sin\left(\frac{x}{2}\right) + c$ **Ans.[B]**

16 $\int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$ equals-

(A) $2\sqrt{\sec x} + c$

(B) $2\sqrt{\tan x} + c$

(C) $2/\sqrt{\tan x} + c$

(D) $2/\sqrt{\sec x} + c$

Sol. $I = \int \frac{\sqrt{\tan x}}{\tan x} \sec^2 x \, dx$

$= \int \frac{\sec^2 x}{\sqrt{\tan x}} dx = 2\sqrt{\tan x} + c$ **Ans. [B]**

17 $\int \sin^5 x. \cos^3 x \, dx$ is equal to-

(A) $\frac{\sin^6 x}{6} - \frac{\sin^8 x}{8} + c$

(B) $\frac{\cos^6 x}{6} - \frac{\cos^8 x}{8} + c$

(C) $\frac{\cos^6 x}{6} - \frac{\sin^8 x}{8} + c$

(D) None of these

Sol. $\int \sin^5 x . \cos^3 x \, dx$

Assumed that $\sin x = t$

$\therefore \cos x \, dx = dt$

$= \int t^5(1 - t^2) \, dt = \int (t^5 - t^7) \, dt$

$= \frac{t^6}{6} - \frac{t^8}{8} + c = \frac{\sin^6 x}{6} - \frac{\sin^8 x}{8} + c$ **Ans.[A]**

18 $\int \frac{x^2}{1+x^6} dx$ is equal to-

(A) $\tan^{-1} x^3 + c$

(B) $\tan^{-1} x^2 + c$

(C) $\frac{1}{3} \tan^{-1} x^3 + c$

(D) $3 \tan^{-1} x^3 + c$

Sol. Put $x^3 = t \Rightarrow x^2 \, dx = \frac{1}{3} \, dt$

$\therefore I = \frac{1}{3} \int \frac{dt}{1+t^2} = \frac{1}{3} \tan^{-1} x^3 + c$ **Ans. [C]**

19 $\int \sqrt{\frac{1+x}{1-x}} \, dx$ equals-

(A) $\sin^{-1} x + \sqrt{1-x^2} + c$

(B) $\sin^{-1} x + \sqrt{x^2 - 1} + c$

(C) $\sin^{-1} x - \sqrt{1-x^2} + c$

(D) $\sin^{-1} x - \sqrt{x^2 - 1} + c$

Sol $I = \int \sqrt{\frac{1+x}{1-x}} \, dx$

$= \int \frac{dx}{\sqrt{1-x^2}} - \frac{1}{2} \int \frac{-2x \, dx}{\sqrt{1-x^2}}$

$= \sin^{-1} x - \sqrt{1-x^2} + c$ **Ans. [C]**

20 The primitive of log x will be-

(A) $x \log (e + x) + c$ (B) $x \log\left(\frac{e}{x}\right) + c$

(C) $x \log\left(\frac{x}{e}\right) + c$ (D) $x \log (ex) + c$

Sol. $\int \log x\, dx = \int \log x.1\, dx$

[Integrating by parts, taking log x as first part and 1 as second part]

$= (\log x).x - \int\left\{\frac{d(\log x)}{dx}\right\}. x\, dx$

$= x \log x - \int \frac{1}{x} .x\, dx = (x \log x - x) + c$

$= x (\log x - 1) + c = \log\left(\frac{x}{e}\right) + c$ **Ans. [C]**

21 $\int x \tan^{-1} x$ is equal to-

(A) $\frac{1}{2}(x^2 + 1) \tan^{-1} x - x + c$ (B) $\frac{1}{2}(x^2 + 1) \tan^{-1} x + x + c$

(C) $\frac{1}{2}(x^2 + 1) \tan^{-1} x - \frac{1}{2}x + c$ (D) $\frac{1}{2}(x^2 - 1) \tan^{-1} x - \frac{1}{2}x + c$

Sol. Integrating by parts taking x as second part

$I = \frac{x^2}{2} \tan^{-1} x - \int \frac{1}{1+x^2} . \frac{x^2}{2}\, dx$

$= \frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2}\left(1 - \frac{1}{1-x^2}\right)dx$

$= \frac{1}{2}x^2 \tan^{-1} x - \frac{1}{2}x + \frac{1}{2}\tan^{-1} x + c$

$= \frac{1}{2}(x^2 + 1) \tan^{-1} x - \frac{1}{2} x + c$ **Ans. [C]**

22 $\int \sin (\log x)\, dx$ equals-

(A) $\frac{x}{\sqrt{2}} \sin (\log x + \frac{\pi}{8}) + c$ (B) $\frac{x}{\sqrt{2}} \sin (\log x - \frac{\pi}{4}) + c$

(C) $\frac{x}{\sqrt{2}} \cos (\log x - \frac{\pi}{4}) + c$ (D) None of these

Sol. $\int \sin (\log x)\, dx$, assumed that $x = e^t$

$\therefore dx = e^t\, dt$

$= \int \sin t.e^t.dt$

$= \frac{e^t}{\sqrt{1+1}} \sin(t - \tan^{-1} 1) + c$

$\Rightarrow \int \sin (\log x)\, dx$

$= \frac{x}{\sqrt{2}} \sin (\log x - \frac{\pi}{4}) + c$ **Ans. [B]**

23 $\int \frac{\sqrt{x}-\sqrt{a}}{\sqrt{x+a}}\,dx$ equals-

(A) $\sqrt{x^2+ax} - 2\sqrt{ax+a^2} - a\cosh^{-1}\left(\sqrt{\frac{x+a}{a}}\right) + c$

(B) $\sqrt{x^2+ax} + \sqrt{ax+a^2} - a\cosh^{-1} + \left(\sqrt{\frac{x+a}{a}}\right)c$

(C) $\sqrt{x^2+ax} - 2\sqrt{ax+a^2} + a\cosh^{-1}\left(\sqrt{\frac{x+a}{a}}\right) + c$

(D) None of these

Sol. Let $x = a\tan^2\theta \Rightarrow dx = 2a\tan\theta\sec^2\theta\,d\theta$

$\therefore I = \int \frac{\sqrt{a}(\tan\theta-1).2a\tan\theta\sec^2\theta}{\sqrt{a}\sec\theta}\,d\theta$

$= 2a\left[\int \tan^2\theta\sec\theta\,d\theta - \int \sec\theta\tan\theta\,d\theta\right]$

$= 2a\left[\int \sqrt{\sec^2\theta-1}\tan\theta\sec\theta\,d\theta - \sec\theta\right] = 2a\int\sqrt{t^2-1}\,dt - 2a\sec\theta + c$ [Where $\sec\theta = t$]

$= 2a\left[\frac{t}{2}\sqrt{t^2-1} - \frac{1}{2}\cosh^{-1}(t)\right] - 2a\sqrt{\frac{a+x}{a}} + c$

$= a\sqrt{\frac{x+a}{a}.\frac{x}{a}} - a\cosh^{-1}\left(\sqrt{\frac{x+a}{a}}\right) - 2\sqrt{ax+a^2} + c$

$= \sqrt{x^2+ax} - 2\sqrt{ax+a^2} - a\cosh^{-1}\left(\sqrt{\frac{x+a}{a}}\right) + c$

Ans. [A]

24 $\int x^3(\log x)^2\,dx$ equals-

(A) $\frac{1}{32}x^4[8(\log x)^2 - 4\log x + 1] + c$

(B) $\frac{1}{32}x^4[8(\log x)^2 - 4\log x - 1] + c$

(C) $\frac{1}{32}x^4[8(\log x)^2 + 4\log x + 1] + c$

(D) None of these

Sol. Integrating by parts taking x^3 as second part

$I = \frac{1}{4}x^4(\log x)^2 - \frac{1}{2}\int x^3\log x\,dx$

$= \frac{1}{4}x^4(\log x)^2 - \frac{1}{2}\left(\frac{1}{4}x^4\log x - \frac{1}{16}x^4\right) + c$

$= \frac{1}{32}x^4[8(\log x)^2 - 4\log x + 1] + c$ **Ans. [A]**

25 The value of $\int x\sec x\tan x\,dx$ is-

(A) $x\sec x + \log(\sec x + \tan x) + c$

(B) $x\sec x - \log(\sec x - \tan x) + c$

(C) $x\sec x + \log(\sec x - \tan x) + c$

(D) None of the above

Sol. $\int x.(\sec x\tan x)\,dx$

$= (x.\sec x) - \int (1.\sec x)\,dx$

(Integrating by parts, taking x as first function)

$= x \sec x - \log(\sec x + \tan x) + c$

$= x \sec x - \log\left\{(\sec x + \tan x)\dfrac{\sec x - \tan x}{\sec x - \tan x}\right\} + c$

$= x \sec x - \log\left(\dfrac{\sec^2 x - \tan^2 x}{\sec x - \tan x}\right) + c$

$= x \sec x + \log(\sec x - \tan x) + c$ **Ans. [C]**

26 $\int \dfrac{\sin^{-1}\sqrt{x}}{\sqrt{1-x}}\,dx$ equals-

(A) $2[\sqrt{x} - \sqrt{1-x}\,\sin^{-1}\sqrt{x}\,] + c$

(B) $2[\sqrt{x} + \sqrt{1-x}\,\sin^{-1}\sqrt{x}\,] + c$

(C) $[\sqrt{x} - \sqrt{1-x}\,\sin^{-1}\sqrt{x}\,] + c$

(D) None of these

Sol. Let $x = \sin^2 t$, then

$dx = 2 \sin t \cos t\, dt$

$\therefore I = \int \dfrac{t}{\cos t} \,.\, 2 \sin t \cos t\, dt$

$= 2\int t \sin t\, dt$

$= 2[-t \cos t + \sin t] + c \quad = 2[\sqrt{x} - \sqrt{1-x}\,\sin^{-1}\sqrt{x}\,] + c$ **Ans. [A]**

27 $\int e^x \dfrac{x-1}{(x+1)^3}\,dx$ equals-

(A) $-\dfrac{e^x}{x+1} + c$

(B) $\dfrac{e^x}{x+1} + c$

(C) $\dfrac{e^x}{(x+1)^2} + c$

(D) $-\dfrac{e^x}{(x+1)^2} + c$

Sol. $I = \int e^x\left[\dfrac{x+1-2}{(x+1)^3}\right]dx$

$= \int e^x\left(\dfrac{1}{(x+1)^2} - \dfrac{2}{(x+1)^3}\right)dx$

Thus the given integral is of the form

$= \int e^x\{f(x) + f'(x)\}\,dx$

$\therefore I = e^x f(x) = \dfrac{e^x}{(x+1)^2} + c$ **Ans.[C]**

28 $\int \sec^3\theta\, d\theta$ is equal to-

(A) $\frac{1}{2}[\tan\theta \sec\theta + \log(\tan\theta + \sec\theta)] + c$

(B) $\frac{1}{2}\tan\theta \sec\theta + \log(\tan\theta + \sec\theta) + c$

(C) $\frac{1}{2}$ [tan θ sec θ – log (tanθ + secθ)] + c

(D) None of these

Sol. $I = \int$ sec. $\sec^2\theta$. dθ

$= \int \sqrt{\tan^2\theta + 1}\,\sec^2\theta\, d\theta$

$= \int \sqrt{t^2+1}\, dt$, where t = tan θ

$= \frac{t}{2}\sqrt{t^2+1} + \frac{1}{2}\log\left(t + \sqrt{t^2+1}\right) + c$

$= \frac{1}{2}$ [tanθ secθ + log (tanθ + secθ)] + c

Ans. [A]

29 $\int \frac{\cos x + x\sin x}{x(x+\cos x)}\, dx$ is equal to-

(A) log {x (x + cos x)}+ c

(B) $\log\left(\frac{x}{x+\cos x}\right) + c$

(C) $\log\left(\frac{x+\cos x}{x+\cos x}\right) + c$

(D) None of these

Sol. $I = \int \frac{(x+\cos x) - x + x\sin x}{x(x+\cos x)}\, dx$

$= \int \frac{1}{x}\, dx - \int \frac{1-\sin x}{x+\cos x}\, dx$

= log x – log (x + cos x) + c

$= \log\left(\frac{x}{x+\cos x}\right) + c$ **Ans. [B]**

30 $\int \sqrt{\sec x - 1}\, dx$ is equal to-

(A) $2\sin^{-1}(\sqrt{2}\cos x/2) + c$

(B) $-2\sinh^{-1}(\sqrt{2}\cos x/2) + c$

(C) $-2\cosh^{-1}(\sqrt{2}\cos x/2) + c$

(D) None of these

Sol. $I = \int \sqrt{\frac{1-\cos x}{\cos x}}\, dx$

$= \int \frac{\sqrt{2}\sin x/2}{\sqrt{2\cos^2 x/2 - 1}}\, dx$

$= -2\int \frac{dt}{\sqrt{t^2-1}}$ where $t = \sqrt{2}\cos x/2$

$= -2\cosh^{-1} t + c$

$= -2\cosh^{-1} + (\sqrt{2}\cos x/2) + c$ **Ans. [C]**

31 $\int \frac{x^2+1}{(x-1)(x-2)}\, dx$ equals-

(A) $\log\left[\frac{(x-2)^5}{(x-1)^2}\right] + c$

(B) $x + \log\left[\frac{(x-2)^5}{(x-1)^2}\right] + c$

(C) $x + \log\left[\frac{(x-1)^5}{(x-2)^5}\right] + c$ (D) None of these

Sol. Here since the highest powers of x in Num^r and Den^r are equal and coefficients of x^2 are also equal,

therefore $\frac{x^2+1}{(x-1)(x-2)} \equiv 1 + \frac{A}{x-1} + \frac{B}{x-2}$

On solving we get A = – 2, B = 5

Thus $\frac{x^2+1}{(x-1)(x-2)} \equiv 1 - \frac{2}{x-1} + \frac{5}{x-2}$

The above method is used to obtain the value of constant corresponding to non repeated linear factor in the Den^r.

Now $I = \left(1 - \frac{2}{x-1} + \frac{5}{x-2}\right)dx$

$= x - 2\log(x-1) + 5\log(x-2) + c$

$= x + \log\left[\frac{(x-2)^5}{(x-1)^2}\right] + c$ **Ans.[B]**

32 The value of $\int \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$ is-

(A) $\frac{1}{b^2-a^2}\left[b\tan^{-1}\frac{x}{b} - a\tan^{-1}\frac{x}{a}\right] + c$ (B) $\frac{1}{b^2-a^2}\left[a\tan^{-1}\frac{x}{b} - b\tan^{-1}\frac{x}{a}\right] + c$

(C) $\frac{1}{b^2-a^2}\left[b\tan^{-1}\frac{x}{b} + a\tan^{-1}\frac{x}{a}\right] + c$ (D) None of these

Sol. Putting $x^2 = y$ in integrand, we obtain

$\frac{y}{(y+a^2)(y+b^2)} = \frac{1}{b^2-a^2}\left[\frac{b^2}{y+b^2} - \frac{a^2}{y+a^2}\right]$

$\therefore I = \frac{1}{b^2-a^2} \cdot \left[\int \frac{b^2}{x^2+b^2}dx - \int \frac{a^2}{x^2+a^2}dx\right]$

$= \frac{1}{b^2-a^2}\left[b\tan^{-1}\frac{x}{b} - a\tan^{-1}\frac{x}{a}\right] + c$

Ans.[A]

33 $\int \frac{dx}{3x^2+2x+1}$ equals-

(A) $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{3x+1}{\sqrt{2}}\right) + c$ (B) $\frac{1}{\sqrt{2}}\sin^{-1}\left(\frac{3x+1}{\sqrt{2}}\right) + c$

(C) $\frac{1}{\sqrt{2}}\cot^{-1}\left(\frac{3x+1}{\sqrt{2}}\right) + c$ (D) None of these

Sol. $I = \frac{1}{3}\frac{dx}{x^2 + \frac{2}{3}x + \frac{1}{3}}$

$= \frac{1}{3}\int \frac{dx}{\left(x+\frac{1}{3}\right)^2 + \frac{2}{9}}$

$$= \frac{1}{3} \times \frac{3}{\sqrt{2}} \tan^{-1} + \left(\frac{x + \left(\frac{1}{3}\right)}{\sqrt{2}/3} \right) c$$

$$= \frac{1}{\sqrt{2}} \tan^{-1} \left(\frac{3x+1}{\sqrt{2}} \right) + c$$

Ans.[A]

34 $\int \sqrt{1 + x - 2x^2}\, dx$ equals-

(A) $\frac{1}{8}(4x - 1)\sqrt{1 + x - 2x^2} + \frac{9\sqrt{2}}{32} \sin^{-1}\left(\frac{4x-1}{3}\right) + c$

(B) $\frac{1}{8}(4x + 1)\sqrt{1 + x - 2x^2} - \frac{9\sqrt{2}}{32} \sin^{-1}\left(\frac{4x-1}{3}\right) + c$

(C) $\frac{1}{8}(4x - 1)\sqrt{1 + x - 2x^2} + \frac{9\sqrt{2}}{32} \cos^{-1}\left(\frac{4x-1}{3}\right) + c$

(D) None of these

Sol. $I = \sqrt{2} \int \sqrt{\frac{1}{2} - \left(x^2 - \frac{x}{2}\right)}\, dx$

$$= \sqrt{2} \int \sqrt{\left\{\frac{9}{16} - \left(x - \frac{1}{4}\right)^2\right\}}\, dx$$

$$= \sqrt{2} \left[\frac{1}{2}\left(x - \frac{1}{4}\right) \sqrt{\left\{\frac{9}{16} - \left(x - \frac{1}{4}\right)^2\right\}} \right.$$

$$\left. + \frac{9}{32} \sin^{-1}\left\{\frac{4}{3}\left(x - \frac{1}{4}\right)\right\} \right] + c$$

$$= \frac{1}{8}(4x - 1)\sqrt{1 + x - 2x^2} + \frac{9\sqrt{2}}{32} \sin^{-1}\left(\frac{4x-1}{3}\right) + c$$

Ans. [A]

35 $\int \frac{dx}{\sqrt{3 - 5x - x^2}}$ equals-

(A) $\sin^{-1}\left(\frac{2x+5}{\sqrt{37}}\right) + c$

(B) $\cos^{-1}\left(\frac{2x+5}{\sqrt{37}}\right) + c$

(C) $\sin^{-1}(2x + 5) + c$

(D) None of these

Sol. $I = \int \frac{dx}{\sqrt{\frac{37}{4} - \left(x + \frac{5}{2}\right)^2}}$

$$= \sin^{-1}\left(\frac{x + 5/2}{\sqrt{37}/2}\right) + c = \sin^{-1}\left(\frac{2x+5}{\sqrt{37}}\right) + c$$

Ans. [A]

36 $\int \sqrt{e^{2x} - 1}\, dx$ is equal to-

(A) $\sqrt{e^{2x} - 1} + \sec^{-1} e^{2x} + c$

(B) $\sqrt{e^{2x} - 1} - \sec^{-1} e^{2x} + c$

(C) $\sqrt{e^{2x} - 1} - \sec^{-1} e^{x} + c$

(D) None of these

Sol. $\int \frac{e^{2x}-1}{\sqrt{e^{2x}-1}}\,dx$

$= \frac{1}{2}\int \frac{2e^{2x}}{\sqrt{e^{2x}-1}}\,dx - \int \frac{e^{x}}{e^{x}\sqrt{e^{2x}-1}}\,dx$

$= \sqrt{e^{2x}-1} - \sec^{-1} e^{x} + c$ **Ans.[C]**

37 $\int \sqrt{\frac{e^{x}+a}{e^{x}-a}}\,dx$ is equal to-

(A) $\cos h^{-1}\left(\frac{e^{x}}{a}\right) + \sec^{-1}\left(\frac{e^{x}}{a}\right) + c$

(B) $\sin h^{-1}\left(\frac{e^{x}}{a}\right) + \sec^{-1}\left(\frac{e^{x}}{a}\right) + c$

(C) $\tan h^{-1}\left(\frac{e^{x}}{a}\right) + \cos^{-1}\left(\frac{e^{x}}{a}\right) + c$

(D) None of these

Sol. $\int \frac{e^{x}+a}{\sqrt{e^{2x}-a^{2}}}\,dx$

$= \int \frac{e^{x}}{\sqrt{e^{2x}-a^{2}}}\,dx + a\int \frac{e^{x}}{e^{x}\sqrt{e^{2x}-a^{2}}}\,dx$

$= \cosh^{-1}\left(\frac{e^{x}}{a}\right) + \sec^{-1}\left(\frac{e^{x}}{a}\right) + c$ **Ans.[A]**

38 $\int \frac{dx}{4\sin^{2}x + 4\sin x\cos x + 5\cos^{2}x}$ is equal to-

(A) $\tan^{-1}\left(\tan x + \frac{1}{2}\right) + c$

(B) $\frac{1}{4}\tan^{-1}\left(\tan x + \frac{1}{2}\right) + c$

(C) $4\tan^{-1}\left(\tan x + \frac{1}{2}\right) + c$

(D) None of these

Sol. After dividing by $\cos^{2} x$ to numerator and denominator of integration

$I = \int \frac{\sec^{2}x\,dx}{4\tan^{2}x + 4\tan x + 5}$

$= \int \frac{\sec^{2}x\,dx}{(2\tan x + 1)^{2} + 4}$

$= \frac{1}{2.2}\tan^{-1}\left(\frac{2\tan x + 1}{2}\right) + c$ **Ans. [B]**

39 $\int \left(\frac{1-x}{1+x}\right)^{2} dx$ is equal to-

(A) $x - 4\log(x+1) + \frac{4}{x+1} + c$

(B) $x - \log(x+1) + \frac{4}{x+1} + c$

(C) $x - 4\log(x+1) - \frac{4}{x+1} + c$

(D) $x + \log(x+1) - \frac{4}{x+1} + c$

Sol. $\int \frac{[2-(x+1)]^{2}}{(x+1)^{2}}\,dx$

$= \int\left[\frac{4}{(x+1)^2} - \frac{4}{x+1} + 1\right]dx$

$= -\frac{4}{x+1} - 4\log(x+1) + x + c$ **Ans. [C]**

40 $\int \frac{e^x}{e^{2x} + 5e^x + 6}$ equals-

(A) $\log\left(\frac{e^x+3}{e^x+2}\right) + c$

(B) $\log\left(\frac{e^x+2}{e^x+3}\right) + c$

(C) $\frac{1}{2}\log\left(\frac{e^x+2}{e^x+3}\right) + c$

(D) None of these

Sol. Put $e^x = t \Rightarrow e^x\,dx = dt$

$\therefore I = \int \frac{dt}{t^2 + 5t + 6} = \int \frac{dt}{(1+2)(t+3)}$

$= \int\left(\frac{1}{t+2} - \frac{1}{t+3}\right)dt$

$= \log\left(\frac{t+2}{t+3}\right) + c = \log\left(\frac{e^x+2}{e^x+3}\right) + c$

Ans. [B]

41 $\int \frac{dx}{x + \sqrt{x}}$ equals-

(A) $2\log(\sqrt{x} - 1) + c$

(B) $2\log(\sqrt{x} + 1) + c$

(C) $\tan^{-1} x + c$

(D) None of these

Sol. $I = \int \frac{dx}{x + \sqrt{x}}$

$= \int \frac{2t\,dt}{t^2 + t}$ where $t^2 = x$

$= 2\int \frac{dt}{t+1} = 2\log(\sqrt{x} + 1) + c$ **Ans.[B]**

42 $I = \int \frac{4e^x + 6e^{-x}}{9e^x - 4e^{-x}}\,dx$ is equal to-

(A) $\frac{19}{36}x + \frac{35}{36}\log(9e^x - 4e^{-x}) + c$

(B) $-\frac{19}{36}x + \frac{35}{36}\log(9e^x - 4e^{-x}) + c$

(C) $\frac{1}{36}x + \frac{1}{36}\log(9e^x - 4e^{-x}) + c$

(D) None of these

Sol. Suppose $4e^x + 6e^{-x} = A(9e^x - 4e^{-x}) + B(9e^x + 4e^{-x})$

By comparing $4 = 9A + 9B$,
$6 = -4A + 4B$

or $A + B = \frac{4}{9}$, $-A + B = \frac{3}{2}$

After solving $A = -\frac{19}{36}$, $B = \frac{35}{36}$

$$\therefore I = \int\left[-\frac{19}{36}+\frac{35}{36}\left(\frac{9e^{x}+4e^{-x}}{9e^{x}-4e^{-x}}\right)\right]dx$$

$$= -\frac{19}{36}x + \frac{35}{36}\log(9e^{x} - 4e^{-x}) + c$$ **Ans.[B]**

DEFINITE INTEGRATION

43 $\int_{0}^{\pi/2} |\sin x - \cos x|\, dx$ equals-

(A) $2\sqrt{2}$ (B) $2(\sqrt{2}+1)$

(C) $2(\sqrt{2}-1)$ (D) 0

Sol. $\because |\sin x - \cos x|$

$$= \begin{cases} -(\sin x - \cos x), & 0 < x < \pi/4 \\ (\sin x - \cos x), & \pi/4 < x < \pi/2 \end{cases}$$

$$\therefore I = \int_{0}^{\pi/4} -(\sin x - \cos x)\,dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)\,dx$$

$$= [\cos x + \sin x]_{0}^{\pi/4} + [-\cos x - \sin x]_{\pi/4}^{\pi/2}$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - 1 - 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$$

$$= 2\sqrt{2} - 2$$ **Ans.[C]**

44 The value of $\lim_{x\to 0} \frac{\int_{0}^{x} \cos t^{2}\,dt}{x}$ is-

(A) 0 (B) 1

(C) –1 (D) None of these

Sol. Let $f(x) = \int_{0}^{x} \cos t^{2}\,dt$ and $g(x) = x$,

then $f(0) = g(0) = 0$

$$\therefore \lim_{x\to 0}\frac{f(x)}{g(x)} = \lim_{x\to 0}\frac{f'(x)}{g'(x)}$$

$$\therefore \text{Given limit} = \lim_{x\to 0}\frac{\cos x^{2}.1 - \cos 0.0}{1}$$

$$\left[\text{since} \frac{d}{dx}\int_{\phi(x)}^{\psi(x)} f(t)\,dt = \int_{\phi(x)}^{\psi(x)} \frac{d}{dx}(f(t))\,dt \right.$$

$$\left. = f(\psi(x)\psi'(x) - f\{(\phi(x)\phi'(x)\}\right]$$

$\therefore$ Given limit

$= \cos 0 = 1.$ **Ans.[B]**

45 If $n \in Z$, then

$$\int_0^{\pi} e^{\sin^2 x} \cos^3 (2n+1)x \, dx =$$

(A) –1 (B) 0

(C) 1 (D) π

Sol. Let $f(x) = e^{\sin^2 x} \cos^3 (2n+1)x \, dx$

$\Rightarrow f(\pi - x) = e^{\sin^2 (\pi - x)} \cos^3 (2n+1)(\pi - x) \, dx$

$= - e^{\sin^2 x} \cos^3 (2n+1)x$

$[\because (2n + 1)$ is odd$]$

$= - f(x)$

So by P – 8, I = 0 **Ans.[B]**

46 $\int_0^1 \frac{6x^2+1}{4x^3+2x+3} dx$ is equal to-

(A) $-\frac{1}{2}\log 3$ (B) $\frac{1}{2}\log 3$

(C) 2 log 3 (D) None of these

Sol. Let $4x^3 + 2x + 3 = t$ $\therefore 2(6x^2 + 1)dx = dt$

Limits –at x = 0; t = 3, at x = 1; t = 9

$\therefore I = \int_3^9 \frac{1}{2}\frac{dt}{t} = \frac{1}{2}[\log t]_3^9$

$= \frac{1}{2}[\log 9 - \log 3] = \frac{1}{2}\log 3$ **Ans.[B]**

47 $\int_0^1 \frac{x}{1+x^4} dx$ is equal to -

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{8}$ (D) π

Sol. $I = \frac{1}{2}\int_0^1 \frac{2x}{1+(x^2)^2} dx$

$= \frac{1}{2}[\tan^{-1} x^2]_0^1$

$= \frac{1}{2}[\tan^{-1} 1 - \tan^{-1} 0]$

$= \frac{1}{2}\left[\frac{\pi}{4} - 0\right] = \frac{\pi}{8}$ **Ans.[C]**

48 $\int_2^4 \frac{\sqrt{x^2-4}}{x} dx$ is equal to

(A) $2(3\sqrt{3} - \pi)$ (B) $2\sqrt{3} - \pi$

(C) $\frac{2}{3}(3\sqrt{3} - \pi)$ (D) π

Sol. Put x = 2 sec t, then

$$I = \int_0^{\pi/3} \frac{2\tan t}{2\sec t}.2\sec t \tan t\, dt$$

$$= 2\int_0^{\pi/3} \tan^2 t\, dt$$

$$= 2\int_0^{\pi/3} (\sec^2 t - 1)\, dt = 2[\tan t - t]_0^{\pi/3}$$

$$= 2[\sqrt{3} - \pi/3] = \frac{2}{3}(3\sqrt{3} - \pi)$$ **Ans. [C]**

49 $\int_0^{\pi^2/4} \frac{\sin\sqrt{x}}{\sqrt{x}}\, dx$ is equal to

(A) 2 (B) 1
(C) $\pi/4$ (D) $\pi^2/8$

Sol. $\sqrt{x} = t$, $\frac{1}{\sqrt{x}}\, dx = 2dt$

$$\therefore I = 2\int_0^{\pi/2} \sin t\, dt = 2(-\cos t)_0^{\pi/2} = 2(0 + 1) = 2$$

Ans. [A]

50 If $f(x) = \begin{cases} 2x+1, & 0 < x < 1 \\ x^2 + 2, & 1 \le x < 2 \end{cases}$, then the value of $\int_0^2 f(x)\, dx$ is-

(A) $-\frac{19}{3}$ (B) $\frac{19}{3}$

(C) $\frac{3}{19}$ (D) None of these

Sol. $\int_0^2 f(x)\, dx = \int_0^1 f(x)\, dx + \int_1^2 f(x)\, dx$

$$= \int_0^1 (2x+1)\, dx + \int_1^2 (x^2+2)\, dx$$

$$= \left[x^2 + x\right]_0^1 + \left[\frac{x^3}{3} + 2x\right]_1^2$$

$$= 2 - 0 + \left(\frac{20}{3} - \frac{7}{3}\right) = \frac{19}{3}$$

Ans.[B]

51 $\int_{-4}^{-5} e^{(x+5)^2}\, dx + 3\int_{1/3}^{2/3} e^{9(x-2/3)^2}\, dx$ is equal to-

(A) e^5 (B) e^4
(C) $3e^2$ (D) 0

Sol. Putting x = –t –4 in first integral and

$x = \frac{t}{3} + \frac{1}{3}$ in second integral

$I_1 = \int_{-4}^{-5} e^{(x+5)^2} dx = -\int_0^1 e^{(-t+1)^2} dt = -\int_0^1 e^{(t-1)^2} dt$

$I_2 = 3\int_{1/3}^{2/3} e^{9(x-2/3)^2} dx$

$= 3\int_0^1 e^{9(t/3-1/3)^2} dt = \int_0^1 e^{(t-1)^2} dt$

$\therefore I = I_1 + I_2 = 0.$ **Ans.[D]**

52 $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}} dx$ is equal to

(A) $\pi/2$ (B) $\pi/4$
(C) π (D) 2π

Sol. Using prop. P–4, we have

$I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx$

Adding it to given integral we have

$2I = \int_0^{\pi/2} dx = [x]_0^{\pi/2} = \pi/2$

$\therefore I = \pi/4$ **Ans.[B]**

53 If f(x) is an odd function of x, then $\int_{-\pi/2}^{\pi/2} f(\cos x) dx$ is equal to

(A) 0 (B) $\int_0^{\pi/2} f(\cos x) dx$

(C) $2\int_0^{\pi/2} f(\sin x) dx$ (D) $\int_0^{\pi} f(\cos x) dx$

Sol. Here f(cos x) will be even function of x,

$I = \int_{-\pi/2}^{\pi/2} f(\cos x) dx = 2\int_0^{\pi/2} f(\cos x) dx$

$= 2\int_0^{\pi/2} f(\sin x) dx$ **Ans.[C]**

54 The value of the integral $\int_{-4}^{4} (ax^3 + bx + c) dx$ depend on-

(A) b and c (B) a, b and c
(C) only c (D) a and c

Sol. $I = \int_{-4}^{4} (ax^3 + bx) dx + \int_{-4}^{4} c\, dx$

$$= 0 + 2\int_0^4 c\,dx \quad (\text{by } P-5)$$

$$= 2c\,[x]_0^4 = 8c$$

Hence the value of I depends on c.

Ans.[C]

55 If $f(x) = \dfrac{x\cos x}{1+\sin^2 x}$, then $\int_{-\pi}^{\pi} f(x)\,dx$ equals-

(A) $\pi/4$ (B) $\pi/2$

(C) π (D) 0

Sol. Since $f(-x) = \dfrac{-x\cos(-x)}{1+\sin^2(\pi-x)}$

$$= \frac{-x\cos x}{1+\sin^2 x} = -f(x)$$

$$\therefore I = \int_{-\pi}^{\pi} f(x)\,dx = 0$$

Ans.[D]

56 $\int_0^{\pi/2} \sin^2 x\cos^3 x\,dx$ equals-

(A) 1 (B) 2/5

(C) 2/15 (D) 4/15

Sol. Using Walli's formula, we get

$$I = \frac{1.2}{5.3.1} = \frac{2}{15}$$

Ans.[C]

57 $\int_{\pi/4}^{3\pi/4} \dfrac{\phi}{1+\sin\phi}\,d\phi$ equals-

(A) $\pi(\sqrt{2}-1)$ (B) $\pi(\sqrt{2}+1)$

(C) $\pi(2-\sqrt{2})$ (D) None of these

Sol. $I = \int_{\pi/4}^{3\pi/4} \dfrac{\phi}{1+\sin\phi}\,d\phi$ ...(1)

$$\Rightarrow I = \int_{\pi/4}^{3\pi/4} \frac{\pi-\phi}{1+\sin(\pi-\phi)}\,d\phi \quad (\text{by } P-8)$$

$$= \int_{\pi/4}^{3\pi/4} \frac{\pi-\phi}{1+\sin\phi}\,d\phi \quad ...(2)$$

$$2I = \int_{\pi/4}^{3\pi/4} \frac{\pi}{1+\sin\phi}\,d\phi = \pi\int_{\pi/4}^{3\pi/4} \frac{1-\sin\phi}{\cos^2\phi}\,d\phi$$

$$= \pi\,[\tan\phi - \sec\phi]_{\pi/4}^{3\pi/4} = 2\pi\,(-\sqrt{2}-1)$$

$$I = \pi(-\sqrt{2}-1)$$

Ans.[A]

58 $\int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos x}$ is equal to-

(A) 2 (B) – 2

(C) 1/2 (D) – 1/2

Sol. By property [P–8]

$$I = \int_{\pi/4}^{3\pi/4} \frac{dx}{1+\cos x(\pi - x)} = \int_{\pi/4}^{3\pi/4} \frac{dx}{1-\cos x}$$

Adding it with the given integral

$$2I = \int_{\pi/4}^{3\pi/4} \frac{2dx}{1-\cos^2 x} = 2\int_{\pi/4}^{3\pi/4} \text{cosec}^2 x\, dx$$

$$= -2\,[\cot x]_{\pi/4}^{3\pi/4} = 4$$

$\Rightarrow I = 2$ **Ans.[A]**

59 The value of $\int_0^{\pi/2} \sin^3 x\, dx$ is -

(A) 2/3 (B) 3/2 (C) 0 (D) $4\pi/3$

Sol. We have $I = \int_0^{\pi/2} \sin^3 x\, dx = \frac{(3-1)}{3}\,.\,1$

$= 2/3$.(Since $n = 3$ is odd). **Ans.[A]**

60 $\lim_{n\to\infty} \left[\frac{n+1}{n^2+1^2} + \frac{n+2}{n^2+2^2} + ... + \frac{1}{n}\right]$ is equal to-

(A) $\frac{\pi}{4} + \frac{1}{2}\log 2$ (B) $\frac{\pi}{4} - \frac{1}{2}\log 2$

(C) $\frac{\pi}{4} - 2\log\frac{1}{2}$ (D) None of these

Sol. $T_r = \frac{n+r}{n^2+r^2} = \frac{1}{n}\left[\frac{\left(1+\frac{r}{n}\right)}{1+\left(\frac{r}{n}\right)^2}\right]$

$\therefore$ given limit $= \int_0^1 \frac{1+x}{1+x^2} dx$

$= \left[\tan^{-1} x\right]_0^1 + \left[\frac{1}{2}\log(1+x^2)\right]_0^1 = \frac{\pi}{4} + \frac{1}{2}\log 2$ **Ans.[A]**

61 $\int_0^{\infty} \frac{x^3}{(1+x^2)^{9/2}} dx$ is equal to-

(A) 2/35 (B) 3/35

(C) 4/35 (D) None of these

Sol. Put $x = \tan t$, then

$$I = \int_0^{\pi/2} \frac{\tan^3 t}{\sec^9 t} \sec^2 t\, dt = \int_0^{\pi/2} \sin^3 t \cos^4 t\, dt = \frac{2.3.1}{7.5.3.1} = \frac{2}{35}$$ **Ans.[A]**

62 $\int_0^{\infty} \frac{dx}{1+e^x}$ is equal to-

(A) log 2 – 1 (B) log 2

(C) log 4 – 1 (D) –log 2

Sol. $I = \int_0^{\infty} \frac{e^{-x}}{e^{-x}+1} dx = -\left[\log(e^{-x}+1)\right]_0^{\infty}$

$= -[\log 1 - \log 2] = \log 2$ **Ans.[B]**

63 $\int_0^{\pi/2} \frac{\cos x - \sin x}{1+\sin x \cos x} dx$ is equal to-

(A) 0 (B) 1

(C) π/2 (D) π/4

Sol. Using P–4, given integral becomes

$I = \int_0^{\pi/2} \frac{\cos(\pi/2-x)-\sin(\pi/2-x)}{1+\sin(\pi/2-x)\cos(\pi/2-x)} dx = \int_0^{\pi/2} \frac{\sin x - \cos x}{1+\cos x \sin x} dx = -I$

$\Rightarrow 2I = 0 \Rightarrow I = 0$ **Ans.[A]**

64 $\int_0^{\infty} \frac{x \ln x}{(1+x^2)^2}$ dx equals

(A) 0 (B) log 7

(C) 5 log 13 (D) None of these

Sol. Here $\int_0^{\infty} \frac{x \log x}{(1+x^2)^2} dx = \int_0^1 \frac{x \log x}{(1+x^2)^2} dx + \int_1^{\infty} \frac{x \log x}{(1+x^2)^2} dx$

$I = I_1 + I_2$

Putting $x = \frac{1}{t}$ in second integrand

$dx = -\frac{1}{t^2} dt$

$\therefore I_2 = \int_1^0 \frac{\frac{1}{t}\log\left(\frac{1}{t}\right)}{\left(1+\frac{1}{t^2}\right)^2} \left(-\frac{1}{t^2}\right) dt = -\int_0^1 \frac{t \log t}{(1+t^2)^2} dt = -I_1$

$I = I_2 + I_1 = -I_1 + I_1 = 0$ **Ans.[A]**

65 $\int_0^{\pi} x \sin^4 x \, dx$ is equal to-

(A) 3π/16 (B) 3π²/16

(C) 16π/3 (D) 16π²/3

Sol. $I = \int_0^{\pi} x \sin^4 x \, dx$ ...(1)

$I = \int_0^{\pi} (\pi - x)\sin^4(\pi - x) \, dx$

$$I = \int_0^{\pi} (\pi - x)\sin^4 x\, dx \qquad ...(2)$$

$$\therefore\ 2I = \pi \int_0^{\pi} \sin^4 x\, dx$$

$$\Rightarrow 2I = \pi \int_0^{\pi} \sin^4 x\, dx \text{ [from property P-6]}$$

$$\Rightarrow I = \pi.\ \frac{3.1}{4.2}.\frac{\pi}{2} = \frac{3\pi^2}{16}$$ **Ans.[B]**

66 $\int_1^2 \log x\, dx$ equals-

(A) 2 log 2

(B) $\log\left(\frac{2}{e}\right)$

(C) $\log\left(\frac{4}{e}\right)$

(D) None of these

Sol. $I = \int_1^2 1.\log x\, dx$ equals

(Intetgrating by parts by taking 1 as a second function)

$$= \{x.\log x\}_1^2 - \int_1^2 \left(\frac{1}{x}.\ x\right) dx$$

$= (2 \log 2 - - 1 \log 1) - [x]_1^2$

$= (2 \log 2 -- 0) - (2 - 1)$

$= \log 4 - \log e = \log\left(\frac{4}{e}\right)$ **Ans.[C]**

67 $\int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}}\, dx$ equals-

(A) 2

(B) π

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{2}$

Sol. $$I = \int_0^{\pi/2} \frac{2^{\sin x}}{2^{\sin x} + 2^{\cos x}}\, dx$$

$$I = \int_0^{\pi/2} \frac{2^{\sin(\pi/2 - x)}}{2^{\sin(\pi/2 - x)} + 2^{\cos(\pi/2 - x)}}\, dx$$

$$= \int \frac{2^{\cos x}}{2^{\cos x} + 2^{\sin x}}\, dx$$

$$2I = \int_0^{\pi/2} dx = \frac{\pi}{2} \Rightarrow I = \frac{\pi}{4}$$ **Ans.[C]**

68 $\int_0^x f(t) = x + \int_x^1 t\,f(t)\,dt$ then f(1) is equal to-

(A) $\frac{1}{2}$ (B) 0

(C) 1 (D) $-\frac{1}{2}$

Sol. $\int_0^x f(t)\,dt = x + \int_x^1 t\,f(t)\,dt$

$\Rightarrow$ f(x) = 1 + (0 – xf(x)) [diff. w.r.t. x]

$\Rightarrow$ f(x) = 1 – xf(x)

$\Rightarrow$ f(1) = 1– 1.f(x)

$\Rightarrow$ $f(1) = \frac{1}{2}$ **Ans.[A]**

69 If f(3 – x) = f(x) then $\int_1^2 x f(x)\,dx$ equals-

(A) $\frac{3}{2}\int_1^2 f(2-x)\,dx$ (B) $\frac{3}{2}\int_1^2 f(x)\,dx$

(C) $\frac{1}{2}\int_1^2 f(x)\,dx$ (D) None of these

Sol. Let x = 3 – y

$$I = \int_2^1 (3-y)f(3-y)(-dy)$$

$$= \int_1^2 (3-x)f(3-x)\,dx$$

$$= \int_1^2 (3-x)f(x)\,dx \qquad [\because f(3-x) = f(x)]$$

$$= 3\int_1^2 f(x)\,dx - I$$

$$I = \frac{3}{2}\int_1^2 f(x)\,dx$$

Ans.[B]

70 $\int_0^1 \frac{x\sin^{-1}x}{\sqrt{1-x^2}}\,dx$ is equal to-

(A) π/2 (B) π /4

(C) 0 (D) 1

Sol. Put $\sin^{-1} x = t$, $\frac{dx}{\sqrt{1-x^2}} = dt$ then

$$\therefore I = \int_0^{\pi/2} t\sin t\,dt = [t(-\cos t)]_0^{\pi/2} + [\sin x]_0^{\pi/2} = 1$$

Ans.[C]

71 $\int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x}$ is equal to-

(A) 0 (B) 2

(C) 1 (D) None of these

Sol. $I = \int_{-\pi/2}^{0} \frac{\cos x}{1+e^x} dx + \int_{0}^{\pi/2} \frac{\cos x}{1+e^x} dx = -\int_{\pi/2}^{0} \frac{\cos y}{1+e^{-y}} dy + \int_{0}^{\pi/2} \frac{\cos x}{1+e^x}$

(putting x = – y in first integral)

$= \int_{0}^{\pi/2} \frac{e^y \cos y}{1+e^y} dy + \int_{0}^{\pi/2} \frac{\cos x}{1+e^x} dx$

$= \int_{0}^{\pi/2} \frac{e^x \cos x}{1+e^x} dx + \int_{0}^{\pi/2} \frac{\cos x}{1+e^x} dx$

$= \int_{0}^{\pi/2} \frac{(e^x+1)\cos x}{1+e^x} dx$

$= \int_{0}^{\pi/2} \cos x \, dx = [\sin x]_0^{\pi/2} = 1$ **Ans.[C]**

72 $\int_{-1}^{1} \frac{\sin x - x^2}{3-|x|} dx$ is equal to-

(A) 0 (B) $2\int_{0}^{1} \frac{\sin x}{3-|x|} dx$

(C) $\int_{0}^{1} \frac{-2x^2}{3-|x|} dx$ (D) $2\int_{0}^{1} \frac{\sin x - x^2}{3-|x|} dx$

Sol. $I = \int_{-1}^{1} \frac{\sin x - x^2}{3-|x|} dx$

$= \int_{-1}^{1} \frac{\sin x}{3-|x|} dx - \int_{-1}^{1} \frac{x^2}{3-|x|} dx$

$= 0 - 2\int_{0}^{1} \frac{x^2}{3-|x|} dx$

[$\because \frac{\sin x}{3-|x|}$ is an odd and $\frac{x^2}{3-|x|}$ is an even function]

$= -2\int_{0}^{1} \frac{x^2}{3-|x|} dx$ **Ans.[C]**

73 $\int_{0}^{2a} \frac{f(x)}{f(x)+f(2a-x)} dx$ is equal to-

(A) a (B) –a

(C) 0 (D) None of these

Sol. Using P–4, given integral becomes

$I = \int_{0}^{2a} \frac{f(2a-x)}{f(2a-x)+f(x)} dx$

Adding it with the given integral, we get

$$2I = \int_0^{2a} 1\,dx = 2a \Rightarrow I = a$$ **Ans.[A]**

74 If $g(x) = \int_0^x \cos^4 t\,dt$, then $g(x+\pi)$ is equal to-

(A) $g(x) + g(\pi)$ (B) $g(x) - g(\pi)$

(C) $g(x)\,g(\pi)$ (D) $g(x)/g(\pi)$

Sol. $g(x+\pi) = \int_0^{\pi+x} \cos^4 t\,dt$

$= \int_0^{\pi} \cos^4 t\,dt + \int_{\pi}^{\pi+x} \cos^4 t\,dt$ [by P–3]

$= \int_0^{\pi} \cos^4 t\,dt + I_2$

Now in I_2, put $t = \pi + \theta$, then

$$I_2 = \int_0^x \cos^4(\pi+\theta)\,d\theta = \int_0^x \cos^4\theta\,d\theta = \int_0^x \cos^4 t\,dt$$

$\therefore g(x+\pi) = \int_0^{\pi} \cos^4 t\,dt + \int_0^x \cos^4 t\,dt = g(x) + g(\pi)$ **Ans.[A]**

75 The value of $\int_0^{100\pi} \sqrt{1-\cos 2x}\,dx$ is

(A) $100\sqrt{2}$ (B) $200\sqrt{2}$

(C) $50\sqrt{2}$ (D) 0

Sol. $I = \sqrt{2} \int_0^{100\pi} |\sin x|\,dx$

$= 100\sqrt{2} \int_0^{\pi} |\sin x|\,dx$

$= 100\sqrt{2} \int_0^{\pi} \sin x\,dx = 100\sqrt{2}\,[-\cos x]_0^{\pi}$

$= 200\sqrt{2}$ **Ans.[B]**

76 $\int_0^{\pi/4} (\sqrt{\tan x} + \sqrt{\cot x})\,dx$ is equal to-

(A) $\pi/2$ (B) $\pi/\sqrt{2}$

(C) $-\pi/2$ (D) $-\pi/\sqrt{2}$

Sol. Putting $\tan x = t^2$, then

$\sec^2 x\,dx = 2t\,dt \Rightarrow dx = \frac{2t\,dt}{1+t^4}$

$$\therefore I = \int_0^1 \left(t + \frac{1}{t}\right)\frac{2t\,dt}{1+t^4}$$

$$= 2\int_0^1 \frac{t^2+1}{t^4+1}dt = 2\int_0^1 \frac{1+1/t^2}{t^2+1/t^2}dt = 2\int_0^1 \frac{d(t-1/t)}{(t-1/t)^2+2}$$

$$= \sqrt{2}\left[\tan^{-1}\frac{1}{\sqrt{2}}\left(t-\frac{1}{t}\right)\right]_0^1 = \sqrt{2}\,[\tan^{-1}0 - \tan^{-1}(-\infty)] = \sqrt{2}\,(\pi/2) = \pi/\sqrt{2}$$ **Ans.[B]**

77 $\int_0^{\pi/2} \frac{dx}{1+2\sin x+\cos x}$ equals-

(A) (1/2) log 3 (B) log 3

(C) (4/3) log 3 (D) None of these

Sol. Here

$$I = \int_0^{\pi/2} \frac{dx}{1+2\frac{2\tan(x/2)}{1+\tan^2(x/2)}+\frac{1-\tan^2(x/2)}{1+\tan^2(x/2)}}$$

$$= \int_0^{\pi/2} \frac{\sec^2(x/2)}{2\{1+2\tan(x/2)\}}dx$$

Let $1+2\tan(x/2) = t$, then

$\sec^2(x/2)\,dx = dt$

$$\therefore I = \frac{1}{2}\int_1^3 \frac{dt}{t} = \frac{1}{2}(\log t)_1^3$$

$$= \frac{1}{2}\log 3$$ **Ans.[A]**

78 $\int_0^{\pi/2} \frac{\sin 2x}{a\cos^2 x+b\sin^2 x}dx$ -

(A) $\frac{1}{b-a}\log\left(\frac{b}{a}\right)$ (B) $\frac{1}{b+a}\log\left(\frac{b}{a}\right)$

(C) $\frac{1}{b-a}\log\left(\frac{a}{b}\right)$ (D) $\frac{1}{b+a}\log\left(\frac{a}{b}\right)$

Sol. $I = \left(\frac{1}{b-a}\right)\int_0^{\pi/2} \frac{(b-a)2\sin x\cos x}{a\cos^2 x+b\sin^2 x}dx$

$$= \frac{1}{b-a}\left[\log(a\cos^2 x+b\sin^2 x)\right]_0^{\pi/2} = \frac{1}{(b-a)}(\log b - \log a)$$

$$= \frac{1}{b-a}\log\left(\frac{b}{a}\right)$$ **Ans.[A]**

79 $\int_0^{\pi/2} (2\log\sin x - \log\sin 2x)\,dx$ equals-

(A) π log 2 (B) $-\pi$ log 2

(C) $(\pi/2)$ log 2 (D) $-(\pi/2)$ log 2

Sol. $I = \int_0^{\pi/2} (2\log\sin x - \log 2\sin x\cos x)\, dx$

$= \int_0^{\pi/2} (2\log\sin x - \log 2 - \log\sin x - \log\cos x)\, dx$

$= \int_0^{\pi/2} \log\sin x\, dx - \int_0^{\pi/2} \log 2 dx - \int_0^{\pi/2} \log\cos x\, dx = -(\pi/2)\log 2.$ **Ans.[D]**

80 $\int_0^1 \cot^{-1}(1 - x + x^2)\, dx$ equals-

(A) $\frac{\pi}{2} + \log 2$ (B) $\frac{\pi}{2} - \log 2$

(C) $\pi - \log 2$ (D) None of these

Sol. $I = \int_0^1 \tan^{-1}\left(\frac{1}{1 - x - x^2}\right) dx$

$= \int_0^1 \tan^{-1}\left(\frac{x + (1 - x)}{1 - x(1 - x)}\right) dx$

$= \int_0^1 [\tan^{-1} x + \tan^{-1}(1 - x)]\, dx$

$= \int_0^1 \tan^{-1} x\, dx + \int_0^1 \tan^{-1}(1 - x)\, dx$

$= 2\int_0^1 \tan^{-1} x\, dx$ [By prov. IV]

$= 2\left[x\tan^{-1} - \frac{1}{2}\log(1 + x^2)\right]_0^1$

$= 2\frac{\pi}{4} - \log 2 = \frac{\pi}{2} - \log 2$ **Ans.[B]**

Exercise 2(A)

1 [Hint: $I = \int_1^{\infty} \frac{dx}{(e \cdot e^x + e^3 \cdot e^{-x})} = \int_1^{\infty} \frac{e^x\,dx}{e(e^{2x} + e^2)}$ (multiply N^r and D^r by e^x)

put $e^x = t \Rightarrow e^x\,dx = dt$

$I = \frac{1}{e}\int_e^{\infty} \frac{dt}{t^2 + e^2} = \frac{1}{e^2}\tan^{-1}\frac{t}{e}\Big|_e^{\infty} = \frac{1}{e^2}\left[\frac{\pi}{2} - \frac{\pi}{4}\right] = \frac{\pi}{4e^2}$ Ans.]

2 [Hint: put $e^{x^2} = t$; $e^{x^2} \cdot 2x\,dx = dt$; $\int_1^{\pi/2} \cos t\,dt = [\sin t]_1^{\pi/2} = 1 - (\sin 1)$]

3 [Hint: Note that in $\left(-\frac{1}{2}, \frac{1}{2}\right)$, $\sin^{-1}(3x - 4x^3) = 3\sin^{-1}x$ and $\cos^{-1}(4x^3 - 3x) = 2\pi - 3\cos^{-1}x$

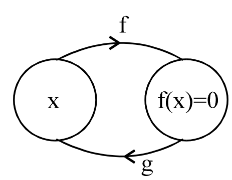
hence $f(x) = 3\sin^{-1}x - 2\pi + 3\cos^{-1}x = -\frac{\pi}{2}$

$\therefore I = -\frac{\pi}{2}\int_{-1/2}^{1/2} dx = -\frac{\pi}{2}$]

[Alternate: $f(x) = \sin^{-1}(3x - 4x^3) - [\pi - \cos^{-1}(3x - 4x^3)]$

$= -\pi + (\sin^{-1}(3x - 4x^3) + \cos^{-1}(3x - 4x^3)) = -\frac{\pi}{2}$]

4 [Sol. $f'(x) = \frac{1}{\sqrt{1 + x^4}} = \frac{dy}{dx}$



now $g'(x) = \frac{dx}{dy} = \sqrt{1 + x^4}$

when $y = 0$ i.e. $\int_2^{x} \frac{dt}{\sqrt{1 + t^4}} = 0$ then $x = 2$ (think !)

hence $g'(0) = \sqrt{1 + 16} = \sqrt{17}$]

5 [Sol. $l = \ln \lim_{t \to 0} \frac{\int_0^{t} (1 + a\sin bx)^{c/x}\,dx}{t} = \ln \lim_{t \to 0} (1 + a\sin bt)^{c/t}$ (using L'Hospital's rule)

$$= \ln e^{\lim_{t\to 0}\frac{c}{t}(a\sin bt)} = \lim_{t\to 0}\frac{abc\sin bt}{bt} = abc \text{ Ans.]}$$

6 [Sol. $\sin nx - \sin(n-2)x = 2\cos(n-1)x\sin x$

$$\int \frac{\sin nx}{\sin x}dx = \int 2\cos(n-1)\,dx + \int \frac{\sin(n-2)x}{\sin x}dx$$

$$\therefore \quad \int_0^{\pi/2}\frac{\sin 5x}{\sin x}dx = \int_0^{\pi/2} 2\cos 4x\,dx + \int_0^{\pi/2}\frac{\sin 3x}{\sin x}dx = 0 + \int_0^{\pi/2}\frac{\sin 3x}{\sin x}dx = \int_0^{\pi/2}dx = \frac{\pi}{2} \text{ Ans.]}$$

7 [Sol. $F(x) = \frac{1}{2}\int \frac{(x^2+1)-(x-1)^2}{(x^2+1)(x-1)}dx = \frac{1}{2}\ln|x-1| - \frac{1}{2}\int \frac{x-1}{x^2+1}dx$

$$= \frac{1}{2}\ln|x-1| + \frac{1}{4}\ln(x^2+1) + \frac{1}{2}\tan^{-1}x + C$$

$\therefore$ discontinuous at $x = 1$

note that $f(x) = \int \frac{dx}{x^{1/3}} = \frac{3}{2}x^{2/3} + C$ is continuous although $\frac{1}{x^{1/3}}$ is discontinuous at $x = 0$]

8 [Sol. $T_r = \dfrac{1}{\sqrt{\frac{r}{n}}\cdot n\left(3\sqrt{\frac{r}{n}}+4\right)^2}$

$$S = \frac{1}{n}\sum_{1}^{4n}\frac{1}{\left(3\sqrt{\frac{r}{n}}+4\right)^2\cdot\sqrt{\frac{r}{n}}} = \int_0^4 \frac{dx}{\sqrt{x}(3\sqrt{x}+4)^2}$$

put $3\sqrt{x}+4 = t \quad \Rightarrow \quad \frac{3}{2}\frac{1}{\sqrt{x}}dx = dt$

$$= \frac{2}{3}\int_4^{10}\frac{dt}{t^2} = \frac{2}{3}\left[\frac{1}{t}\right]_{10}^{4} = \frac{2}{3}\left[\frac{1}{4}-\frac{1}{10}\right] = \frac{2}{3}\cdot\frac{6}{40} = \frac{1}{10} \quad]$$

9 [Sol. $f'(x) = f(x) \Rightarrow f(x) = Ce^x$ and since $f(0) = 1$

$\therefore \quad 1 = f(0) = C \quad \therefore \quad f(x) = e^x$ and hence $g(x) = x^2 - e^x$

Thus $\int_0^1 f(x)g(x)\,dx = \int_0^1 (x^2e^x - e^{2x})\,dx$

$$= x^2e^x\Big|_0^1 - 2\int_0^1 xe^x\,dx - \left[\frac{e^{2x}}{2}\right]_0^1 = (e-0) - 2\left[xe^x\Big|_0^1 - e^x\Big|_0^1\right] - \frac{1}{2}(e^2-1)$$

$$= (e - 0) - 2\,[(e - 0) - (e - 1)] - \frac{1}{2}(e^2 - 1)$$

$$= e - \frac{1}{2}e^2 - \frac{3}{2} \qquad]$$

10 [Sol. $I = \int_{-\pi/2}^{\pi/2} \frac{\cos\theta\, d\theta}{(2-\sin\theta)\cos\theta}$ (putting $x = \sin\theta$)

$$= \int_0^{\pi/2} \left(\frac{1}{2-\sin\theta} + \frac{1}{2+\sin\theta}\right) d\theta \qquad \left[\text{using } \int_{-a}^{a} f(x)\,dx = \int_0^{a} [f(x) + f(-x)\ dx\right]$$

$$= 4\int_0^{\pi/2} \frac{d\theta}{4-\sin^2\theta} = \frac{4}{3}\int_0^{\pi/2} \frac{\sec^2\theta\, d\theta}{\frac{4}{3}+\tan^2\theta} = \frac{4}{3}\int_0^{\infty} \frac{d\theta}{t^2+\frac{4}{3}} = \frac{4}{\sqrt{3}}\cdot\frac{\sqrt{3}}{2}\cdot\tan^{-1}\frac{\sqrt{3}\,t}{2}\Bigg|_0^{\infty} = \frac{2}{\sqrt{3}}\cdot\frac{\pi}{2} = \frac{\pi}{\sqrt{3}} \quad]$$

11 [Sol. $T_r = \frac{\pi}{6n}\sec^2\frac{r\pi}{6n}$

$$S = \sum T_r = \frac{\pi}{6n}\sum_{r=1}^{n}\sec^2\frac{r\pi}{6n} = \frac{\pi}{6}\int_0^1 \sec^2\frac{\pi x}{6}\,dx = \tan\frac{\pi x}{6}\Bigg|_0^1 = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \quad]$$

12 [Sol. Clearly f is an even function, hence

$$I_1 = \int_0^{\pi} f\big(\cos(\pi - x)\big)dx = \int_0^{\pi} f(-\cos x)\,dx = \int_0^{\pi} f(\cos x)\,dx$$

$\therefore$ $I_1 = 2\int_0^{\pi/2} f(\cos x)\,dx = 2I_2$ $\Rightarrow$ $\frac{I_1}{I_2} = 2$ **Ans.**

<u>Alternatively</u>: let $u = \cos x$ $\Rightarrow$ $du = -\sin x\,dx$

$\therefore$ $I_1 = \int_{-1}^{1} \frac{f(u)}{\sqrt{1-u^2}}\,du$ $\Rightarrow$ $2\int_0^{1} \frac{f(u)}{\sqrt{1-u^2}}\,du$ ....(1)

|||ly with $\sin t = t$, $I_2 = \int_0^{1} \frac{f(t)}{\sqrt{1-t^2}}\,dt$ ....(2)

from (1) and (2) $\frac{I_1}{I_2} = 2$ Ans.]

13 [Hint: $\int_2^4 \left(\frac{\ln 2}{\ln x} - \frac{\ln 2}{\ln^2 x}\right) dx$ if $f(x) = \frac{1}{\ln x}$ $\Rightarrow$ $x f'(x) = -\frac{1}{\ln^2 x}$

$$\Rightarrow \quad I = \ln 2\left(\frac{x}{\ln x}\right)_2^4 = \ln 2\left[\frac{4}{\ln 4} - \frac{2}{\ln 2}\right] = 0\ ]$$

14 [Hint: On rationalisation,

$$\int_{-1}^{1}\frac{(1+x^3)-\sqrt{1+x^6}}{1+x^6+2x^3-1-x^6}dx = \int_{-1}^{1}\frac{(1+x^3)-\sqrt{1+x^6}}{2x^3}dx = \underbrace{\frac{1}{2}\int_{-1}^{1}\frac{1}{x^3}dx}_{\text{odd} \Rightarrow \text{zero}} + \frac{1}{2}\int_{-1}^{1}dx - \underbrace{\int_{-1}^{1}\frac{\sqrt{1+x^6}}{2x^3}dx}_{\text{odd} \Rightarrow \text{zero}}$$

$$\frac{1}{2}\int_{-1}^{1}dx = \frac{1}{2}\cdot 2 = 1 \text{ Ans.]}$$

15 [Sol. at $y = 0$, $x = 2$

$f'(x) = \sqrt{9+x^4}\cdot 2x$

$$\therefore \quad g'(y) = \left.\frac{1}{f'(x)}\right|_{x=2} = \frac{1}{2x\sqrt{9+x^4}} = \frac{1}{20}\ ]$$

16 [Sol. $\left.\frac{t^3}{3}\right|_0^{f(x)} = x\cos\pi x \quad \Rightarrow [f(x)]^3 = 3x\cos\pi x$ ....(1)

$$[f(9)]^3 = -27 \Rightarrow f(9) = -3$$

also differentiating $\int_0^{f(x)} t^2\,dt = x\cos\pi x$

$$[f(x)]^2 \cdot f'(x) = \cos\pi x - x\pi\sin\pi x$$

$\therefore \quad [f(9)]^2 \cdot f'(9) = -1$

$$\Rightarrow \quad f'(9) = -\frac{1}{(f(9))^2} = -\frac{1}{9} \qquad f'(9) = -\frac{1}{9} \quad \Rightarrow \text{(A)]}$$

17 [Hint: $\lim_{x\to\infty}\frac{x^{3/2}}{(x-1)}\cdot\frac{1}{2\sqrt{x}} = \frac{1}{2[1-(1/x)]} = \frac{1}{2}$ Ans.]

18 [Sol. $I = \int_1^e \underbrace{f''(x)}_{II}\ \underbrace{\ln x}_{I}\,dx = \ln x\cdot f'(x)\Big|_1^e - \int_1^e \frac{f'(x)}{x}dx$

$I = 1 - I_1$

$$I_1 = \int_1^e \frac{1}{x}f'(x)\,dx = \left.\frac{1}{x}\cdot f(x)\right|_1^e + \int_1^e\frac{f(x)}{x^2}dx$$

$$= \left(\frac{1}{e}-1\right) + \frac{1}{2}$$

$$= \frac{1}{e} - \frac{1}{2}$$

$\therefore \quad I = 1 - \frac{1}{e} + \frac{1}{2} = \frac{3}{2} - \frac{1}{e}$ Ans.]

19 [Sol. $f'(x)\frac{dy}{dx} = \frac{1}{\sqrt{x^4+3x^2+13}}$ when $y = f(x)$

$\therefore \quad g'(y) = \frac{1}{dy/dx} = \sqrt{x^4+3x^2+13}$

when y = 0 then x = 3

hence $g'(0) = \sqrt{3^4+27+13} = \sqrt{121} = 11$ Ans.]

20 [Hint: $I = \int \sqrt{1+2\,\text{cosec}\ x\ \cot x + 2\cot^2 x}$

$$= \int \sqrt{\text{cosec}^2 x + 2\,\text{cosec} x \cot x + \cot^2 x}\, dx$$

$$= \int (\text{cosec} x + \cot x)\, dx\ ]$$

21 [Hint: $\left.\frac{t^2}{2} - \log_2 a \cdot t\right|_0^2 = 2 - \log_2(a^2)$

$(2 - 2\log_2 a) = 2 - 2\log_2 a$

$2\log_2 a = 2\log_2 a \quad \Rightarrow \quad a \in R^+$]

22 [Hint: Put $4x - 5 = 5t^2 \Rightarrow 4dx = 10t\ dt$ or <u>better will be $5(4x-5) = t^2$</u>]

$$I = \frac{5}{2}\int_{\sqrt{\frac{3}{5}}}^{\sqrt{\frac{7}{5}}} \sqrt{\frac{5}{2}(1+t^2)-5t} + \sqrt{\frac{5}{2}(1+t^2)+5t}\ t\ dt = \left(\frac{5}{2}\right)^{3/2} \int_{\sqrt{\frac{3}{5}}}^{\sqrt{\frac{7}{5}}} \left(|t-1| + |(t+1)|\right) t\ dt$$

$$= \left(\frac{5}{2}\right)^{3/2} \left[\int_{\sqrt{\frac{3}{5}}}^{1} \left((1-t) + |(1+t)|\right) t\ dt + \int_{1}^{\sqrt{\frac{7}{5}}} \left((t-1) + (t+1)\right) t\ dt\right]$$

$$= \left(\frac{5}{2}\right)^{3/2} \left[2\int_{\sqrt{\frac{3}{5}}}^{1} t\ dt + \int_{1}^{\sqrt{\frac{7}{5}}} t^2\ dt\right]]$$

23 [Hint: $\frac{dy}{dx} = \frac{1}{\sqrt{y^2+1}}$

$$\frac{dy}{dx} = \sqrt{y^2+1}\,; \quad \frac{d^2y}{dx^2} = \frac{y}{\sqrt{y^2+1}}\sqrt{y^2+1} = y \text{ Ans.]}$$

24 [Hint: $f(x) = \sqrt{1+x^2} - x\,;\ \lim_{x\to-\infty} x\left(\sqrt{1+x^2} - x\right) \to -\infty \Rightarrow$ DNE]

25 [Sol. $x = \frac{1}{t} \Rightarrow dx = \frac{1}{t^2}\,dt$

$$I = \int_2^{1/2} t\,\sin\left(\frac{1}{t} - t\right)\left(-\frac{1}{t^2}\right) dt = \int_2^{1/2} \frac{1}{t}\sin\left(t - \frac{1}{t}\right) dt = -\int_{1/2}^{2} \frac{1}{t}\sin\left(t - \frac{1}{t}\right) dt = -I$$

$\Rightarrow \quad 2I = 0 \Rightarrow I = 0$

Alternatively : put $x = e^t \Rightarrow I = \int_{-\ln 2}^{\ln 2} \sin(e^t - e^{-t})dt = 0$ (odd function)]

26 [Sol. $f'(\ln x) = \begin{cases} 1 & \text{for } 0 < x \le 1 \\ x & \text{for } x > 1 \end{cases}$

put $\ln x = t \Rightarrow x = e^t$

for $x > 1\,;\ f'(t) = e^t$ for $t > 0$

integrating $f(t) = e^t + C\,;\ f(0) = e^0 + C \Rightarrow C = -1$

$\therefore\ f(t) = e^t - 1$ for $t > 0$ (corresponding to $x > 1$)

$\therefore\ f(x) = e^x - 1$ for $x > 0$ ....(1)

again for $0 < x \le 1$

$f'(\ln x) = 1 \qquad (x = e^t)$

$f'(t) = 1$ for $t \le 0$

$f(t) = t + C$

$f(0) = 0 + C \Rightarrow C = 0 \Rightarrow f(t) = t$ for $t \le 0 \Rightarrow f(x) = x$ for $x \le 0$]

27 [Sol. $\int \frac{1}{x}\ln\frac{x}{e^x}\,dx = \int \frac{1}{x}(\ln x - \ln e^x)\,dx$

$$= \int \frac{\ln x - x}{x}\,dx = \left[\int \frac{1}{x}\ln x\,dx - \int \frac{1}{x}\,dx\right] \quad \left(\text{put } \ln x = u\,;\ \frac{1}{x}dx = du\right)$$

$$= \int u\,dx - \int 1\,dx = \frac{1}{2}\ln^2 x - x + C$$]

28 [Sol. $\int_1^e e^x[x\ln x + 1 + \ln x - 1]\,dx = \int_1^e e^x[(\underbrace{x\ln x}_{f(x)}) + (\underbrace{\ln x + 1}_{f'(x)})]\,dx - \int_1^e e^x dx$

$= e^x\cdot(x\ln x)\Big|_1^e - \left[e^x\right]_1^e = (e^e\cdot e - 0) - [e^e - e]$

$= e^e(e - 1) + e$ Ans.]

29 [Hint: $\left|\int_{10}^{19}\frac{\sin x}{1+x^8}\right| \le \int_{10}^{19}\frac{|\sin x|}{1+x^8}\,dx \le \int_{10}^{19}\frac{dx}{1+x^8} < \int_{10}^{19}\frac{dx}{x^8} = \left[\frac{x^{-7}}{-7}\right]_{10}^{19}$

$= -\frac{1}{7}\,[19^{-7} - 10^{-7}] = \frac{1}{7}\,[10^{-7} - 19^{-7}] < 10^{-7}$]

30 [Sol. $\lim_{n\to\infty}\int_0^2\left(1+\frac{t}{n+1}\right)^n dt = \lim_{n\to\infty}\left[\left(1+\frac{t}{n+1}\right)^{n+1}\right]_0^2 = \lim_{n\to\infty}\left(1+\frac{2}{n+1}\right)^{n+1} - 1 = e^2 - 1$

note that $\left[\left(1+\frac{t}{n+1}\right) \textit{is a linear function } a+bt \textit{ type}\right]$]

31 [Sol. $I = \int x\,2^{\ln(x^2+1)}dx$ let $x^2 + 1 = t$; $x\,dx = \frac{dt}{2}$

Hence $I = \frac{1}{2}\int 2^{\ln t}dt = \frac{1}{2}\int t^{\ln 2}dt = \frac{1}{2}\cdot\frac{t^{\ln 2+1}}{\ln 2+1} + C = \frac{1}{2}\cdot\frac{(x^2+1)^{\ln 2+1}}{\ln 2+1} + C \Rightarrow$ (C)]

32 [Hint: $\int_0^1(1+\cos^8 x)\,f(x)\,dx = \int_0^2(1+\cos^8 x)\,f(x)\,dx =$

$\int_0^1(1+\cos^8 x)\,f(x)\,dx + \int_1^2(1+\cos^8 x)\,f(x)\,dx$

Hence $\int_1^2(1+\cos^8 x)\,f(x)\,dx = 0$

$\Rightarrow (1+\cos^8 x)\,f(x) = 0$ at least once in (1,2)

but $1 + \cos^8 x \ne 0$

$\Rightarrow f(x) = ax^2 + bx + c$ vanishes at least once in (1,2)]

33 [Hint: $I = \int_0^{\pi/4}(1 - 2\sin^2 x)^{3/2}\cos x\,dx$. Put $\sqrt{2}\sin x = \sin\theta$

$\Rightarrow I = \frac{1}{\sqrt{2}}\int_0^{\pi/2}\cos^4\theta\,d\theta = \frac{3\pi}{16\sqrt{2}}$]

34 [Sol. Given $\int f(x)\,dx = g(x) \Rightarrow g'(x) = f(x)$

now $\frac{d}{dx}\left(\ln(1+g^2(x))\right) = \frac{2g(x)g'(x)}{1+g^2(x)} = \frac{2f(x)g(x)}{1+g^2(x)} \Rightarrow$ (B)]

35 [Sol. $\displaystyle \lim_{x\to 0} \frac{\int_0^x \sin t^2 dt}{x^3 \frac{(1-\cos x)}{x^2}}$ (using $\displaystyle \lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$)

$$= \lim_{x\to 0} \frac{\int_0^x \sin t^2 dt}{x^3} \quad \text{(Using L'Hospital Rule)}$$

$$2 \lim_{x\to 0} \frac{\sin x^2}{3x^2} = \frac{2}{3} \quad \text{Ans.]}$$

36 [Sol. $I = \int_{-1}^{1} f(x)\,dx = \int_{-1}^{1} f(-x)\,dx$ (using K)

$$2I = \int_{-1}^{1} \left(f(x) + f(-x)\right) dx = \int_{-1}^{1} (x^2)\,dx$$

$$2I = 2\int_{0}^{1} (x^2)\,dx \quad \Rightarrow \quad I = \int_{0}^{1} (x^2)\,dx = \frac{1}{3} \quad \text{Ans.]}$$

37 [Sol. $I = \displaystyle\int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{x^4}{1-x^4} \cos^{-1} \frac{2x}{1+x^2}\,dx$ ....(1)

$$I = \int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{x^4}{1-x^4} \cos^{-1}\left(\frac{-2x}{1-x^4}\right) dx \quad \text{(using King)}$$

$$I = \int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{x^4}{1-x^4} \left(\pi - \cos^{-1} \frac{2x}{1-x^4}\right) dx \quad ....(2)$$

add (1) and (2)

$$\therefore \quad 2I = \pi \int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{x^4}{1-x^4}\,dx$$

$$2I = 2\pi \int_{0}^{1/\sqrt{3}} \frac{x^4}{1-x^4}\,dx$$

$\therefore$ $k = \pi$ Ans.]

38 [Sol. $I = \int_0^{\pi/2} \sqrt{\tan x}\, dx$ ....(1) ; $I = \int_0^{\pi/2} \sqrt{\cot x}\, dx$ ....(2)

adding (1) and (2), we get

$$2I = \int_0^{\pi/2} \left(\sqrt{\tan x} + \sqrt{\cot x}\right) dx = \sqrt{2}\int_0^{\pi/2} \frac{\sin x + \cos x}{\sqrt{\sin 2x}}\, dx = \sqrt{2}\int_0^{\pi/2} \frac{\sin x + \cos x}{\sqrt{1-(\sin x - \cos x)^2}}\, dx$$

$$= \sqrt{2}\int_{-1}^{1} \frac{dt}{\sqrt{1-t^2}} = 2\sqrt{2}\int_0^{1} \frac{dt}{\sqrt{1-t^2}} = \sqrt{2}\,\pi \quad \text{(where } \sin x - \cos x = t)$$

$\therefore$ $I = \dfrac{\pi}{\sqrt{2}}$ Ans.]

39 [Hint: $I_1 = \int_{-\pi/4}^{\pi/4} \ln(\sin x + \cos x)\, dx = \int_{-\pi/4}^{\pi/4} \ln(\cos x - \sin x)\, dx$ (using king)

$\Rightarrow 2 I_1 = \int_{-\pi/4}^{\pi/4} \ln \cos 2x\, dx = 2\int_0^{\pi/4} \ln(\cos 2x) = \int_0^{\pi/2} \ln(\cos t)\, dt$ where $2x = t$

$\int_0^{\pi/2} \ln(\sin t)\, dt = I \Rightarrow I_1 = I/2$]

40 [Hint: $f'(x) = \dfrac{1}{x} + \pi\cos(\pi x) + C$

$f'(2) = \dfrac{1}{2} + \pi + C = \dfrac{1}{2} + \pi \quad \Rightarrow \quad C = 0$

$f(x) = \ln|x| + \sin(\pi x) + C'$

$f(1) = C' = 0$

$f(x) = \ln|x| + \sin(\pi x)$]

41 [Hint: $f'(x) = 1 + \ln^2 x + 2\ln x = 0 \Rightarrow (1 + \ln x)^2 = 0 \Rightarrow x = \dfrac{1}{e}$

Hence $f\left(\dfrac{1}{e}\right) = 1 + \dfrac{1}{e} + \int_1^{\frac{1}{e}} (\ln^2 t + 2\ln t)\, dt = 1 + \dfrac{1}{e} + t\ln^2 t\Big]_1^{\frac{1}{e}} = 1 + \dfrac{1}{e} + \dfrac{1}{e} = 1 + 2e^{-1} \Rightarrow$ [D]

42 [Sol. $I = \int_{-\infty}^{\infty} \underbrace{h'(x)}_{II} \cdot \underbrace{\sin x}_{I}\, dx = \sin x \cdot h(x)\Big]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \cos x \cdot h(x)\, dx = 0 - \cos 0 = -1 \Rightarrow$ (A)

note that here $\cos x = f(x)$]

43 [Sol. $I = \int_0^{\infty} (x^2)^n . x\, e^{-x^2}\, dx$ put $x^2 = t \Rightarrow x\, dx = -dt/2$

$$= \frac{1}{2}\int_0^\infty t^n\ e^{-t}\ dt \ = \frac{1}{2}\left[\ t^n\, e^{-t}\Big]_0^\infty \ + n\int_0^\infty t^{n-1}\ e^{-t}\ dt\ \right] = \frac{1}{2}\left[\ 0 + n\int_0^\infty t^{n-1}\ e^{-t}\ dt\ \right]$$

Hence $I = \frac{n!}{2}$]

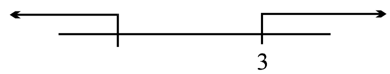
44 [Sol. $\int_a^0 3^{-x}\,(3^{-x} - 2)\ dx \geq 0$ put $3^{-x} = t \Rightarrow 3^{-x}\ln 3\ dx = -dt$

$$\ln 3\int_1^{3^{-a}} (t-2)\ dt\ \geq 0 \quad \Rightarrow \quad \left.\frac{t^2}{2} - 2t\right]_1^{3^{-a}} \geq 0$$

$$\left(\frac{3^{-2a}}{2} - 2.3^{-a}\right) - \left(\frac{1}{2} - 2\right) \geq 0$$

$3^{-2a} - 4\times 3^{-a} + 3 > 0$

$(3^{-a} - 3)\,(3^{-a} - 1) > 0$



$3^{-a} > 3^1 \Rightarrow a < 1$

or $3^{-a} < 3^0 \Rightarrow a > 0$

Hence $a \in (-\infty, -1) \cup [0, \infty)$]

45 [Sol. $\left.\sin(x + \alpha^2)\right]_0^\alpha = \sin\alpha$

$\sin(\alpha^2 + \alpha) - \sin\alpha^2 = \sin\alpha$

$2\cos(\alpha^2 + \alpha/2)\sin\alpha/2 = \sin\alpha$

now proceed and get

$\sqrt{2\pi}\ , \frac{-1 + \sqrt{1 + 8\pi}}{2} \Rightarrow$ 2 solutions]

46 Let $A = \int_0^1 \frac{e^t\ dt}{1+t}$ then $\int_{a-1}^{a} \frac{e^{-t}\,dt}{t-a-1}$ has the value

(A) Ae^{-a} (B*) $-Ae^{-a}$ (C) $-ae^{-a}$ (D) Ae^a

[Hint : $I = \int_{a-1}^{a} \frac{e^{-t}}{t-a-1}\ dt$ put $t = a-1+y$ (so that lower limit becomes zero)

$\therefore I = \int_0^1 \frac{e^{1-a-y}}{y-2}\ dy$ (now using king)

$$I = \int_0^1 \frac{e^{1-a-1+y}}{1-y-2}\ dy = -e^{-a}\int_0^1 \frac{e^y}{1+y}\ dy = -e^{-a}A \Rightarrow \text{(B)} \ ]$$

47 [Hint: $I = \int_0^1 \frac{e^t\left(t+1-t\right)}{\left(1+t\right)^2}\ dt = \int_0^1 \frac{e^t}{1+t}\ dt - \int_0^1 e^t\left(\frac{1}{1+t} - \frac{1}{(1+t)^2}\right)\ dt$

$$= A - \left.\frac{e^t}{1+t}\right]_0^1 = A - \frac{e}{2} + 1 \;;\; \text{Alternatively I. B. P. directly}]$$

48 [Hint : $\beta + \int_0^1 \underbrace{x}_{I}\underbrace{2xe^{-x^2}}_{II} dx = \int_0^1 e^{-x^2}\,dx$

$$\beta + \left[-xe^{-x^2}\right]_0^1 - \int_0^1 -e^{-x^2}\,dx = \int_0^1 e^{-x^2}\,dx \qquad \beta = \frac{1}{e}\;]$$

49 [Sol. $g(x) = \int_0^x t\,\sin\frac{1}{t}\,dt$

$g'(x) = x\sin(1/x)$ which is diff $\Rightarrow$ g is cont. in $(0, \pi)$

$$l(x) = \begin{cases} x\,\sin x & 0 < x < \pi/2 \\ -\dfrac{\pi\sin x}{2} & \pi/2 < x < \pi \end{cases}$$

obvious discontinuity at $x = \pi/2 \Rightarrow$ (D)]

50 [Sol. $f(x) = \int_0^{\pi} \frac{t\sin t}{\sqrt{1+\tan^2 x\sin^2 t}}\,dt$

Using king and add.

$$f(x) = \frac{\pi}{2}\int_0^{\pi}\frac{\sin t}{\sqrt{1+\tan^2 x\sin^2 t}}\,dt \qquad = \pi\int_0^{\pi/2}\frac{\sin t}{\sqrt{1+\tan^2 x(1-\cos^2 t)}}\,dt$$

$$= \pi\int_0^{\pi/2}\frac{\sin t}{\sqrt{\sec^2 x-\tan^2 x\cos^2 t}}\,dt \;=\; \pi\int_0^{1}\frac{dy}{\sqrt{\sec^2 x-\tan^2 x.y^2}}$$

$$= \frac{\pi}{\tan x}\int_0^1\frac{dy}{\sqrt{\operatorname{cosec}^2 x - y^2}} \quad = \frac{\pi}{\tan x}\left\{\sin^{-1}\frac{y}{\operatorname{cosec} x}\right\}_0^1 = \frac{\pi}{\tan x}\sin^{-1}(\sin x) = \frac{\pi x}{\tan x}\quad]$$

51 [Sol. $I = \int_0^{n\pi+V} |\cos x|\,dx = \underbrace{\int_0^{n\pi} |\cos x|\,dx}_{2n} + \underbrace{\int_{n\pi}^{n\pi+V} |\cos x|\,dx}_{I_1\ (\text{put } x=n\pi+t)}$

$$\text{So, } I_1 = \int_0^V |\cos t|\,dt = \int_0^{\pi/2}\cos t\,dt - \int_{\pi/2}^{V}\cos x\,dx$$

$= 1 - (\sin x)_{\pi/2}^{V} = 1 - \sin V + 1$

$\therefore \quad I = 2n + 2 - \sin V$]

52 [Sol. $\displaystyle\int \frac{p\,x^{p+2q-1} - qx^{q-1}}{(x^{p+q}+1)^2}\,dx = \int \frac{p\,x^{p-1} - qx^{-q-1}}{(x^{p}+x^{-q})^2}\,dx$

taking x^q as x^{2q} common from Denominator and take it in N^r]

53 [Hint: for $0 < x < \ln 2$, $[2e^{-x}] = 1$, otherwise zero $\Rightarrow I = \displaystyle\int_0^{\ell n 2} dx + \int_{\ell n 2}^{\infty} 0\,dx = \ln 2$

Alternatively: Put $e^{-x} = t$; $-x = \ln t$; $dx = -\frac{1}{t}dt$; Hence $I = -\displaystyle\int_1^0 \frac{[2t]dt}{t} = \int_0^1 \frac{[2t]dt}{t}$

$I = \displaystyle\int_0^{1/2} 0\,dt + \int_{1/2}^{1} \frac{dt}{t} = \ln t\Big]_{1/2}^{1} = 0 - \ln\frac{1}{2} = \ln 2$ Ans.]

54 [Sol. $2\displaystyle\int_0^1 \frac{dx}{\sqrt{x}} = \left[\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}\right]_0^1 = 4\left[\sqrt{x}\right]_0^1 = 4 \Rightarrow$ (C)]

55 [Sol. $I = \displaystyle\int_0^1 x \ln\left(\frac{x+2}{2}\right) dx = \int_0^1 x(\ln(x+2) - \ln 2)\,dx$

$\therefore \quad I = \displaystyle\int_0^1 x\ln(x+2)dx - \ln 2\int_0^1 x\,dx$; hence $I = \ln(x+2).\frac{x^2}{2}\Big]_0^1 - \int_0^1 \frac{x^2}{x+2}\,dx - \frac{\ln 2}{2}$

$= \frac{1}{2}\ln 3 - \displaystyle\int_0^1 \frac{x^2-4+4}{x+2}\,dx - \frac{\ln 2}{2} \Rightarrow \frac{1}{2}\ln\frac{3}{2} - \int_0^1\left((x-2) + \frac{4}{x+2}\right)dx$ now proceed]

56 [Sol. $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\left(x+\sqrt{x}\right)dx$; put $x = t^2$; $dx = 2t\,dt$

$= \displaystyle\int e^t(t^2+t)\,dt = e^t(At^2 + Bt + C)$ (Let)

Diffrentiate both the sides

$e^t(t^2+t) = e^t(2At + B) + (At^2 + Bt + C)e^t$

On comparing coefficient we get

$A = 1$; $B = -1$; $C = 1$

57 [Hint: $I = \int_{-1}^{1} \frac{x^3}{x^2+2|x|+1}\,dx + \int_{-1}^{1} \frac{|x|+1}{(|x|+1)^2}\,dx \Rightarrow 2\int_0^1 \frac{dx}{1+x} = 2\ln 2$]

odd $\Rightarrow$ vanishes even]

58 [Hint: Let $I = \int_0^{\pi/2} \frac{\sin x\,dx}{1+\sin x+\cos x}$

$$I = \int_0^{\pi/2} \frac{\cos x}{1+\sin x+\cos x} \Rightarrow 2I = \int_0^{\pi/2} \frac{\sin x+\cos x+1-1}{\sin x+\cos x+1}\,dx$$

$$\Rightarrow 2I = \frac{\pi}{2} - \ln 2 \Rightarrow I = \frac{\pi}{4} - \frac{1}{2}\ln 2\ ]$$

59 [Sol. $\underset{x \to x_1}{\text{Limit}} \frac{\int_{x_1}^{x} f(t)\,dt}{\left(\frac{x-x_1}{x}\right)} = \underset{x \to x_1}{\text{Limit}} \frac{f(x).x^2}{x_1}$ (using Lopital's rule) $= x_1 f(x_1) \Rightarrow$ (B)]

60 [Sol. $I = \int_{-\pi/4}^{\pi/4} \ln(\cos x + \sin x)\,dx$

$I = \int_{-\pi/4}^{\pi/4} \ln(\cos x - \sin x)\,dx$ hence $2I = \int_{-\pi/4}^{\pi/4} \ln(\cos 2x)\,dx$

$= \int_0^{\pi/2} \cos t\,dt = -\frac{\pi}{2}\ln 2 \Rightarrow I = -\frac{\pi}{4}\ln 2$]

61 [Sol. $f(x) = \cos(\tan^{-1}x)$

$$f'(x) = -\frac{\sin(\tan^{-1}x)}{1+x^2}$$

$$I = \int_0^1 x\,f''(x)\,dx = x\,f'(x)\Big]_0^1 - \int_0^1 f'(x)\,dx$$

$$= f'(1) - [f(x)]_0^1 = f'(1) - [f(1) - f(0)] = f'(1) - f(1) + f(0)$$

$$f(0) = 1\ ;\ f'(1) = -\frac{1}{2\sqrt{2}}\ ;\ f(1) = \frac{1}{\sqrt{2}}\ ]$$

62 [Hint: note that $\sec^{-1}\sqrt{1+x^2} = \tan^{-1}x$; $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\tan^{-1}x$ for $x > 0$

$$I = \int \frac{e^{\tan^{-1}x}}{1+x^2}\left((\tan^{-1}x)^2 + 2\tan^{-1}x\right)dx \qquad \text{put } \tan^{-1}x = t$$

$$= \int e^t (t^2 + 2t)\,dt = e^t \cdot t^2 = e^{\tan^{-1}x}\left(\tan^{-1}x\right)^2 + C \;]$$

63 [Hint: $I = \int_1^2 1.(ln x)^2\,dx = ln^2 x . x\Big]_1^2 - 2\int_1^2 \frac{ln x}{x}.x\,dx = 2\ln^2 2 - 2\left[\int_1^2 ln\,x\,dx\right]$

$= 2\ln^2 2 - 2\left[x\,ln x - x\right]_1^2 = 2\ln^2 2 - 2\,[\,(2\ln 2 - 2)\,(0-1)\,)\,]$

$= 2\ln^2 2 - 2\,[\,2\ln 2 - 1] = 2\ln^2 2 - 4\ln 2 + 2 = 2\,[\,\ln^2 2 - 2\ln 2 + 1\,] = 2\left(ln\frac{2}{e}\right)^2 \Rightarrow$ (B)]

64 [Sol. Given $U_n = \int_0^1 x^n.(2-x)^n\,dx$; $V_n = \int_0^1 x^n.(1-x)^n\,dx$

in U_n put $x = 2t \Rightarrow dx = 2dt$

$\therefore \quad U_n = 2\int_0^{1/2} 2^n.t^n\,2^n\,(1-t)^n\,dt$ ....(1)

Now $V_n = 2\int_0^{1/2} x^n\,(1-x)^n\,dx$ (Using Queen) .....(2)

From (1) and (2)

$U_n = 2^{2n}.V_n \Rightarrow$ (C)]

65 [Hint: $S'(x) = \ln x^3 .\,3x^2 - \ln x^2 .\,2x \qquad = 9x^2 \ln x - 4x \ln x$

$= x\,ln\,x\,(9x-4)$. Hence $\frac{S'(x)}{x} = \ln x\,(9x-4)$.

Now it is obvious that $\frac{S'(x)}{x}$ is continuous and derivable in its domain.]

66 [Hint: using L Hospital's rule

$l = \lim_{x\to 0}\frac{-x\sin x}{2-2\cos 2x} = \lim_{x\to 0}\frac{-x\sin x}{2(2\sin^2 x)} = \lim_{x\to 0}\frac{-1}{4\frac{\sin x}{x}} = -\frac{1}{4}$]

67 [Hint: LHS = secx + cosecx $= 2\sqrt{2} \Rightarrow x = \frac{\pi}{4}$ and $\frac{11\pi}{12}$]

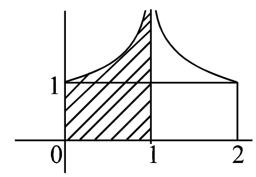
68 [Hint: $\lim_{n\to\infty} \dfrac{\sqrt{1}+\sqrt{2}+\sqrt{3}+\ldots\ldots\ldots\ldots+\sqrt{n}}{n\sqrt{n}} = \int_0^1 \sqrt{x}\,dx = \dfrac{2}{3}$

$\therefore \quad S_n = \dfrac{2}{3} n^{3/2}$]

69 [Sol. $\int_0^2 \dfrac{dx}{(1-x)^2} = \int_0^{1^-} \dfrac{dx}{(1-x)^2} + \int_{1^+}^2 \dfrac{dx}{(1-x)^2}$

$= \left.\dfrac{1}{1-x}\right]_0^{1^-} + \left.\dfrac{1}{1-x}\right]_{1^+}^2$

$= (\infty - 1) + (-1) - (-\infty) \Rightarrow$ indeterminant



Note that the shaded area is divergent]

70 [Hint: $I = \int_0^{\pi/2} \dfrac{\sin x \cos x}{x\left(\dfrac{\pi}{2}-x\right)}dx = \int_0^{\pi/2} \dfrac{\sin 2x}{x(\pi-2x)}dx$; put $2x = t$

$I = \int_0^{\pi} \dfrac{\sin t}{t(\pi-t)}dt = \dfrac{1}{\pi}\int_0^{\pi}\left(\dfrac{\sin t}{t}+\dfrac{\sin t}{(\pi-t)}\right)dt = \dfrac{1}{\pi}\int_0^{\pi}\dfrac{\sin t}{t}dt + \dfrac{1}{\pi}\int_0^{\pi}\dfrac{\sin t}{\pi-t}dt$

$= \dfrac{1}{\pi}\int_0^{\pi}\dfrac{\sin t}{t}dt + \dfrac{1}{\pi}\int_0^{\pi}\dfrac{\sin t}{t}dt = \dfrac{2}{\pi}\int_0^{\pi}\dfrac{\sin t}{t}dt$ Ans.]