EXERCISE - 1

1. (B)

Given $f(4)=6, f'(4)=1$

$$\therefore \lim_{x\to 4}\frac{xf(4)-4f(x)}{x-4}=\lim_{x\to 4}\frac{xf(4)-4f(4)+4f(4)-4f(x)}{x-4}$$

$$=\lim_{x\to 4}\frac{(x-4)f(4)}{x-4}-4\lim_{x\to 2}\frac{f(x)-f(4)}{x-4}$$

$$= f(4)-2f'(4)=4$$

2. (D)

$$f(x)=\begin{cases}x^3-1 & , \quad x\ge 1\\ 1-x^3 & , \quad x<1\end{cases} \quad \text{and} \quad f'(x)=\begin{cases}3x^2 & , \quad x\ge 1\\ -3x^2 & , \quad x<1\end{cases}$$

$$f'(1^+)=3,\ f'(1^-)=-3$$

3. (B)

$$f(x)=\sin 2x.\cos 2x.\cos 3x+\log_2 2^{x+3}\ ,$$

$$\Rightarrow f(x)=\frac{1}{2}\sin 4x\cos 3x+(x+3)\log_2 2\ ,$$

$$\Rightarrow f(x)=\frac{1}{4}[\sin 7x+\sin x]+x+3$$

Differentiate w.r.t. x,

$$f'(x)=\frac{1}{4}[7\cos 7x+\cos x]+1\ ,$$

$$\Rightarrow f'(\pi)=-2+1=-1\,.$$

4. (B)

(b) In neighborhood of $x=\frac{3\pi}{4}$, $|\cos^3 x|=-\cos^3 x$ and $|\sin^3 x|=\sin^3 x$

$$\therefore\ y=-\cos^3 x+\sin^3 x$$

$$\therefore\ \frac{dy}{dx}=3\cos^2 x\sin x+3\sin^2 x\cos x$$

At $x=\frac{3\pi}{4}$, $\frac{dy}{dx}=3\cos^2\frac{3\pi}{4}\sin\frac{3\pi}{4}+3\sin^2\frac{3\pi}{4}\cos\frac{3\pi}{4}=0\,.$

5. (D)

$$f(x)=|\log x|=\begin{cases}-\log x, & \text{if } 0<x<1\\ \log x, & \text{if } x\geq 1\end{cases}$$

$$\Rightarrow f'(x)=\begin{cases}-\dfrac{1}{x}, & \text{if } 0<x<1\\ \dfrac{1}{x}, & \text{if } x>1\end{cases}.$$

Clearly $f'(1^-)=-1$ and $f'(1^+)=1$,

$\therefore$ $f'(x)$ does not exist at $x=1$

6. (C)

$$\text{Let } y=\left[\log\left\{e^x\left(\frac{x-1}{x+1}\right)\right\}\right]=\log e^x+\log\left(\frac{x-1}{x+1}\right)$$

$$\Rightarrow y=x+[\log(x-1)-\log(x+1)]$$

$$\Rightarrow \frac{dy}{dx}=1+\left[\frac{1}{x-1}-\frac{1}{x+1}\right]=1+\frac{2}{(x^2-1)}$$

$$\Rightarrow \frac{dy}{dx}=\frac{x^2+1}{x^2-1}.$$

7. (A)

$$y=\sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)+\sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)$$

$$=\cos^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)+\sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right)=\frac{\pi}{2}$$

$$\Rightarrow \frac{dy}{dx}=0$$

8. (D)

$$\frac{d}{dx}\tan^{-1}\left[\frac{\cos x-\sin x}{\cos x+\sin x}\right]=\frac{d}{dx}\tan^{-1}\left[\tan\left(\frac{\pi}{4}-x\right)\right]=-1$$

9. (A)

$$y=\frac{(1-x)(1+x)(1+x^2)(1+x^4)(1+x^8)}{1-x}=\frac{1-x^{16}}{1-x}$$

$$\therefore \frac{dy}{dx}=\frac{-16x^{15}(1-x)+1-x^{16}}{(1-x)^2}, \quad \therefore \text{ At } x=0,\ \frac{dy}{dx}=1.$$

10. (A)

$$f(x)=\frac{2\sin x.\cos x.\cos 2x.\cos 4x}{2\sin x}=\frac{\sin 8x}{8\sin x}$$

$$\therefore f'(x)=\frac{1}{8}.\frac{8\cos 8x.\sin x-\cos x.\sin 8x}{\sin^2 x}$$

$$\therefore f'\left(\frac{\pi}{4}\right)=\sqrt{2}$$

11. (B)

$$\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}$$

$$=\frac{a[\cos\theta-\theta(-\sin\theta)-\cos\theta]}{a[-\sin\theta+\theta\cos\theta+\sin\theta]}=\frac{\theta\sin\theta}{\theta\cos\theta}=\tan\theta$$

12. (D)

Obviously $x=\cos^{-1}\frac{1}{\sqrt{1+t^2}}$ and $y=\sin^{-1}\frac{t}{\sqrt{1+t^2}}$

$\Rightarrow x=\tan^{-1}t$ and $y=\tan^{-1}t$

$\Rightarrow y=x \Rightarrow \frac{dy}{dx}=1$.

13. (C)

$x=\frac{1-t^2}{1+t^2}$ and $y=\frac{2t}{1+t^2}$

Put $t=\tan\theta$ in both the equations to get

$x=\frac{1-\tan^2\theta}{1+\tan^2\theta}=\cos 2\theta$ and $y=\frac{2\tan\theta}{1+\tan^2\theta}=\sin 2\theta$.

Differentiating both the equations, we get $\frac{dx}{d\theta}=-2\sin 2\theta$ and $\frac{dy}{d\theta}=2\cos 2\theta$.

Therefore $\frac{dy}{dx}=-\frac{\cos 2\theta}{\sin 2\theta}=-\frac{x}{y}$.

14. (D)

$$y=\sqrt{x+1+\sqrt{x+1+\sqrt{x+1 \ldots \text{to } \infty}}} \Rightarrow y=\sqrt{x+1+y}$$

$$\Rightarrow y^2=x+y+1 \Rightarrow 2y\frac{dy}{dx}=1+\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx}(2y-1)=1 \Rightarrow \frac{dy}{dx}=\frac{1}{2y-1}$$

15. (B)

$$y=(x+1)^{(x+1)^{(x+1)\ldots\ldots\infty}} \Rightarrow y=(x+1)^y$$

$$\Rightarrow \log_e y = y\log_e(x+1)$$

$$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx}=\frac{y}{(x+1)}+\ln(x+1)\frac{dy}{dx}$$

$$\Rightarrow \left(\frac{1}{y}-\ln(x+1)\right)\frac{dy}{dx}=\frac{y}{x+1}$$

$$\Rightarrow (x+1)(1-\ln y)\frac{dy}{dx}=y^2$$

16. (A)

$$\sqrt{1-x}+\sqrt{1-y}=1 \Rightarrow y=x+2\sqrt{1-x}-1$$

$$\Rightarrow \frac{dy}{dx}=1-\frac{1}{\sqrt{1-x}}=\frac{\sqrt{1-x}-1}{\sqrt{1-x}}$$

17. (B)

for $f(x)$, $y=x+\ln x$

then for $f^{-1}(x)$, $x=y+\ln y$

$$\Rightarrow \frac{dx}{dy}=1+\frac{1}{y} \text{ or } \frac{dy}{dx}=\frac{y}{1+y}$$

Further $x+\ln x=1 \Rightarrow x=1$, hence for $f^{-1}(x)$, $y=1$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{1+1}=\frac{1}{2}.$$

Ans.[B]

18. (A)

$$x = a\sec\theta \Rightarrow \frac{dx}{d\theta} = a\sec\theta\tan\theta,\ y = b\tan\theta \Rightarrow \frac{dy}{d\theta} = b\sec^2\theta$$

$$\Rightarrow \frac{dy}{dx} = \frac{b\sec^2\theta}{a\sec\theta\tan\theta} = \frac{b^2x}{a^2y}$$ **Ans.[A]**

19. (C)

$$\frac{dx}{d\theta} = a\,(1+\cos),\ \frac{dx}{d\theta} = a\sin\theta$$

$$\therefore \frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{a\sin\theta}{a(1+\cos\theta)} = \tan\frac{1}{2}\theta$$ **Ans.[C]**

20. (A)

$$y = \log e^x - \log(e^x + 1)$$
$$= x - \log(e^x + 1)$$

$$\therefore \frac{dy}{dx} = 1 - \frac{e^x}{e^x+1} = \frac{1}{e^x+1}$$ **Ans.[A]**

21. (B)

$$y = \frac{\sec x - \tan x}{\sec x + \tan x}\cdot\frac{\sec x - \tan x}{\sec x - \tan x}$$
$$= (\sec x - \tan x)^2/\,1$$

$$\therefore \frac{dy}{dx} = 2(\sec x - \tan x)\,(\sec x\tan x - \sec^2 x)$$
$$= -2\sec x\,(\sec x - \tan x)^2$$ **Ans.[B]**

22. (C)

$$y = e^{x+y}$$

$$\Rightarrow \log y = x + y \Rightarrow \frac{1}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{1-y}$$ **Ans.[C]**

23. (B)

When $x = 0$, $e^y = e \Rightarrow y = 1$

Differentiating w.r.t. x, we get

$$e^y \frac{dy}{dx} + y + x\frac{dy}{dx} = 0 \qquad ....(1)$$

$$e^y \frac{d^2y}{dx^2} = e^y\left(\frac{dy}{dx}\right)^2 + \frac{dy}{dx} + \frac{dy}{dx} + x\frac{d^2y}{dx^2} = 0 \qquad ....(2)$$

When $x = 0, y = 1$ $\quad \therefore$ From (1) $\frac{dy}{dx} = -\frac{1}{e}$

Putting the data in (2), we get

$$e \cdot \frac{d^2y}{dx^2} + e \cdot \frac{1}{e^2} - \frac{2}{e} = 0 \qquad \therefore \frac{d^2y}{dx^2} = \frac{1}{e^2}$$]

24. (A)

$$y = \tan^{-1}\left\{\sqrt{\frac{1+\cos x}{1-\cos x}}\right\}$$

$$= \tan^{-1}\left\{\sqrt{\frac{2\cos^2 x/2}{2\sin^2 x/2}}\right\}$$

$$= \tan^{-1}\left|\cot\frac{x}{2}\right| = \tan^{-1}\left(\cot\frac{x}{2}\right)$$

$$\Rightarrow y = \tan^{-1}\left\{\tan\left(\frac{\pi}{2} - \frac{x}{2}\right)\right\} = \frac{\pi}{2} - \frac{x}{2}$$

$$\therefore \frac{dy}{dx} = 0 - \frac{1}{2} = -\frac{1}{2}$$

25. (B)

$$f(x) = |x^2 - 5x + 6| = \begin{cases} x^2 - 5x + 6 & \text{if } x \geq 3 \text{ or } x \leq 2 \\ -(x^2 - 5x + 6), & \text{if } 2 < x < 3 \end{cases}$$

$$\Rightarrow f'(x) = \begin{cases} (2x-5), & \text{if } x > 3 \text{ or } x < 2 \\ -(2x-5), & \text{if } 2 < x < 3 \end{cases}$$

26. (C)

$$y'(x) = f'(f(f(f(x))))f'(f(f(x)))f'f(x)f'(x)$$

$$\Rightarrow y'(0) = f'(f(f(f(0))))f'(f(f(0)))f'(f(0))f'(0)$$

$$= f'(f(f(0)))f'(f(0))f'(0)f'(0)$$

$$= f'(f(0))f'(0)f'(0)f'(0)$$

$$= f'(0)f'(0)f'(0)f'(0)$$
$$= (f'(0))^4 = 2^4 = 16$$

27. (C)

$$y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!}$$
$$\Rightarrow \frac{dy}{dx} = 0 + 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^{n-1}}{(n-1)!}$$
$$\Rightarrow \frac{dy}{dx} + \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!}$$
$$\Rightarrow \frac{dy}{dx} = y - \frac{x^n}{n!}$$

28. (D)

7.d. $y = a\sin x + b\cos x$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = a\cos x - b\sin x$$

Now, $\left(\frac{dy}{dx}\right)^2 = (a\cos x - b\sin x)^2$

$$= a^2\cos^2 x + b^2\sin^2 x - 2ab\sin x\cos x, \text{ and}$$
$$y^2 = (a\sin x + b\cos x)^2$$
$$= a^2\sin^2 x + b^2\cos^2 x + 2ab\sin x\cos x$$

So, $\left(\frac{dy}{dx}\right)^2 + y^2 = a^2(\sin^2 x + \cos^2 x) + b^2(\sin^2 x + \cos^2 x)$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 + y^2 = (a^2 + b^2) = \text{constant.}$$

29. (B)

$$y = \sqrt{\frac{1-\sin 2x}{1+\sin 2x}} = \frac{\cos x - \sin x}{\cos x + \sin x} = \frac{1-\tan x}{1+\tan x} = \tan\left(\frac{\pi}{4} - x\right)$$
$$\Rightarrow \frac{dy}{dx} = -\sec^2\left(\frac{\pi}{4} - x\right)$$

30. (A)

$$\frac{dy}{dx}=\frac{d}{dx}\left[\left(x+\sqrt{x^2+a^2}\right)^n\right]$$

$$=n\left(x+\sqrt{x^2+a^2}\right)^{n-1}\cdot\frac{d}{dx}\left(x+\sqrt{x^2+a^2}\right)$$

$$=n(x+\sqrt{x^2+a^2})^{n-1}\left(\frac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a}}\right)$$

$$=\frac{n(x+\sqrt{x^2+a^2})^n}{\sqrt{x^2+a^2}}$$

$$=\frac{ny}{\sqrt{x^2+a^2}}$$

31. (B)

).b. $f(x)=\sqrt{1+\cos^2(x^2)}$

$$\Rightarrow f'(x)=\frac{1}{2\sqrt{1+\cos^2(x)^2}}(2\cos x^2)(-\sin x^2)(2x)$$

$$\Rightarrow f'(x)=\frac{-x\sin 2x^2}{\sqrt{1+\cos^2(x^2)}}$$

$$\Rightarrow f'\left(\frac{\sqrt{\pi}}{2}\right)=\frac{-\frac{\sqrt{\pi}}{2}\sin\frac{2\pi}{4}}{\sqrt{1+\cos^2\frac{\pi}{4}}}=\frac{-\frac{\sqrt{\pi}}{2}\cdot 1}{\sqrt{\frac{3}{2}}}$$

$$\therefore f'\left(\frac{\sqrt{\pi}}{2}\right)=-\sqrt{\frac{\pi}{6}}$$

32. (A)

$$\frac{d}{dx}\cos^{-1}\sqrt{\cos x}=\frac{\sin x}{2\sqrt{\cos x}\sqrt{1-\cos x}}$$

$$=\frac{\sqrt{1-\cos^2 x}}{2\sqrt{\cos x}\sqrt{1-\cos x}}=\frac{1}{2}\sqrt{\frac{1+\cos x}{\cos x}}$$

33. (C)

i.c. $y = \dfrac{\log \tan x}{\log \sin x}$

$$\Rightarrow \frac{dy}{dx} = \frac{(\log \sin x)\left(\dfrac{\sec^2 x}{\tan x}\right) - (\log \tan x)(\cot x)}{(\log \sin x)^2}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)_{\pi/4} = \frac{-4}{\log 2} \qquad \text{(On simplification)}$$

34. (B)

i.b. $y = \dfrac{\sin^{-1} x}{\sqrt{1-x^2}}$

$$\frac{dy}{dx} = \frac{\sqrt{1-x^2}\dfrac{1}{\sqrt{1-x^2}} - (\sin^{-1} x)\dfrac{1}{2}\dfrac{(-2x)}{\sqrt{1-x^2}}}{1-x^2}$$

$$\Rightarrow (1-x^2)\frac{dy}{dx} = 1 + x\left(\frac{\sin^{-1} x}{\sqrt{1-x^2}}\right) = 1 + xy$$

35. (A)

$$y = \cot^{-1}\left[\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right]$$

$$= \cot^{-1}\left[\frac{2+2\cos x}{2\sin x}\right] = \cot^{-1}\left[\frac{1+\cos x}{\sin x}\right]$$

$$= \cot^{-1}\left[\cot\frac{x}{2}\right] = \frac{x}{2}$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}$$

36. (C)

$$y = x^{\left(x^x\right)}$$

$$\Rightarrow \log y = x^x \log x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{dz}{dx}\log x + \frac{1}{x}z \ \text{(where } x^x = z\text{)}$$

$$\Rightarrow \frac{dy}{dx} = x^{\left(x^x\right)}\left[x^x(\log ex)\log x + x^{x-1}\right] \qquad \left(\because \frac{dz}{dx} = x^x \log ex\right)$$

38. (C)

$$y = ae^{mx} + be^{-mx}$$

$$\Rightarrow \frac{dy}{dx} = ame^{mx} - mbe^{-mx}$$

Again $\frac{d^2y}{dx^2} = am^2 e^{mx} + m^2 be^{-mx}$

$$\Rightarrow \frac{d^2y}{dx^2} = m^2\left(ae^{mx} + be^{-mx}\right) \Rightarrow \frac{d^2y}{dx^2} = m^2 y$$

$$\Rightarrow \frac{d^2y}{dx^2} - m^2 y = 0$$

EXERCISE - 2

1. (C)

$f(x+2y) = 2f(x)f(y) \Rightarrow 2f'(x+2y) = 2f(x)f'(y)$ {partially differentiating w.r.to y}

For x = 5 & y = 0, $f'(5) = f(5)f'(0) \Rightarrow f'(5) = 6$

2. (C)

By L'hospital's rule

$$\lim_{x\to 2}\frac{g^2(x)f^2(2) - f^2(x)g^2(2)}{x^2-4} = \lim_{x\to 2}\frac{g(x)g'(x)f^2(2) - f(x)f'(x)g^2(2)}{x}$$

$$= \frac{(-1)\times 4\times 9 - 3\times(-2)\times 1}{2} = -15$$

3. (B)

Given $5f(2x) + 3f\left(\frac{2}{x}\right) = 2x + 2$ ......(i)

Replacing x by $\frac{1}{x}$ in (i), $5f\left(\frac{2}{x}\right) + 3f(2x) = \frac{2}{x} + 2$ ......(ii)

On solving equation (i) and (ii), we get, $8f(2x) = 5x - \frac{3}{x} + 2$,

$$\Rightarrow 8f(x) = \frac{5x}{2} - \frac{6}{x} + 2$$

$$\therefore\ 8f'(x) = \frac{5}{2} + \frac{6}{x^2}$$

$$\because\ y = xf(x)\ \Rightarrow \frac{dy}{dx} = f(x) + xf'(x)$$

$$= \frac{1}{8}\left(\frac{5x}{2} - \frac{6}{x} + 2\right) + \frac{x}{8}\left(\frac{5}{2} + \frac{6}{x^2}\right)$$

at $x = 1$, $\frac{dy}{dx} = \frac{1}{8}\left(\frac{5}{2} - 6 + 2\right) + \frac{1}{8}\left(\frac{5}{2} + 6\right) = \frac{7}{8}$

4. (A)

$$x = \exp\left\{\tan^{-1}\left(\frac{y-x}{x}\right)\right\} \Rightarrow \log x = \tan^{-1}\left(\frac{y-x}{x}\right)$$

$$\Rightarrow \frac{y-x}{x} = \tan(\log x) \Rightarrow y = x\tan(\log x) + x$$

$$\Rightarrow \frac{dy}{dx} = \tan(\log x) + x\frac{\sec^2(\log x)}{x} + 1$$

$$\Rightarrow \frac{dy}{dx} = \tan(\log x) + \sec^2(\log x) + 1$$

At $x = 1$, $\frac{dy}{dx} = 2$.

5. (B)

Let $y = \sin^2\left(\cot^{-1}\sqrt{\frac{1-x}{1+x}}\right)$

Put $x = \cos\theta \Rightarrow \theta = \cos^{-1} x$

$$\Rightarrow y = \sin^2\left(\cot^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right) = \sin^2\left(\cot^{-1}\left(\tan\frac{\theta}{2}\right)\right)$$

$$\Rightarrow y = \sin^2\left(\frac{\pi}{2} - \frac{\theta}{2}\right) = \cos^2\frac{\theta}{2} = \frac{1}{2}(1+\cos\theta) = \frac{1}{2}(1+x)$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}$$

6. (A)

Let $\cos\alpha = \frac{5}{13}$. Then $\sin\alpha = \frac{12}{13}$. So, $y = \cos^{-1}\{\cos\alpha.\cos x - \sin\alpha.\sin x\}$

$\therefore y = \cos^{-1}\{\cos(x+\alpha)\} = x + \alpha$ ($\because$ $x + \alpha$ is in the first or the second quadrant)

$$\therefore \frac{dy}{dx} = 1.$$

7. (C)

$$y\left(\frac{\tan^2 2x - \tan^2 x}{1 - \tan^2 2x \tan^2 x}\right)\cot 3x = \left(\frac{\tan 2x - \tan x}{1 + \tan 2x \tan x}\right)\left(\frac{\tan 2x + \tan x}{1 - \tan 2x \tan x}\right)\cot 3x$$

$$\Rightarrow y = \tan x \tan 3x \cot 3x = \tan x$$

$$\Rightarrow \frac{dy}{dx} = \sec^2 x$$

8. (A)

$$f(x)=\cot^{-1}\left(\frac{x^x-x^{-x}}{2}\right)$$

Put $x^x=\tan\theta$, $\therefore\ y=f(x)=\cot^{-1}\left(\frac{\tan^2\theta-1}{2\tan\theta}\right)$

$= \cot^{-1}(-\cot 2\theta)\ = \pi-\cot^{-1}(\cot 2\theta)$

$\Rightarrow y\ =\ \pi-2\theta\ \ =\pi-2\tan^{-1}(x^x)$

$\Rightarrow \frac{dy}{dx}=\frac{-2}{1+x^{2x}}.x^x(1+\log x)$

$\Rightarrow f'(1)=-1$.

9. (A)

$xe^{x+y}=y+2\sin x\Rightarrow e^{x+y}+xe^{x+y}(1+y')=y'+2\cos x$

Now $x=0$ gives $y=0$, hence $\frac{dy}{dx}=-1$.

10. (D)

$\sin(3x-2y)=\log(3x-2y)\Rightarrow\left(3-2\frac{dy}{dx}\right)\cos(3x-2y)=\left(3-2\frac{dy}{dx}\right)\frac{1}{3x-2y}$

$\Rightarrow\frac{dy}{dx}=\frac{3}{2}$

11. (C)

$x^4y^5=2(x+y)^9\Rightarrow 4x^3y^5+5x^4y^4\frac{dy}{dx}=18(x+y)^8\left(1+\frac{dy}{dx}\right)$

$\Rightarrow 4\frac{2(x+y)^9}{x}+5\frac{2(x+y)^9}{y}\frac{dy}{dx}=18(x+y)^8\left(1+\frac{dy}{dx}\right)$

$\Rightarrow\frac{4}{x}-\frac{9}{x+y}=\left(\frac{9}{x+y}-\frac{5}{y}\right)\frac{dy}{dx}$

$\Rightarrow\frac{dy}{dx}=\frac{y}{x}$

12. (A)

$$y = x^2 + \frac{2}{y} \Rightarrow y^2 = x^2y + 2$$

$$\Rightarrow 2y\frac{dy}{dx} = y.2x + x^2\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2xy}{2y - x^2}$$

13. (C)

$$x = e^{2y+x}$$

Taking log both sides, $\log x = (2y + x)\log e = 2y + x$

$$\Rightarrow 2y + x = \log x \Rightarrow 2\frac{dy}{dx} + 1 = \frac{1}{x} \Rightarrow \frac{dy}{dx} = \frac{1-x}{2x}$$

14. (C)

$$x = \ln\left(y + \sqrt{1+y^2}\right) \Rightarrow \sqrt{1+y^2} + y = e^x \;\&\; \sqrt{1+y^2} - y = e^{-x}$$

$$\Rightarrow y = \frac{e^x - e^{-x}}{2}$$

$$\Rightarrow \frac{dy}{dx} = \frac{e^x + e^{-x}}{2} = \frac{e^{2x} + 1}{2e^x}$$ **Ans.[C]**

15. (B)

for $x > \frac{1}{2}$, $\sin^{-1}\left(3x - 4x^3\right) = \pi - 3\sin^{-1}x$

Now $y = \pi - 3\sin^{-1}x \Rightarrow \frac{dy}{dx} = -\frac{3}{\sqrt{1-x^2}}$ **Ans.[B]**

16. (C)

$$\frac{dy}{dx} = \frac{-2x}{(x^2 - a^2)^2} \Rightarrow \frac{d^2y}{dx^2}$$

$$= -\frac{(x^2-a^2)^2.2 - 2x.2(x^2-a^2).2x}{(x^2-a^2)^4}$$

$$= \frac{2(3x^2 + a^2)}{(x^2-a^2)^3}$$ **Ans.[C]**

17. (B)

Let us first express y in terms of x because all alternatives are in terms of x. So

$x\sqrt{1+y} = -y\sqrt{1+x}$

$\Rightarrow x^2 (1+y) = y^2 (1+x)$

$\Rightarrow x^2 - y^2 + x^2y - y^2 x = 0$

$\Rightarrow (x-y)(x+y+xy) = 0$

$\Rightarrow x + y + xy = 0 \qquad (\because x \neq y)$

$\Rightarrow y = -\dfrac{x}{1-x}$

$\therefore \dfrac{dy}{dx} = -\dfrac{(1+x)1 - x.1}{(1+x)^2} = -\dfrac{1}{(1+x)^2}$ **Ans.[B]**

18. (D)

Taking log on both sides, we have

$y \log x + x \log y = 0$

Now using partial derivatives, we have

$\dfrac{dy}{dx} = -\dfrac{y/x + \log y}{\log x + x/y} = -\dfrac{y(y + x\log y)}{x(x + y\log x)}$ **Ans [D]**

19. (B)

Here $y = \sqrt{\sin x + y} \quad \Rightarrow y^2 = \sin x + y$

$\therefore 2y\dfrac{dy}{dx} = \cos x + \dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = \dfrac{\cos x}{2y-1}$ **Ans.[B]**

20. (C)

$\lim\limits_{x\to 0} x^x = 1$; let $l = x^{x^x}$ hence as $x \to 0$, $x^x \to 1$

$\therefore \qquad L = (0)' - 1 = -1 \qquad \Rightarrow \qquad$ (C)

21. (D)

$f(x) = \dfrac{\ln(\ln x)}{\ln x}$

Now use Quotient Rule.

22. (D)

$y^4 = x^2 - 6$

$4y^3\frac{dy}{dx} = 2x \Rightarrow y^3\frac{dy}{dx} = \frac{x}{2} \Rightarrow y^3\frac{d^2y}{dx^2} + 3y^2\left(\frac{dy}{dx}\right)^2 = \frac{1}{2}$

$y^3\frac{d^2y}{dx^2} + 3y^2\left(\frac{x}{2y^3}\right)^2 = \frac{1}{2} \Rightarrow y^3\frac{d^2y}{dx^2} + 3y^2\frac{x^2}{4y^6} = \frac{1}{2} \Rightarrow y^3\frac{d^2y}{dx^2} + \frac{3x^2}{4y^4} = \frac{1}{2}$

$\Rightarrow y^3\frac{d^2y}{dx^2} = \frac{1}{2} - \frac{3x^2}{4y^4} \Rightarrow y^3\frac{d^2y}{dx^2} = \frac{2y^4 - 3x^2}{4y^4} \Rightarrow \frac{d^2y}{dx^2} = \frac{2y^4 - 3x^2}{4y^7}$

23. (C)

Let $f(x) = y \Rightarrow x = f^{-1}(y) = g(y) \Rightarrow x = e^{e^y}$

$\Rightarrow \frac{dx}{dy} = e^{e^y}\cdot e^y = e^{e^y + y} = g'(y)$

hence $g'(x) = e^{e^x + x}$

24. (D)

Let $y = \log x$

$\Rightarrow y_1 = \frac{1}{x}, y_2 = \frac{-1}{x^2}, y_3 = \frac{2}{x^3}, \ldots, y_n = \frac{(-1)^{n-1}(n-1)!}{x^n}$.

25. (C)

i.e. $y = \sqrt{\log x + y}$

$\Rightarrow y^2 = \log x + y$

$\Rightarrow 2y\frac{dy}{dx} = \frac{1}{x} + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{x(2y-1)}$

26. (C)

$f(\log_e x) = \log_e(\log_e x)$

$\frac{df(\ln x)}{dx} = \frac{1}{\ln x}\cdot\frac{1}{x}$

27. (A)

$$y=\sec(\tan^{-1}x)=\sec\left(\sec^{-1}\sqrt{1+x^2}\right)=\sqrt{1+x^2}$$

Differentiating w.r.t. x, we have $\frac{dy}{dx}=\frac{x}{\sqrt{1+x^2}}$

$$\Rightarrow\left(\frac{dy}{dx}\right)_{x=1}=\frac{1}{\sqrt{2}}$$

28. (A)

$$y=f(x^2)\Rightarrow\frac{dy}{dx}=f'(x^2)2x=2x\sqrt{2(x^2)^2-1}$$

At $x=1$, $\frac{dy}{dx}=2\times1\times\sqrt{2-1}=2$

29. (A)

$$\frac{du}{dv}=\frac{\frac{du}{dx}}{\frac{dv}{dx}}=\frac{f'(x^3)3x^2}{g'(x^2)2x}=\frac{\cos x^3 3x^2}{\sin x^2 2x}$$

$$=\frac{3}{2}x\cos x^3\operatorname{cosec}x^2$$

30. (B)

24.b. $\frac{dx}{dy}=\frac{\frac{dx}{dt}}{\frac{dy}{dt}}=\frac{\cos t-t\sin t}{1+\cos t}$

$$\therefore\ \frac{d^2x}{dy^2}=\frac{\frac{d}{dt}\left(\frac{dx}{dy}\right)}{\frac{dy}{dt}}$$

$$=\frac{\frac{(-2\sin t-t\cos t)(1+\cos t)-(\cos t-t\sin t)(-\sin t)}{(1+\cos t)^2}}{1+\cos t}$$

Now, put $t=\pi/2$

31. (C)

.c. $f(x)=\sqrt{1-\sin 2x}=\sqrt{(\cos x-\sin x)^2}$

$=|\cos x-\sin x|$

$=\begin{cases}\cos x-\sin x, & \text{for } 0\le x\le \pi/4\\ -(\cos x-\sin x), & \text{for } \pi/4<x\le\pi/2\end{cases}$

$\therefore\ f'(x)=\begin{cases}-(\cos x+\sin x), & \text{for } 0<x<\pi/4\\ (\cos x+\sin x), & \text{for } \pi/4<x<\pi/2.\end{cases}$

32. (D)

26.d. Let $u=y^2$ and $v=x^2$

$\therefore\ \frac{du}{dx}=\frac{d}{dx}y^2=\left(\frac{d}{dy}y^2\right)\left(\frac{dy}{dx}\right)$

$=2y(1-2x)=2(x-x^2)(1-2x)=2x(1-x)(1-2x)$ (1)

and $\frac{dv}{dx}=2x$ (2)

Hence, $\frac{du}{dv}=\frac{\left(\frac{du}{dx}\right)}{\left(\frac{dv}{dx}\right)}=\frac{2x(1-x)(1-2x)}{2x}$ (from (1) and (2))

$=(1-x)(1-2x)=1-3x+2x^2$

33. (A)

7.a. $f(x)=\cos^{-1}\left[\cos\left(\frac{\pi}{2}-\sqrt{\frac{1+x}{2}}\right)\right]+x^x=\frac{\pi}{2}-\sqrt{\frac{1+x}{2}}+x^x$

$\Rightarrow f'(x)=-\frac{1}{\sqrt{2}}\times\frac{1}{2\sqrt{1+x}}+x^x(1+\log x)$

$\Rightarrow f'(1)=-\frac{1}{4}+1=\frac{3}{4}$

34. (B)

$2xf'(x^2)=3x^2\ \Rightarrow\ 4f'(4)=12\ \Rightarrow\ f'(4)=3$

35. (C)

$(a^2-2a-15)e^{ax}+(b^2-2b-15)e^{bx}=0$
$\Rightarrow (a^2-2a-15)=0$ and $b^2-2b-15=0$
$\Rightarrow (a-5)(a+3)=0$ and $(b-5)(b+3)=0$
$\Rightarrow a=5$ or -3 and $b=5$ or -3
$\therefore a\neq b$ hence $a=5$ and $b=-3$
or $a=-3$ and $b=5$
$\Rightarrow ab=-15$

36. (B)

b. $y=\dfrac{(a-x)^{3/2}+(x-b)^{3/2}}{\sqrt{a-x}+\sqrt{x-b}}$

$$=\frac{\left(\sqrt{a-x}+\sqrt{x-b}\right)\left(a-x-\sqrt{a-x}\sqrt{x-b}+x-b\right)}{\sqrt{a-x}+\sqrt{x-b}}$$

$$=a-b-\sqrt{a-x}\sqrt{x-b}$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{2\sqrt{a-x}}\sqrt{x-b}-\frac{1}{2\sqrt{x-b}}\sqrt{a-x}$$

$$=\frac{2x-a-b}{2\sqrt{a-x}\sqrt{x-b}}$$

37. (B)

2.b. $f(g(x))=x$
$\Rightarrow f'(g(x))g'(x)=1$
$\Rightarrow (e^{g(x)}+1)g'(x)=1$
$\Rightarrow (e^{g(f(\log 2))}+1)g'(f(\log 2))=1$
$\Rightarrow (e^{\log 2}+1)g'(f(\log 2))=1$
$\Rightarrow g'(f(\log 2))=1/3$

38. (C)

.c. $f'(x)=(kx+e^x)h'(x)+h(x)(k+e^x)$
$f'(0)=h'(0)+h(0)(k+1)$

1. **(B)**

Let $f(x)=3x^{10}-7x^{8}+5x^{6}-21x^{3}+3x^{2}-7$

$f'(x)=30x^{9}-56x^{7}+30x^{5}-63x^{2}+6x$

$f'(1)=30-56+30-63+6$

$=66-63-56=-53$

Consider $\lim\limits_{\alpha\to 0}\dfrac{f(1-\alpha)-f(1)}{\alpha^{3}+3\alpha}$

$=\lim\limits_{\alpha\to 0}\dfrac{f'(1-\alpha)(-1)-0}{3\alpha^{2}+3}$ (By using L'Hospital rule)

$=\dfrac{f'(1-0)(-1)}{3(0)^{2}+3}=\dfrac{-f'(1)}{3}=\dfrac{53}{3}$

2. **(B)**

Let f(x) = $\tan^{-1}\left(\dfrac{6x\sqrt{x}}{1-9x^{3}}\right)$ where $x\in\left(0,\dfrac{1}{4}\right)$.

$=\tan^{-1}\left(\dfrac{2.\left(3x^{3/2}\right)}{1-\left(3x^{3/2}\right)^{2}}\right)=2\tan^{-1}(3x^{3/2})$

As $3x^{3/2}\in\left(0,\dfrac{3}{8}\right)$ $\left[\because 0<x<\dfrac{1}{4}\Rightarrow 0<x^{3/2}<\dfrac{1}{8}\Rightarrow 0<3x^{3/2}<\dfrac{3}{8}\right]$

So $\dfrac{dx\,f(x)}{dx}=2\times\dfrac{1}{1+9x^{3}}\times 3\times\dfrac{3}{2}\times x^{1/2}=\dfrac{9}{1+9x^{3}}\sqrt{x}$

On comparing

$\therefore g(x)=\dfrac{9}{1+9x^{3}}$

3. **(D)**

$y=\left[x+\sqrt{x^{2}-1}\right]^{15}+\left[x-\sqrt{x^{2}-1}\right]^{15}$

Differentiate w.r.t. 'x'

$\dfrac{dy}{dx}=15\left(x+\sqrt{x^{2}-1}\right)^{14}\left[1+\dfrac{x}{\sqrt{x^{2}-1}}\right]$

$=15\left(x-\sqrt{x^{2}-1}\right)^{14}\left[1-\dfrac{x}{\sqrt{x^{2}-1}}\right]$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{15}{\sqrt{x^2-1}}.y \qquad ...(i)$$

$$\Rightarrow \quad \sqrt{x^2-1}.\frac{dy}{dx} = 15y$$

Again differentiating both sides w.r.t. x

$$\frac{x}{\sqrt{x^2-1}}.\frac{dy}{dx} + \sqrt{x^2-1}\frac{d^2y}{dx^2} = 15\frac{dy}{dx}$$

$$\Rightarrow x\frac{dy}{dx} + \left(x^2-1\right)\frac{d^2y}{dx^2}$$

$$= \quad 15\sqrt{x^2-1}.\frac{15}{\sqrt{x^2-1}}.y = 225y$$

4. (D)

$\because$ f(x) has extremum values of $x = 1$ and $x = 2$

$\because$ f'(1) = 0 and f'(2) = 0

As, f(x) is a polynomial of degree 4.

Suppose $f(x) = Ax^4 + Bx^3 + Cx^2 + Dx + E$

$$\because \quad \lim_{x\to 0}\left(\frac{f(x)}{x^2}+1\right) = 3$$

$$\Rightarrow \lim_{x\to 0}\left(\frac{Ax^4+Bx^3+Cx^2+Dx+E}{x^2}+1\right) = 3$$

$$\Rightarrow \lim_{x\to 0}\left(Ax^2+Bx+C+\frac{D}{x}+\frac{E}{x^2}+1\right) = 3$$

As limit has finite value, so D = 0 and E = 0

Now $A(0)^2 + B(0) + C + 0 + 0 + 1 = 3$

$\Rightarrow c + 1 = 3 \Rightarrow c = 2$

$f'(x) = 4Ax^3 + 3Bx^2 + 2Cx + D$

$f'(1) = 0 \Rightarrow 4A(1) + 3B(1) + 2C(1) + D = 0$

$\Rightarrow 4A + 3B = -4$ ...(i)

$f'(2) = 0 \Rightarrow 4A(8) + 3B(4) + 2C(2) + D = 0$

$\Rightarrow 8A + 3B = -2$ ...(ii)

From equation (i) and (ii) we get

$A = \frac{1}{2}$ and $B = -2$

So, $f(x) = \frac{x^4}{2} - 2x3 + 2x^2$

Therefore, $f(-1)=\frac{(-1)^4}{2}-2(-1)^3+2(-1)^2$

$=\frac{1}{2}+2+2=\frac{9}{2}$. Hence $f(-1)=\frac{9}{2}$

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5. **(B)**

Here, $\frac{dx}{dt}=\frac{1}{2\sqrt{2^{\operatorname{cosec}^{-1}}t}}2^{\operatorname{cosec}^{-1}}t\log 2.\frac{-1}{x\sqrt{x^2-1}}$

$\frac{dy}{dt}=\frac{1}{2\sqrt{2^{\sec^{-1}}t}}2^{\sec^{-1}}t\log 2.\frac{-1}{x\sqrt{x^2-1}}$

$\therefore \quad \frac{dy}{dt}=\frac{\frac{dy}{dx}}{\frac{dx}{dy}}=\frac{1-\sqrt{2^{\operatorname{cosec}^{-1}}t}}{\sqrt{2^{\sec^{-1}}t}}\frac{2^{\sec^{-1}}t}{2^{\operatorname{cosec}^{-1}}t}$

$\frac{dy}{dt}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=-\sqrt{\frac{2^{\sec^{-1}}t}{2^{\operatorname{cosec}^{-1}}t}}=\frac{-y}{x}$

6. **(C)**

$(a+\sqrt{2}b\cos x)(a-\sqrt{2}b\cos x)=a^2-b^2$

Differentiating both sides,

$\left(-\sqrt{2}b\sin x\right)\left(a-\sqrt{2}b\cos y\right)+\left(a+\sqrt{2}b\cos x\right)\ \left(\sqrt{2}b\sin y\right)\frac{dy}{dx}=0$

$\Rightarrow \frac{dy}{dx}=\frac{\left(\sqrt{2}b\sin x\right)\left(a-\sqrt{2}b\cos y\right)}{\left(a+\sqrt{2}b\cos x\right)\left(\sqrt{2}b\sin y\right)}$

$\therefore \left[\frac{dy}{dx}\right]_{\left(\frac{\pi}{4},\frac{\pi}{4}\right)}=\frac{a-b}{a+b}\Rightarrow\frac{dx}{dy}=\frac{a+b}{a-b}$

7. **(A)**

$\lim_{x\to a}\frac{xf(a)-af(x)}{x-a} \qquad \left[\frac{0}{0}\text{form}\right]$

On applying L' Hospital rule, we get

$= \quad \lim_{x\to a}\frac{f(a)-af'(x)}{1}$

$f(a) - af'(a) = 4 - 2a$ $\quad [\because f'(a) = 2 \text{ and } f(a) = 4]$

8. (A)

$$y(x) = \cot^{-1}\left[\frac{\cos\frac{x}{2} + \sin\frac{x}{2} + \sin\frac{x}{2} - \cos\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2} - \sin\frac{x}{2} + \cos\frac{x}{2}}\right]$$

$$y(x) = \cos^{-1}\left(\tan\frac{x}{2}\right) = \frac{\pi}{2} - \frac{x}{2}$$

9. (B)

Given function is $y - \tan^{-1}(\sec x^3 - \tan x^3)$.

$$= \tan^{-1}\left(\frac{1 - \sin x^3}{\cos x^3}\right) = \tan^{-1}\left(\frac{1 - \cos\left(\frac{\pi}{2} - x^3\right)}{\sin\left(\frac{\pi}{2} - x^3\right)}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} - \frac{x^3}{2}\right)\right); \text{ we have , } \frac{\pi}{4} - \frac{x^3}{2} \in \left(\frac{\pi}{4}, 0\right)$$

Let $y = \left(\frac{\pi}{4} - \frac{x^3}{2}\right)$ ...(i)

Differnetiate w.r.t.x.

$y' = \frac{-3x^2}{2}, y'' = -3x$ [from equation (i),]

$$4y = \pi - 2x^3 \Rightarrow 4y = \pi - 2x^2\left(\frac{-y''}{3}\right)$$

$$12y = 3\pi + 2x^2y''$$

Required equation, $x^2y'' - 6y + \frac{3\pi}{2} = 0$.

10. (D)

Given function is

$$\cos^{-1}\left(\frac{y}{2}\right) = \log_e\left(\frac{x}{5}\right)^5 \Rightarrow \cos^{-1}\left(\frac{y}{2}\right) = 5\log_e\left(\frac{x}{5}\right)$$

Differentiate w.r.t.x.

$$\frac{-1}{\sqrt{1-\frac{y^2}{4}}}.\frac{y'}{2}=5.\frac{1}{\frac{x}{5}}\times\frac{1}{5}\Rightarrow\frac{-y'}{\sqrt{4-y^2}}=\frac{5}{x} \quad ...(i)$$

$$-xy'=5\sqrt{4-y^2}$$

Again, differentiate w.r.t.x.

$$-xy''-y'=5.\frac{1}{2\sqrt{4-y^2}}\left(-2yy'\right)$$

$$\Rightarrow \quad xy''+y'=\frac{5y'.y}{\sqrt{4-y^2}}\Rightarrow xy''+y'=5.\left(\frac{-5}{x}\right)y \quad \{\text{From (i)}\}$$

$$x^2y''+xy'=-25y$$

Required differential equation is $x^2y''+xy'=-25y$.

11. **(D)**

Take $\log_e 2\frac{d}{dx}(\log_{\cos x}\cos ecx)$

Let, $y=\log_{\cos x}\cos ex$

$$y=-\frac{\ln(\sin x)}{\ln(\cos x)}$$

Diff. w.r.t.x both sides,

$$\frac{dy}{dx}=-\frac{\left[\cot x.\ln(\cos x)+\tan x.\ln(\sin x)\right]}{\left(\ln(\cos x)\right)^2}$$

$$\left.\frac{dy}{dx}\right)_{x=\frac{\pi}{4}}=\frac{4}{\ln 2}$$

Now, $\Rightarrow \log_e 2.\frac{4}{\ln 2}=4$

12. **(C)**

Since, $f(x)=x^3-x^2f'(1)+xf'(2)-f''(3), x\in R$

Let $f'(1)=a, f''(2)=b, f'''(3)=c$

Now $f(x)=x^3-ax^2+bx-c$

$\Rightarrow f'(x)=3x^2-2ax+b, f''(x)=6x-2a, f'''(x)=6$

So, $c = 6, a = 3, b = 6; f(x) = x^3 - 3x^2 + 6x - 6$

$f(1)=-2, f(2)=2, f(3)=12, f(0)=-6$

thus, $2f(0)-f(1)+f(3)=2=f(2)$

13. (B)

Since, given

$$f(\theta)=3\left(\sin^4\left(\frac{3\pi}{2}-\theta\right)+\sin^4(3x+\theta)\right)-2\left(1-\sin^2 2\theta\right)$$

and $S=\left\{\theta\in[0,\pi]: f'(\theta)=-\frac{\sqrt{3}}{2}\right\}$

Now $f(\theta)=3\left(\cos^4\theta+\sin^4\theta\right)-2\cos^2 2\theta$

$$\Rightarrow f(\theta)=3\left(1-\frac{1}{2}\sin^2 2\theta\right)-2\cos^2 2\theta$$

$$\Rightarrow f(\theta)=3-\frac{3}{2}\sin^2 2\theta-2\cos^2\theta$$

$$=\frac{3}{2}-\frac{1}{2}\cos^2 2\theta=\frac{3}{2}-\frac{1}{2}\left(\frac{1+\cos 4\theta}{2}\right)$$

$$f(\theta)=\frac{5}{4}-\frac{\cos 4\theta}{4}$$

Since $f'(\theta)=\sin 4\theta$

Given, $f'(\theta)=\sin 4\theta=-\frac{\sqrt{3}}{2}$

$$\Rightarrow 4\theta=n\pi+(-1)^n\frac{\pi}{3}\Rightarrow\theta=\frac{n\pi}{4}+(-1)^n\frac{\pi}{12}$$

$$\theta=\frac{\pi}{12},\left(\frac{\pi}{4}-\frac{\pi}{12}\right),\left(\frac{\pi}{2}-\frac{\pi}{12}\right),\left(\frac{3\pi}{4}-\frac{\pi}{12}\right)$$

$$4\beta=\frac{\pi}{4}+\frac{\pi}{2}+\frac{3\pi}{4}=\frac{3\pi}{2}\Rightarrow\beta=\frac{3\pi}{8}$$

$$\Rightarrow f(\beta)=\frac{5}{4}-\frac{\cos\frac{3\pi}{2}}{4}=\frac{5}{4}$$

14. (B)

Let $y\sin^3\left(\pi/3\cos g(x)\right)$

where $g(x)=\frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{3/2}$

and $g(1)=2\pi/3$

Now, $y' = 3\sin^2\left(\frac{\pi}{3}\cos g(x)\right)\times\left(\frac{\pi}{3}\cos g(x)\right)\times\frac{\pi}{3}\left(-\sin(x)\right)g'(x)$

At x = 1

$$y'(1) = 3\sin^2\left(-\frac{\pi}{6}\right).\cos\left(\frac{\pi}{6}\right).\frac{\pi}{3}\left(-\sin\frac{2\pi}{6}\right)g'(1)$$

and $g'(x) = \frac{\pi}{3\sqrt{2}}\left(-4x^3+5x^2+1\right)^{3/2}\left(-12x^2+10x\right)$

$$\Rightarrow g'(1) = \frac{\pi}{2\sqrt{2}}\left(\sqrt{2}\right)(-2) = -\pi$$

So, $\Rightarrow y'(1) = \frac{\cancel{3}}{4}.\frac{\sqrt{3}}{2}.\frac{\pi}{\cancel{3}}\left(\frac{-\sqrt{3}}{2}\right)(-\pi) = \frac{3\pi^2}{16}$

and $y(1) = \sin^3(\pi/3\cos 2\pi/3) = -\frac{1}{8}$

Thus, $2y'(1)+3\pi^2 y(1) = 0$

15. **(B)**

Let $x^y = u \Rightarrow y\text{lm}\,x = \ln u$

$\Rightarrow \frac{du}{dx} = x^y\left[\frac{y}{x}+\ln x\frac{dy}{dx}\right]$ and $y^x = v \Rightarrow x\ln y = \ln v$

$$\Rightarrow \frac{dv}{dx} = y^x\left[\ln y+\frac{x}{y}\frac{dy}{dx}\right]$$

Now, $2x^y+3y^x = 20 \Rightarrow 2u+3v = 20 \Rightarrow \frac{2du}{dx}+\frac{3dv}{dx} = 0$

$$2x^y\left[\frac{y}{x}+\ln x\frac{dy}{dx}\right]+3y^x\left[\ln y+\frac{x}{y}\frac{dy}{dx}\right] = 0$$

At (x, y) = (2,2); $\frac{dy}{dx} = \frac{-(12\ln 2+8)}{12+8\ln 2} = -\left(\frac{2+\log_e 8}{3+\log_e 4}\right)$

16. (B)

$$f(x)=\frac{\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x\right)-1}{\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x-\frac{1}{\sqrt{2}}\cos x\right)}$$

$$=\frac{\sin\left(x(x+\pi/4\right)-1}{\sin\left(x-\pi/4\right)}$$

$$f'(x)=\frac{\cos(x+\pi/4)\sin\left(x-\pi/4\right)-\cos\left(x-\pi/4\right)\left(\sin\left(x+\pi/4\right)-1\right)}{\sin^2\left(x-\pi/4\right)}$$

$$f'(x)=\frac{-(1-\cos(x+\pi/4))}{1-\cos^2\left(x-\pi/4\right)}$$

$$f'(x)=\frac{1}{1+\cos\left(x-\pi/4\right)}\Rightarrow\quad f'(x)=\frac{-\sin\left(x-\pi/4\right)}{\left(1+\cos\left(x-\pi/4\right)\right)^2}\Rightarrow$$

$$\Rightarrow\quad f\left(\frac{7\pi}{12}\right)=\frac{-1}{\sqrt{3}},f'(7\pi/12)=-2\sqrt{3}/9$$

$$f\left(\frac{7\pi}{12}\right).f''\left(\frac{7\pi}{12}\right)=\frac{2}{9}$$

17. (40)

Put $=0$

$\ln\ y=0\Rightarrow y=1$

$\ln(x+y)=4xy$

Diff. w.r.t.x $\frac{1}{x+y}\left(1+\frac{dy}{x}\right)=4\left(\frac{dy}{dx}+x\frac{dy}{dx}\right)$

at $x=0,\ y=1\ \Rightarrow\frac{dy}{dx}=3$

$1+\frac{dy}{dx}=4\left(x+y\right)\left(x\frac{dy}{dx}+y\right)$ ....(ii)

Diff eq (ii) w.r.t. x $\frac{d^2y}{dx^2}=40$

18. (248)

Given

$f(x+y)=2^x.\ f(y)+4y.\ f(x)$.

Put $y=2$

$2^x\ f(2)+4^2\ f(x)$

$f(x + 2) = 2^x . 3 + 16 f(x)$

$f'(x+2) = 16f'(x) + 3.2^x \ln 2$

$f'(4) = 16f'(2) + 12\ln 2$ ...(i)

$f(y + 2) = 4 f(y) + 3. 4^y$

$f'(4) = 4f'(2) + 96\ln 2$ ...(ii)

On solving eqs. (i) and (ii) , we get

$f'(2) = 7\ln 2$

From equation (i), we get

$f'(4) = 2^4.3\ln 2$

Now , $14.\dfrac{f'(4)}{f'(2)}$

$14 \times \dfrac{2^4.31\ln 2}{7\ln 2}$

or $\dfrac{14 \times 124}{7} = 248$

19. **(2)**

Given $f(x)a_0x^2 + a_1x + a^2$

Differentiate w.r.t. x.

$f'(x) = 2a_0x + a_1$; Put $x = 0, 1$

$f'(0) = a_1 = 1; f'(1) = 2a_0 + a_1 = 0$

$2a_0 = -1 \Rightarrow a_0 = \dfrac{-1}{2}$

A G.P series is a , $(a+d)r, (a+2d)r^2, ...(a+(n-1)d)r^{n-1}$

Here, $d = 1, r = 2$ and $a = -\dfrac{1}{2}$

A G.P. series = $\dfrac{-1}{2}, \left(-\dfrac{1}{2}+1\right).2, \left(-\dfrac{1}{2}+2\right)2^2$

$\dfrac{-1}{2}, \left(\dfrac{1}{2}\right).2, \left(\dfrac{3}{2}\right).2^2 = \dfrac{-1}{2}, 1, 6$

So, $f(x) = a_0x^2 + a_1x + a_2 \Rightarrow f(x) = \dfrac{-1}{2}x^2 + x + 6$

Put x = 4 in above f(x).

$f(4) = \dfrac{-1}{2} \times 16 + 4 + 6 \Rightarrow f(4) = 2$

Therefore, the value of f(4) is 2.

20. **(16)**

Given function is

$y(x)=\left(x^x\right)^x=x^{x^2}$

Take log both sides,

$l\,n\,y(x)=x^2.l\,n\,x$

Differentiate w.r.t. x both sides,

$\frac{1}{y(x)}y'(x)=\frac{x^2}{x}+2x.l\,n\,x$

$y'(x)=y(x)[x+2x l\,n\,x]$

21. **(14)**

Given $f(x)=x^2+g'(1)x+g''(2)$ ...(i)

$f'(x)=2x\_g'(1)$

$f'(x)=2$

$g(x)=f(1)x^2+xf'(x)+f''(x)$

$g'(x)=2f(1)x+4x+g'(1)\Rightarrow \text{put } x=1, f(1)=-2$

$g''(x)=2f(1)+4\Rightarrow g''(x)=0$ [from (i)]

$g'(1)=-3$

So, $f'(x)=2x-3; f(x)=x^2-3x+c; c=0$

$f(x)=x^2-3x; g(x)=-3x+2; f(4)-g(4)=14$

22. **(10)**

Given, $f(x)=\sum_{k=1}^{10}kx^k$

Now, $f(x)=x+2x^2+......+10x^{10}$

$f(x).x=x^2+2x^3+.....+9x^{10}+10x^{11}$

$f(x)\left(1-x\right)=x+x^2+x^3+.....+x^{10}-10x^{11}$

$f(x)=\frac{x\left(1-x^{10}\right)}{\left(1-x\right)^2}-\frac{10x^{11}}{\left(1-x\right)}$