Elasticity

1. (B)

$$\Delta l = \frac{Fl}{\pi r^2 Y} \Rightarrow \frac{\Delta l_1}{\Delta l_2} = \left(\frac{F_1}{F_2}\right)\left(\frac{l_1}{l_2}\right)\left(\frac{r_2}{r_1}\right)^2$$

$$\Rightarrow \frac{l}{\Delta l_2} = \left(\frac{F}{2F}\right)\left(\frac{L}{2L}\right)\left(\frac{2r}{r}\right)^2$$

$$\Rightarrow \Delta l_2 = l$$

2. (C)

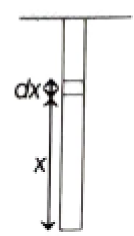
Slope $= \frac{F}{\Delta l} = \frac{AY}{l} \Rightarrow \text{Slope} \propto A$

Slope is least for wire *A*. So, wire *A* will be thinnest.

3. (A)

$$\Delta l = \frac{Fl}{AY},$$

Slope $= \frac{\Delta l}{F} = \frac{(4-1)\times 10^{-4}}{80-20}$



$$\Rightarrow \frac{l}{AY} = \frac{1}{20}\times 10^{-4}$$

$$\Rightarrow \frac{1}{10^{-6}Y} = \frac{1}{20}\times 10^{-4}$$

$$\Rightarrow Y = 2\times 10^{11}\ \text{N/m}^2$$

4. (B)

$$\Delta l = \frac{Fl}{AY}$$

$$\Rightarrow \frac{\Delta l_1}{\Delta l_2} = \left(\frac{F_1}{F_2}\right)\left(\frac{l_1}{l_2}\right)\left(\frac{r_2}{r_1}\right)^2\left(\frac{Y_2}{Y_1}\right)$$

$$= \left(\frac{3mg}{2mg}\right)(a)\left(\frac{1}{b}\right)^2\left(\frac{1}{c}\right) = \frac{3a}{2b^2c}$$

5. (C)

Let natural length be *l*.

$$F = kx$$

$$\Rightarrow T_1 = k(l_1 - l) \qquad \text{...(i)}$$

and $T_2 = k(l_2 - l) \qquad \text{...(ii)}$

Dividing Eq. (i) by Eq. (ii), we get

$$l = \frac{l_1 T_2 - l_2 T_1}{T_2 - T_1}$$

6. (A)

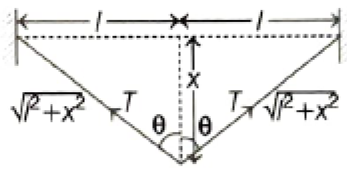
Breaking stress depends only on the material of wire.

7. (C)

$$T = \frac{2m_1m_2g}{m_1+m_2} = \frac{4mg}{3}$$

$$\text{Breaking stress} = \frac{T}{A} \Rightarrow S = \frac{\frac{4mg}{3}}{\pi r^2} \Rightarrow r = \sqrt{\frac{4mg}{3\pi S}}$$

8. (C)



$$\text{Strain } = \frac{\Delta l}{l} = \frac{2\sqrt{l^2+x^2-2l}}{2l} = \left(1+\frac{x^2}{l^2}\right)^{\frac{1}{2}} - 1$$

$$= 1 + \frac{x^2}{2l^2} - 1 \qquad \{x << l\}$$

$$= \frac{x^2}{2l^2}$$

9. (D)

$$2\cos\theta = w$$

$$\Rightarrow \quad T = \frac{w}{2\cos\theta} = \frac{w\sqrt{l^2+x^2}}{2x}$$

$$\text{Stress} = \frac{T}{A} = \frac{w\sqrt{l^2+x^2}}{2xA} = \frac{wl}{2xA} \qquad \because \{x << l\}$$

10. (A)

$$\Delta L = \frac{FL}{A\eta} = \frac{500 \times 0.04}{\left(4\times16\times10^{-4}\right)\left(2\times10^{6}\right)} = 0.156 \times 10^{-2} \text{ m}$$

= 0.156 cm

11. (D)

$$\text{Bulk strain } = \frac{\Delta V}{V} = 3\left(\frac{\Delta l}{l}\right) = 3\left(\frac{2}{100}\right) = 0.06$$

12. (B)

$$B = \frac{\Delta p}{\left(\frac{\Delta V}{V}\right)} \Rightarrow B = \frac{\rho g h}{\left(\frac{\Delta V}{V}\right)} = \frac{10^3 \times 10 \times 200}{\left(\frac{0.1}{100}\right)}$$

$= 2\times10^{9}\ \text{N/m}^2$

13. (B)

$$p = p_0\,\rho^{\alpha V} \Rightarrow \frac{dp}{dV} = p_0 e^{\alpha V}(\alpha)$$

$$B = \frac{dp}{\left(\frac{dV}{V}\right)} = V\left(p_0 e^{\alpha V}\alpha\right) = \alpha p V$$

14. (D)

$$B = \frac{\Delta p}{\left(\frac{\Delta V}{V}\right)} \Rightarrow \frac{\Delta V}{V} = \frac{\Delta p}{B}$$

$$\Rightarrow\ \gamma\Delta T = \frac{p}{B} \ \Rightarrow\ \Delta T = \frac{p}{B\gamma}$$

15. (C)

Steel has the highest value of modulus of elasticity among the given materials.

16. (A)

$$\text{Longitudinal strain } = \frac{\text{Lateral strain}}{\sigma} = \frac{0.01\times10^{-3}}{0.4}$$

$$= 2.5\times10^{-5}$$

$$\text{Longitudinal stress } = \frac{F}{A} = \frac{100}{0.025} = 4\times10^{3}\ \text{N/m}^2$$

$$Y = \frac{4\times10^{3}}{2.5\times10^{-5}} = 1.6\times10^{8}\ \text{Nm}^{-2}$$

17. (A)

$$\frac{\Delta V}{V} = (1-2\sigma)\frac{\Delta l}{l} = (1-2(0.2))\left(4\times10^{-3}\right) = 0.24\times10^{-2}$$

$$\frac{\Delta V}{V}\times100 = \left(0.24\times10^{-2}\right)\times100 = 0.24\%$$

18. (C)

$$\sigma_x = \sigma_y = \sigma_z = \frac{F}{A}$$

$$\text{In } x\text{-direction, Strain } = \frac{\sigma_x}{Y} - \sigma\left(\frac{\sigma_y}{Y}\right) - \sigma\left(\frac{\sigma_z}{Y}\right)$$

$$\Rightarrow\ \frac{\Delta l}{1} = \frac{F}{AY}(1-2\sigma)$$

$$\Rightarrow\ \Delta l = \frac{F}{AY}(1-2\sigma)$$

19. (A)

Energy per unit volume

$$=\frac{1}{2}Y(\text{Strain})^2=\frac{1}{2}\times 2\times 10^{10}\times\left(\frac{0.06}{100}\right)^2$$

$$=3600\ \text{Jm}^{-3}$$

20. (D)

$$\sigma=\frac{F}{A}=\frac{F}{\pi r^2}\ \Rightarrow\ \sigma\propto\frac{1}{r^2}\Rightarrow\frac{\sigma_1}{\sigma_2}=\left(\frac{r_2}{r_1}\right)^2$$

$$U=\frac{(Stress)^2}{2Y}\Rightarrow U\propto\sigma^2$$

$$\Rightarrow\ \frac{U_1}{U_2}=\left(\frac{\sigma_1}{\sigma_2}\right)^2=\left(\frac{r_2}{r_1}\right)^4$$

$$=\left(\frac{2r}{r}\right)^4=16:1$$

21. (B)

$$k_{\text{eq}}=k_1+k_2\Rightarrow\frac{(2A)Y_{\text{eq}}}{L}=\frac{AY_1}{L}+\frac{AY_2}{L}$$

$$\Rightarrow\ Y_{\text{eq}}=\frac{Y_1+Y_2}{2}$$

22. (D)

For no change in the lengths of individual rods, the compressive forces in both rods due to thermal expansion should be equal. So,

$$A_1Y_1\alpha_1\Delta T=A_2Y_2\alpha_2\Delta T$$

$$\Rightarrow\ \frac{A_1}{A_2}=\frac{\alpha_2Y_2}{\alpha_1Y_1}$$

23. (B)

$$E=\frac{\text{Stress}}{\text{Strain}}\qquad\Rightarrow\ E=\frac{\frac{F}{A}}{\left(\frac{2\pi R-2\pi r}{2\pi r}\right)}$$

$$\Rightarrow\ F=AE\left(\frac{R-r}{r}\right)$$

Surface Tension

24. (D)

For equilibrium of wire *ab*,

$$2Tl=mg$$

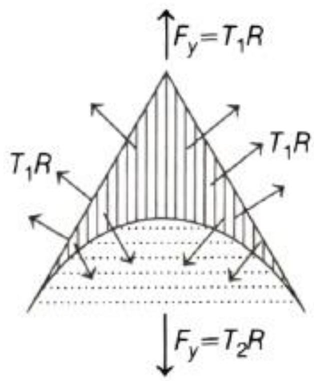
$$\Rightarrow\ m=\frac{2Tl}{g}=\frac{2\times 25\times 10^{-3}\times 0.1}{10}=0.5\text{ g}$$

25. (B)

$F_y = T \times$ Projected length

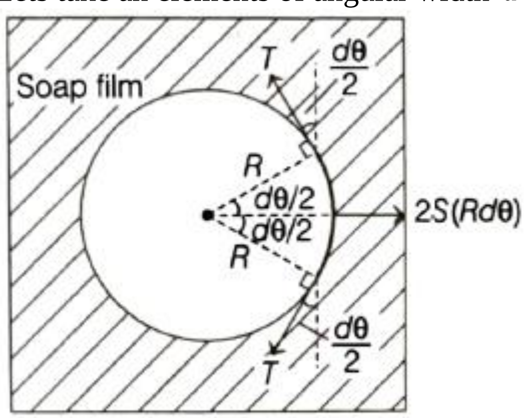
Net force $= T_1R - T_2R$

$= (T_1 - T_2)R$ in $+y$-direction



26. (D)

Lets take an elements of angular width $d\theta$ on the thread.



Force on the element due to surface tension

$$= 2S(Rd\theta)$$

For equilibrium of element,

$$2T\sin\left(\frac{d\theta}{2}\right) = 2SRd\theta$$

$$\Rightarrow \quad 2T\left(\frac{d\theta}{2}\right) = 2SRd\theta \Rightarrow T = 2SR$$

27. (B)

Let radius of single large drop be R.

For volume conservation, $2\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$

$$\Rightarrow \quad R = 2^{1/3}r$$

$$U_1 = T\left(4\pi r^2\right)\times 2$$

$$U_2 = T\left(4\pi R^2\right)$$

$$\frac{U_2}{U_2} = \frac{T\left(8\pi r^2\right)}{T\left(4\pi r^2\right)} = \frac{2r^2}{\left(2^{1/3}r\right)^2} = 2^{1/3} : 1$$

28. (B)

$$U_1 = S\left[2\times 4\pi\left(\frac{d}{2}\right)^2\right], \qquad U_2 = S\left[2\times 4\pi\left(\frac{2d}{2}\right)^2\right]$$

$$W = \Delta U = U_2 - U_1 = 6\pi d^2 S$$

29. (D)

$$W = T\left(4\pi r^2 \times 2\right)$$

$$= T\left[4\pi\left\{\left(\frac{3V}{4\pi}\right)^{1/3}\right\}^2 \times 2\right] \qquad \left\{\because V = \frac{4}{3}\pi r^3 \Rightarrow r = \left(\frac{3V}{4\pi}\right)^{1/3}\right\}$$

$$\Rightarrow \quad W \propto V^{2/3}$$

$$\Rightarrow \quad \frac{W_1}{W_2} = \left(\frac{V_1}{V_2}\right)^{2/3} \Rightarrow \frac{W}{W_2} = \left(\frac{V}{2V}\right)^{2/3}$$

$$\Rightarrow \quad W_2 = 2^{2/3}W = \sqrt[3]{4}\,W$$

30. (B)

From volume conservation, $\frac{4}{3}\pi R^3 = n\left(\frac{4}{3}\pi r^3\right)$

$$\Rightarrow \quad r = \frac{R}{n^{1/3}}$$

$$U_1 = T\left(4\pi R^2\right)$$

$$U_2 = nT\left(4\pi T^2\right) = n4\pi T\left(\frac{R}{n^{1/3}}\right)^2 = n^{1/3}T\left(4\pi R^2\right)$$

$$W = \Delta U = U_2 - U_1 = \left(n^{1/3} - 1\right)T\left(4\pi R^2\right)$$

31. (A)

$$U = T\left(8\pi r^2\right)$$

$$P = \frac{dU}{dt} = T\left(8\pi\right)\left(2r\frac{dr}{dt}\right)$$

$$\Rightarrow \quad P \propto r \qquad \because \left\{\frac{dr}{dt} \text{ is constants}\right\}$$

So, P versus r graph will be straight line passing through origin.

32. (C)

Air bubble inside water has only one free surface.

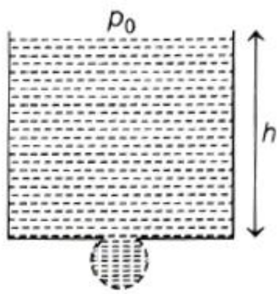
So, excess pressure will be $\frac{2T}{r}$.

$$p=\left(p_0+ghd\right)+\frac{2T}{r}$$

33. (A)

Pressure difference on two sides of a curved surface is inversely proportional to the radius of curvature.

34. (D)



$$p_0+\rho gh-\frac{2T}{r}=p_0$$

$$\Rightarrow \quad h=\frac{2T}{\rho gr}=\frac{2\times 70}{1\times 1000\times 0.005}=28.0 \text{ cm}$$

35. (D)

$$h=\frac{2T\cos\theta}{\rho g_{\text{eff}} R}$$

Fore freely falling elevator, $g_{\text{eff}}=0 \Rightarrow h\to\infty$

So, capillary will be completely filled with water.

36. (B)

$$H=\frac{2T\cos\theta}{\rho gR} \Rightarrow H\propto\frac{1}{R}$$

So, when radius is double, H will be halved.

$$M'=\rho\left[\pi(2R)^2\left(\frac{H}{2}\right)\right] \text{ and } M=\rho\left(\pi R^2 H\right)$$

$$\frac{M'}{M}=2 \Rightarrow M'=2M$$

37. (D)

$$T(2\pi r)\cos\theta=w$$

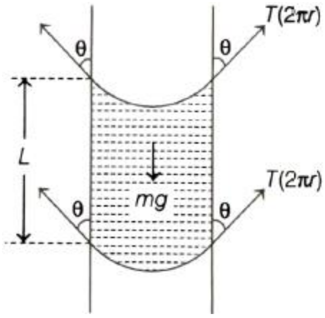
$$\Rightarrow \quad \text{Circumference } =2\pi r=\frac{w}{T\cos\theta}$$

$$=\frac{75\times 10^{-4}}{6\times 10^{-2}\cos 0°}=12.5\times 10^{-2} \text{ m}$$

38. (A)

For equilibrium of water column,

$$T(2\pi r)\cos\theta + T(2\pi r)\cos\theta = mg$$

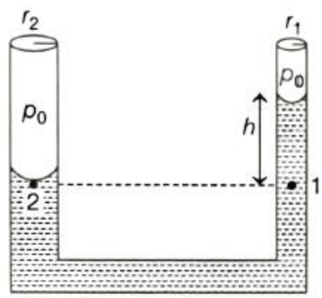


$$\Rightarrow \quad T(4\pi r)\cos 0° = \left(\rho\pi r^2 L\right)g \;\Rightarrow\; O = \frac{4T}{\rho g r}$$

39. (A)

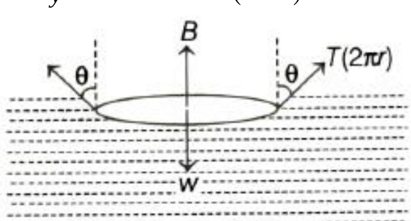
$p_1 = p_2$

$$\Rightarrow \quad p_0 - \frac{2T}{r_2} = p_0 - \frac{2T}{r_1} + \rho g h \Rightarrow T = \frac{\rho g h r_1 r_2}{2(r_2 - r_1)}$$



40. (C)

$$\sum F_y = 0 \Rightarrow B + T(2\pi r)\cos\theta = w_{\text{disc}}$$



$$\Rightarrow \quad w_{\text{disc}} = w + 2\pi r T\cos\theta$$

Viscosity

41. (A)

$$v_T = \frac{2}{9}\frac{(\sigma)r^2 g}{\eta}$$

$$\Rightarrow \; v_T = \frac{2}{9}\times\frac{10^3\times\left(0.3\times10^{-3}\right)^2\times 9.8}{1.8\times10^{-5}}$$

$$= 10.9 \text{ m/s}$$

42. (B)

$m = \sigma\left(\frac{4}{3}\pi r^3\right)$

$\Rightarrow \sigma \propto \frac{1}{r^3} \Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{r_2^3}{r_1^3}$

$\Rightarrow \sigma_2 = 8\sigma_1$

$v_T = \frac{2}{9}\frac{\sigma r^2 g}{\eta}$

$\Rightarrow \frac{v_1}{v_2} = \left(\frac{\sigma_1}{\sigma_2}\right)\left(\frac{r_1}{r_2}\right)^2$

$\Rightarrow \frac{v}{v_2} = \left(\frac{\sigma_1}{8\sigma_1}\right)\left(\frac{r_1}{r_1/r_2}^2\right)$

$\Rightarrow v_2 = 2v$

43. (C)

$mg - B - 6\pi\eta rv = ma$

So, *a versus v* graph will straight line with negative slope.
Hence, option (C) in incorrect.

44. (B)

From volume conservation,

$2\left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$

$\Rightarrow \quad R = (2)^{1/3} r$

$v_T \propto r^2 \Rightarrow \frac{v_1}{v_2} = \left(\frac{r_1}{r_2}\right)^2 \quad \Rightarrow \frac{v}{v_2} = \left(\frac{r}{2^{1/3} r}\right)^2$

$\Rightarrow \quad v_2 = (4^{1/3})v$

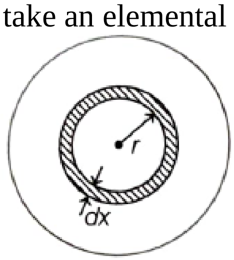
45. (D)

$2\eta A\frac{dv}{dx} = mg \Rightarrow 2\eta A\frac{dv}{dx} = (\rho A d)g$

$\Rightarrow \quad \frac{dv}{dx} = \frac{\rho d g}{2\eta}$

46. (C)

Lets take an elemental ring of radius *r* and thickness *dr* on the disc.



$$F=\eta A\frac{dv}{dx}\Rightarrow dF=\eta(2\pi rdr)\left(\frac{\omega r-0}{t}\right)$$

$$\int d\tau=\int_0^R \eta(2\pi rdr)\left(\frac{\omega r}{t}\right)r$$

$\Rightarrow\quad \tau\propto R^4$

47. (C)

$$R_e=\frac{\rho vd}{\eta}=\frac{1\times6\times2}{0.01}=1200$$

48. (B)

$$v_T=\frac{2}{9}\frac{(\sigma-\rho)r^2g}{\eta}$$

$$\Rightarrow\quad 8=\frac{2}{9}\frac{(10.5-1.5)(0.2)\times980}{\eta}$$

$\Rightarrow\quad \eta=9.8\text{ poise}$

Numerical Value Answer

Elasticity

49. (200)

$$F=\sigma A=\sigma(2\pi rt)$$

$$=3.45\times10^8\left(2\times3.14\times0.73\times10^{-2}\times1.24\times10^{-2}\right)=200\text{ kN}$$

50. (8)

$$k=\frac{(\pi r^2)Y}{L}\Rightarrow\frac{k_1}{k_2}=\left(\frac{r_1}{r_2}\right)^2=\frac{1}{4}$$

$$\Delta l_1+\Delta l_2=10\text{ mm}\qquad ...(\text{i})$$

$$k_1\Delta l_1=k_2\Delta l_2\Rightarrow\frac{\Delta l_1}{\Delta l_2}=\frac{k_2}{k_1}=4\qquad ...(\text{ii})$$

Solving Eqs. (i) and (ii), we get

$\Delta l_1=8\text{ mm}$

51. (1025)

Consider 1m^3 of water at the surface. Let us calculate change in volume ΔV of this water when it is at a pressure of 501 atm.

$$\Delta V=\frac{(\Delta p)V}{B}=\left(\frac{501-1\times10^5\times1}{2\times10^9}\right)=0.025\text{ m}^3$$

$$\text{New density }=\frac{m}{V-\Delta V}=\frac{1000}{1-0.025}=1025\text{ kg m}^{-3}$$

52. (3)

$$\sigma = \frac{T}{A} = \frac{m(g+a)}{A}$$

$$\text{Strain } = \frac{\sigma}{Y} = \frac{m(g+a)}{AY}$$

$$= \frac{10(10+2)}{2\times10^{-4}\times2\times10^{11}} = 3\times10^{-6}$$

53. (4)

$$F = \left(\frac{AY}{l}\right)\Delta l \Rightarrow \text{ Force constant } = \frac{AY}{l}$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{AY}{ml}} = \sqrt{\frac{4.9\times10^{-7}Y}{0.1\times1}} = 140$$

$$\Rightarrow \quad Y = 4\times10^{9} \text{ Nm}^{-2}$$

Surface Tension

54. (4)

$$B = \frac{\Delta p}{\left(\frac{\Delta V}{V}\right)} \Rightarrow \frac{\Delta V}{V} = \frac{\Delta p}{B} \Rightarrow \frac{3\Delta R}{R} = \frac{2T}{BR}$$

$$\Rightarrow \quad \Delta R = \frac{2T}{3B} = \frac{2\times0.075}{3\times1.25\times10^{8}} = 4\times10^{-10}\text{ m} = 4\,\text{Å}$$

55. (450)

$$h = \frac{2T\cos\theta}{\rho gR}$$

$$\Rightarrow \quad 3 = \frac{2T\cos0°}{1.5\times1000\times0.25\times10^{-1}}$$

$$\Rightarrow \quad T = 56.25 \text{ dyne / cm}$$

$$\Delta p = \frac{4T}{r} = \frac{4\times56.25}{0.5} = 450 \text{ dyne/cm}^2$$

56. (3)

For an isothermal situation,

$$p_A V_A + p_B V_B = p_c V_c$$

$$\Rightarrow \quad \left(p_0 + \frac{5T}{a}\right)\left(\frac{4}{3}\pi a^3\right) + \left(p_0 + \frac{4T}{b}\right)\left(\frac{4}{3}\pi b^3\right)$$

$$= \left(p_0 + \frac{4T}{c}\right)\left(\frac{4}{3}\pi c^3\right)$$

$$\Rightarrow \quad p_0\left(\frac{4}{3}\pi a^3 + \frac{4}{3}\pi b^3 - \frac{4}{3}\pi c^3\right) + \frac{4T}{3}\left(4\pi a^2 + 4\pi b^2 - 4\pi c^2\right) = 0$$

$$\Rightarrow \quad p_0 V + \left(\frac{4T}{3}\right)S = 0 \quad \Rightarrow \quad 3p_0 V + 4TS = 0$$

So, $\lambda = 3$

57. (2.4)

$$p = p_0 + \rho g h + \frac{2T}{R}$$

$$= 10^5 + 10^3 \times 10 \times 6 + \frac{2 \times 0.07}{3.5 \times 10^{-6}} = 2 \times 10^5 \text{ Pa}$$

$$\rho = \frac{pM}{RT} \Rightarrow \frac{\rho_1}{\rho_2} = \frac{p_1}{p_2}$$

$$\Rightarrow \frac{1.2}{\rho_2} = \frac{10^5}{2 \times 10^5}$$

$$\Rightarrow \rho_2 = 2.4 \text{ kg/m}^3$$

58. (9)

Initial pressure inside bubble $= p_0 + \dfrac{4T}{r}$

Final pressure inside bubble $= p_{\text{new}} + \dfrac{4T}{(r/2)}$

For isothermal condition,

$$p_1 V_1 = p_2 V_2$$

$$\Rightarrow \left(p_0 + \frac{4T}{r}\right)\left(\frac{4}{3}\pi r^3\right) = \left(p_{\text{new}} + \frac{8T}{r}\right)\frac{4}{3}\pi\left(\frac{r}{2}\right)^3$$

$$\Rightarrow p_{\text{new}} = 8p_0 + \frac{24T}{r} = 8 \times 10^5 + \frac{24 \times 0.08}{1.92 \times 10^{-5}}$$

$$= 9 \times 10^5 \text{ N/m}^2$$

Viscosity

59. (20.4)

$$v_T = \frac{2}{9}\frac{(\sigma - \rho) r^2 g}{\eta} = \sqrt{2gh}$$

$$\Rightarrow \frac{2}{9}\frac{\left(10^4 - 10^3\right)\left(1 \times 10^{-4}\right)^2 (9.8)}{9.8 \times 10^{-6}} = \sqrt{2 \times 9.8\, h}$$

$$\Rightarrow h = 20.4 \text{ m}$$

60. (9)

$F_1 =$ Viscous force on upper surface

$$= \eta A \frac{dv}{dx} = \eta A\left(\frac{v - 0}{d/2}\right) = 2\eta \frac{Av}{d}$$

$F_2 =$ viscous force on lower surface $= \dfrac{2\eta A v}{d}$

$$F_{\text{ext}} = F_1 + F_2 = \frac{4\eta A v}{d}$$

$$= \frac{4\left(150\times10^{-1}\right)(0.1)^2\left(3\times10^{-2}\right)}{2\times10^{-3}} = 9\text{ N}$$

61. (50)

In fluid frame, ball is moving with constant velocity and resultant of all forces acting on it is zero.

$$v_T = \frac{2}{9}\frac{(\rho-\sigma)r^2 g}{\eta}$$

$$= \frac{2}{9}\frac{(1000-100)(0.05)^2(10)}{0.1}$$

$$= 50\text{ m/s}$$

62. (5)

Rate of production of heat $= P = Fv_T = \left(6\pi\eta r V_T\right)v_T$

$P \propto r v_T^2$

$P \propto r^5 \because \left\{v_T \propto r^2\right\}$

1. (D)

(d) $Y_c \times (\Delta L_c / L_c) = Y_s \times (\Delta L_s / L_s)$

$$\Rightarrow \quad 1\times10^{11}\times\left(\frac{1\times10^{-3}}{1}\right) = 2\times10^{11}\times\left(\frac{\Delta L_s}{0.5}\right)$$

$$\therefore \quad \Delta L_s = \frac{0.5\times10^{-3}}{2} = 0.25\ \text{mm}$$

Therefore, total extension of the composite wire

$$= \Delta L_c + \Delta L_s = 1\ \text{mm} + 0.25\ \text{m} = 1.25\ \text{m}$$

2. (C)

(c) According to questions,

$$\frac{\ell_s}{\ell_b} = a, \frac{r_s}{r_b} = b, \frac{y_s}{y_b} = c, \frac{\Delta \ell s}{\Delta \ell_b} = ?$$

$$\text{As, } y = \frac{F\ell}{A\Delta\ell} \Rightarrow \Delta\ell = \frac{F\ell}{Ay}$$

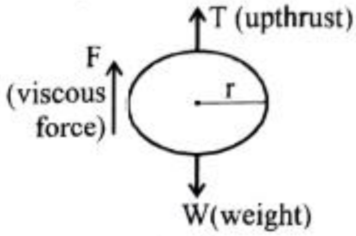
$$\Delta\ell_s = \frac{3mg\ell_s}{\pi r_s^2 \cdot y_s} \qquad [\because F_s = (M+2M)g]$$

$$\Delta\ell_b = \frac{2Mg\ell_b}{\pi r_b^2 \cdot y_b} \qquad [\because F_b = 2Mg]$$

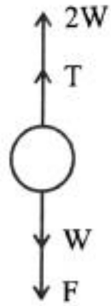
$$\therefore \quad \frac{\Delta\ell_s}{\Delta\ell_b} = \frac{\dfrac{3Mg\ell_s}{\pi r_s^2 \cdot y_s}}{\dfrac{2Mg.\ell_b}{\pi r_b^2 \cdot y_b}} = \frac{3a}{2b^2C}$$

3. (C)

(c) Initially



W = T + F



Finally

Here

2W + T = W + F

$\Rightarrow$ W = T + F

So in both case W = T + F

So, in both case state of body will be same.

4. (B)

(b) $F = \eta A \frac{dv}{dx}$

$$\frac{F}{A} = \eta \frac{dv}{dx} = 10^{-2} \times \frac{5}{5} = 10^{-2}\ \text{N/m}^2$$

5. (B)

(b) According to Toricelli's theorem,

Velocity of efflex,

$$V_{eff} = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 5} \cong 9.8\ \text{ms}^{-1}$$

6. (C)

(c) Poisson's ratio, $\sigma = \frac{\text{lateral strain } (\beta)}{\text{longitudinal strain } (\alpha)}$

For material like copper, $\sigma = 0.33$

And, $Y = 3k(1 - 2\sigma)$ Also, $\frac{9}{Y} = \frac{1}{k} + \frac{3}{\eta}$

$Y = 2\eta(1 + \sigma)$ Hence, $\eta < Y < k$

7. (A)

(a) When the bubble gets detached,

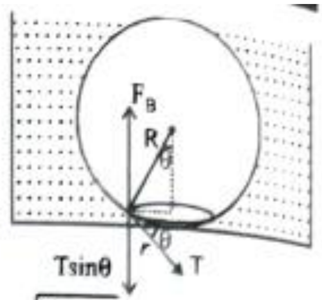
Buoyant force = force due to surface tension from diagram

$\sin\theta = \frac{r}{R}$

$\rho_\omega Vg = (T\sin\theta)\times(2\pi r)$

$\Rightarrow \rho_\omega \times \frac{4}{3}\pi R^3 g = T\cdot\frac{r}{R}\times 2\pi r$

$\Rightarrow \rho_\omega \frac{4}{3}R^3 g = \frac{2T}{R}r^2 \Rightarrow r = R^2\sqrt{\frac{2\rho_\omega g}{3T}}$



8. (A)

Inside a drop or an air bubble, towards concave side

$P = P_0 + \frac{2T}{R} \Rightarrow \Delta P = \frac{2T}{R}$

Here, the figure shows that the water between the plates formed a curved surface, similar to that formed by air bubble inside the water. So, pressure in water between the plates is lowered by $\frac{2T}{R}$.

9. (C)

When a point mass is falling vertically in a viscous medium, the medium or viscous fluid exerts drag force on the body to oppose its motion and at one stage body falling with constant terminal velocity.

10. (A)

Young's modulus $Y = \frac{F}{A}\Big/\frac{\Delta\ell}{\ell}$

$Y = \frac{F\ell}{\pi r^2 \Delta\ell}$

Given, radius r = 5 mm, force $F = 50\pi k$ N,

$\frac{\ell}{\Delta\ell} = 0.01\,\text{mm}$

$\therefore\ Y = \frac{F}{\pi r^2}\frac{\ell}{\Delta\ell} = 2\times10^{14}\ \text{N/m}^2$

11. (D)

$\text{Stress} = \frac{\text{Normal force}}{\text{Area}} = \frac{N}{A} = \frac{N}{(2\pi a)b}$

$V = \pi a^2 b$

$\Delta V = 2\pi ab\,\Delta a$

Stress = $B\times$ strain

$\frac{N}{(2\pi a)b} = B\frac{2\pi a\Delta a\times b}{\pi a^2 b}\ \left[\because\ \text{Strain} = \frac{\Delta V}{V}\right]$

$\Rightarrow\ N = B\frac{(2\pi a)^2\,\Delta ab^2}{\pi a^2 b}$

For needed to push the cork.

$$f = \mu N = \frac{\mu \times B \times 4\pi^2 a^2 b^2 \Delta a}{\pi a^2 b} = (4\pi\mu Bb)\Delta a$$

12. (C)

(c) Bulk modulus, $K = \frac{\text{volumetric stress}}{\text{volumetric strain}}$

$K = \frac{mg}{a\left(\frac{dV}{V}\right)}$ $\left[\because \text{Stress} = \frac{F}{A} = \frac{mg}{a}\right]$

$\Rightarrow \frac{dV}{V} = \frac{mg}{Ka}$ ....(i)

volume of sphere, $V = \frac{4}{3}\pi r^3$

Fractional change in volume $\frac{dV}{V} = \frac{3dr}{r}$ ...(ii)

Using eq. (i) & (ii) $\frac{3dr}{r} = \frac{mg}{Ka}$

$\therefore \frac{dr}{r} = \frac{mg}{3Ka}$ (fractional decrement in radius)

13. (B)

(b) As we know that

$\frac{2T\cos\theta}{r\rho g} = \text{R h}$ or $\frac{T_{Hg}}{T_{Water}} = 7.5$

$\frac{\rho_{Hg}}{\rho_W} = 13.6$ & $\frac{\cos\theta_{Hg}}{\cos\theta_W} = \frac{\cos 135°}{\cos 0°} = \frac{1}{\sqrt{2}}$

$$\frac{R_{Hg}}{R_{Water}} = \left(\frac{T_{Hg}}{T_W}\right)\left(\frac{\rho_W}{\rho_{Hg}}\right)\left(\frac{\cos\theta_{Hg}}{\cos\theta_W}\right)$$

$$= 7.5 \times \frac{1}{13.6} \times \frac{1}{\sqrt{2}} = 0.4 = \frac{2}{5}$$

14. (D)

(d) We have, $h = \frac{2T\cos\theta}{r\rho g}$

Mass of the water in the capillary

$m = \rho V = \rho \times \pi r^2 h = \rho \times \pi r^2 \times \frac{2T\cos\theta}{r\rho g} \Rightarrow m \propto r$

$\therefore \frac{m_1}{m_2} = \frac{r}{2r}$ or $m_2 = 2m_1 = 2m.$

15. (A)

(a) $27 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi r^3$ or $r = \frac{R}{3}$.

Terminal velocity, $v \propto r^2$

$$\therefore \frac{v_1}{v_2} = \frac{r_1^2}{r_2^2} \text{ or } v_2 = \left(\frac{r_2}{r_1}\right)^2 v_1 = \left(\frac{R/3}{R}\right)^2 v_1 = \frac{1}{9}$$

or $\frac{v_1}{v_2} = 9$.

16. (C)

(c) $\Delta_{temp} = \Delta_{force}$

or $L\alpha(\Delta T) = \frac{FL}{AY}$ $\quad\therefore\quad \alpha = \frac{FL}{AYT} = \frac{F}{\pi r^2 YT}$

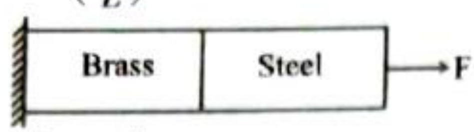
Coefficient of volume expression

$$r = 3\alpha = \frac{3F}{\pi r^2 YT}.$$

17. (None)

(None) Young modulus, $Y = \frac{\text{Stress}}{\left(\frac{\Delta l}{L}\right)}$

Let σ be the stress.



Total elongation $\Delta l_{net} = \frac{\sigma L_1}{Y_1} + \frac{\sigma L_2}{Y_2}$

$$\Delta l_{net} = \sigma\left[\frac{1}{Y_1} + \frac{1}{Y_2}\right] \quad [\because L_1 = L_2 = 1\text{m}]$$

$$\sigma = \Delta l\left(\frac{Y_1 Y_2}{Y_1 + Y_2}\right) = 0.2 \times 10^{-3} \times \left(\frac{120 \times 60}{180}\right) \times 10^9$$

$$= 8 \times 10^6 \frac{N}{m^2}$$

18. (B)

(b) Stress $= \frac{F}{A} = \frac{400 \times 4}{\pi d^2} = 379 \times 10^6\ \text{N/m}^2$

$$\Rightarrow d^2 = \frac{400 \times 4}{379 \times 10^6 \pi} \Rightarrow d = 1.15\ \text{mm}$$

19. (C)

(c) $\Delta_1 = \Delta_2$

or $\frac{Fl_1}{\pi r_1^2 y_1} = \frac{Fl_2}{\pi r_2^2 y_2}$ or $\frac{2}{R^2 \times 7} = \frac{1.5}{2^2 \times 4}$

$\therefore \quad R = 1.75\,\text{mm}$

20. (A)

(a) When a catapult is stretched up to length l, then the stored energy in it $= \Delta$k. E $\Rightarrow \frac{1}{2}\cdot\left(\frac{YA}{L}\right)(\Delta l)^2 = \frac{1}{2}mv^2$

$$\Rightarrow \quad y = \frac{mv^2 L}{A(\Delta l)^2}$$

$m = 0.02\,\text{kg}, v = 20\,\text{ms}^{-1}, L = 0.42\,\text{m}, A = (\pi d^2)/(4)$

$d = 6 \times 10^{-3}\,\text{m}, \Delta l = 0.2\,\text{m}$

$$y = \frac{0.02 \times 400 \times 0.42 \times 4}{\pi \times 36 \times 10^{-6} \times 0.04} = 2.97 \times 10^6\,\text{N/m}^2$$

So, order is 10^6.

21. (A)

(a) If force F acts along the length L of the wire of cross-section A, then energy stored in unit volume of wire is given by

Energy density $= \frac{1}{2}$ stress × strain

$$= \frac{1}{2} \times \frac{F}{A} \times \frac{F}{AY} \quad \left(\because \text{ stress} = \frac{F}{A} \text{ and strain} = \frac{F}{AY}\right)$$

$$= \frac{1}{2}\frac{F^2}{A^2 Y} = \frac{1}{2}\frac{F^2 \times 16}{(\pi d^2)^2 Y} = \frac{1}{2}\frac{F^2 \times 16}{\pi d^4 Y}$$

If u_1 and u_2 are the densities of two wires, then

$$\frac{u_1}{u_2} = \left(\frac{d_2}{d_1}\right)^4 \Rightarrow \frac{d_1}{d_2} = (4)^{1/4} \Rightarrow \frac{d_1}{d_2} = \sqrt{2}:1$$

22. (C)

(c) According to question, pressure inside, 1st soap bubble,

$$\Delta P_1 = P_1 - P_0 = 0.01 = \frac{4T}{R_1} \quad ...(i)$$

And $\Delta P_2 = P_2 - P_0 = 0.02 = \frac{4T}{R_2}$ ...(ii)

Dividing, equation (ii) by (i),

$$\frac{1}{2} = \frac{R_2}{R_1} \Rightarrow R_1 = 2R_2$$

Volume $V = \frac{4}{3}\pi R^3 \Rightarrow \frac{V_1}{V_2} = \frac{R_1^3}{R_2^3} = \frac{8R_2^3}{R_2^3} = \frac{8}{1}$

23. (B)

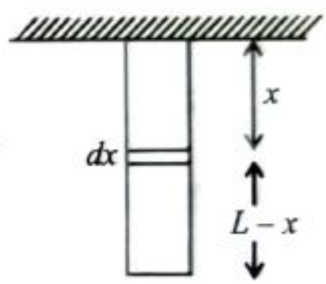
(b) Given, Weight of uniform heavy rod. $W = 10\ \text{kg ms}^{-2}$

Length of rod, $L = 20\ \text{cm} = 0.2\ \text{m}$

Consider an element dx at distance x from top.

Force acting on part below dx

$$= \frac{(L-x)W}{L}$$



Elongation for small element dx, dl

$$= \left(\frac{L-x}{L}\right)\frac{Wdx}{AY}$$

$$\int_0^{\Delta l} dl = \int\left(\frac{L-x}{L}\right)\frac{w\,dx}{AY} \Rightarrow \Delta\ell = \frac{WL}{2AY} = 5\times10^{-9}\text{m}$$

24. (B)

(b) Force exerted on each column F $= \frac{mg}{4}$

$$\because y = \frac{F}{A}\Big/\frac{\Delta\ell}{\ell}$$

$$\therefore \quad \text{Strain}\left(\frac{\Delta\ell}{\ell}\right) = \frac{mg}{4AY}$$

$$= \frac{50\times10^3\times9.8}{4\times\left[\pi\times1^2 - \pi(.5)^2\right]} = 2.6\times10^{-7} \quad \left[\because A = \pi r^2\right]$$

25. (C)

(c) Here, $m_1 = 3kg, m_2 = 5 kg$

Tension in the metal wire.

$$T = \frac{2m_1m_2g}{m_1 + m_2} = \frac{2\times3\times5\times10}{8} = \frac{75}{2}$$

$$\text{Stress} = \frac{T}{A} = \frac{T}{\pi r^2}$$

$$\Rightarrow \frac{24}{\pi}\times10^2 = \frac{75}{2\times\pi r^2} \Rightarrow r^2 = \frac{75}{2\times24\times100} = \frac{3}{8\times24}$$

$$\Rightarrow r = 0.125 \text{ m} = 12.5 \text{ cm}$$

26. (A)

(a) Bulk modulus (K) is given by $K = \dfrac{P}{\left(-\dfrac{\Delta V}{V}\right)}$

Density, $\rho = \dfrac{m}{V}$

So, $\dfrac{\Delta\rho}{\rho} = \dfrac{+\Delta V}{V} \quad \therefore K = \dfrac{P}{\left(\dfrac{\Delta\rho}{\rho}\right)} \Rightarrow \dfrac{\Delta\rho}{\rho} = \dfrac{P}{K} \Rightarrow \Delta\rho = \dfrac{\rho P}{K}$

27. (D)

(d) We know that

$Y = 3K(1-2\sigma)$

Here,

σ = Poisson's ratio

$$\sigma = \frac{1}{2}\left(1 - \frac{Y}{3K}\right)$$

Also,

$Y = 2\eta(1+\sigma)$

$$\sigma = \frac{Y}{2\eta} - 1$$

From equation (i) and equation (ii),

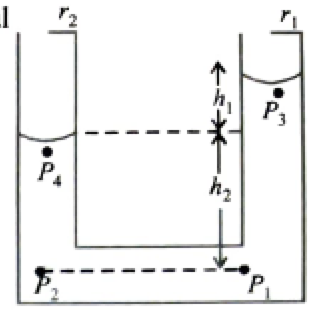
$$\frac{1}{2}\left(1-\frac{Y}{3K}\right) = \frac{Y}{2\eta} - 1$$

$$\Rightarrow 1 - \frac{Y}{3K} = \frac{Y}{\eta} - 2 \Rightarrow \frac{Y}{3K} = 3 - \frac{Y}{\eta} \Rightarrow \frac{Y}{3K} = \frac{3\eta - Y}{\eta}$$

$$\Rightarrow \frac{\eta Y}{3K} = 3\eta - Y \Rightarrow K = \frac{\eta Y}{9\eta - 3Y}$$

28. (B)

(b) Pressure at same horizontal level is equal



So, $P_1 = P_2$

$P_3 = P_{atm} - \frac{2T}{r_1}$

& $P_4 = P_{atm} - \frac{2T}{r_2}$

$P_2 = P_4 + \rho g h_2 = P_{atm} - \frac{2T}{r_2} + \rho g h_2$

& $P_1 = P_3 + \rho g(h_1 + h_2) = P_{atm} - \frac{2T}{r_1} + \rho g(h_1 + h_2)$

$\therefore P_{atm} - \frac{2T}{r_1} + \rho g(h_1 + h_2) = P_{atm} - \frac{2T}{r_2} + \rho g h_2$

$\therefore \rho g h_1 = 2T\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

$= 2 \times 7.3 \times 10^{-2}\left[\frac{1}{2.5 \times 10^{-3}} - \frac{1}{4 \times 10^{-3}}\right]$

$\Rightarrow h_1 = 2.19 \times 10^{-3}\text{ m} = 2.19\text{ mm}$

29. (A)

(a) As volume remain same i.e volume of two smaller drops will be equal to volume of one big drop.

$2 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \Rightarrow 2r^3 = R^3$

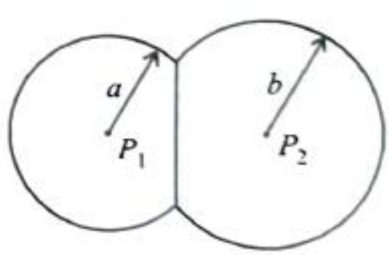
$\Rightarrow 2 = \left(\frac{R}{r}\right)^3 \Rightarrow \frac{R}{r} = 2^{\frac{1}{3}}$

$\frac{U_i}{U_f} = \frac{T \times 2 \times (4\pi r^2)}{T \times 4\pi R^2} = \frac{2r^2}{2^{\frac{2}{3}} R^2} = 2^{\frac{1}{3}}$

30. (A)

(a) Let R be the radius of curvature of common surface

$P_1 = P_0 + \frac{4T}{a}$ and



$P_2 = P_0 + \frac{4T}{b}$

And $P_1 - P_2 = \frac{4T}{R}$

$\left(P_0 + \frac{4T}{a}\right) - \left(P_0 + \frac{4T}{b}\right) = \frac{4T}{R}$

$\Rightarrow \frac{1}{a} - \frac{1}{b} = \frac{1}{R} \qquad \therefore R = \frac{ab}{(b-a)}$

31. (C)

(c) As volume remains unchanged

$$n\times\frac{4}{3}\pi r^3=\frac{4}{3}\pi R^3 \Rightarrow nr^3=R^3$$

Work done = T × Increment in surface area

$$W=T[4\pi nr^2-4\pi R^2] \quad ...(i)$$

By conservation of energy

Loss in thermal energy = work done against surface tension

$$JQ=W$$

Where J = mechanical equivalent of heat

$$Q=\frac{W}{J}=\frac{4\pi T(nr^2-R^2)}{J} \qquad \text{[Using (i)]}$$

Heat energy per unit volume

$$Q=\frac{4\pi T}{Jn\cdot\frac{4}{3}\pi r^3}[nr^2-R^2]=\frac{4\pi T}{J\times\frac{4}{3}\pi}\left[\frac{1}{r}-\frac{R^2}{nR^3}\right]$$

$$=\frac{3T}{J}\left[\frac{1}{r}-R^2\right]=\frac{3T}{J}\left[\frac{1}{r}-\frac{1}{R}\right]$$

32. (A)

Young modulus depends on material of wire not on length and area of wire's cross section.

33. (C)

(c) Force $F=YA\frac{\Delta l}{l}$ Y = young's modulus of the wire

$$=2\times10^{11}\times10^{-4}\left(\frac{2l-l}{l}\right)=2\times10^{7}\,\text{N}$$

34. (D)

(d) Young's modulus,

$$Y=\frac{F/A}{\frac{\Delta\ell}{\ell}}=\frac{F\ell}{A\Delta\ell}\Rightarrow\Delta\ell=\frac{F\ell}{AY}$$

Net elongation, $\Delta\ell=\Delta\ell_1+\Delta\ell_2$.

$$\Rightarrow\ \Delta\ell=\frac{F\ell_1}{A_1Y_s}+\frac{F\ell_2}{A_2Y_c}$$

$$\Rightarrow\ F=\frac{\Delta\ell}{\frac{\ell_1}{A_1Y_s}+\frac{\ell_2}{A_2Y_c}}=\frac{\Delta\ell A}{\frac{\ell_1}{Y_s}+\frac{\ell_2}{Y_c}}\ (\because\ A_1=A_2=A)$$

$$=\frac{1.4\times10^{-3}\times\pi\times(1.4\times10^{-3})^2}{\frac{3.2}{2\times10^{11}}+\frac{4.4}{1.1\times10^{11}}}=1.54\times10^{2}=154\,\text{N}$$

35. (A)

(a) Since breaking stress (Maximum lifting capacity) is the property of material so it will remain same.

$$\text{breaking stress} = \frac{\text{Maximum lifting capacity}}{\text{Area of cross-section of rope}}$$

$$\Rightarrow \frac{10}{2.5\times10^{-4}} = \frac{25}{A}$$

$$\Rightarrow A = \frac{25\times2.5\times10^{-4}}{10} = 625\times10^{-6} = 6.25\times10^{-4}\,m^2$$

36. (C)

(c) When length 'L' is hanging from fixed support

$$\Delta L = \frac{mgL}{\gamma A} \Rightarrow \Delta L \propto m$$

$$\Rightarrow \frac{\Delta L_1}{\Delta L_2} = \frac{m_1}{m_2} = \frac{1}{2} \Rightarrow \frac{L_1 - L}{L_2 - L} = \frac{1}{2}$$

$$\Rightarrow 2L_1 - 2L = L_2 - L \Rightarrow 2L_1 - L_2 = L$$

$$\therefore L = 2L_1 - L_2$$

37. (C)

(c) At equilibrium, $\vec{F} = 0$

$$\Rightarrow \frac{-du}{dr} = 0 \Rightarrow \frac{-d}{dr}\left(\frac{A}{r^{10}} - \frac{B}{r^5}\right) = 0$$

$$\Rightarrow +\frac{A}{r^{11}}\times10 - \frac{5B}{r^6} = 0 \Rightarrow \frac{1}{r^6}\left[\frac{10A}{r^5} - \frac{5B}{1}\right] = 0$$

$$\Rightarrow \frac{10A}{r^5} = 5B \Rightarrow r^5 = \frac{10A}{5B} \Rightarrow r = \left(\frac{2A}{B}\right)^{1/5}$$

38. (C)

(c) Bulk modulus, $B = -\dfrac{\Delta P}{\frac{\Delta V}{V}} \Rightarrow \Delta P = -B\dfrac{\Delta V}{V}$

$$|\Delta P| = +3\times10^{10}\times0.02 = 6\times10^8$$

39. (D)

(d) We know that terminal velocity is given by

$$V_T = \frac{2gr^2}{g\eta}(\rho - \rho_\ell)$$

Here, we have no involvement of buoyant force. So remove ρ_ℓ.

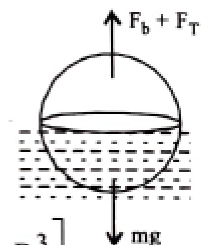
Then, $v_T = \frac{2gr^2\rho}{9\eta} = \frac{2\times10\times10^{-12}\times10^3}{9\times1.8\times10^{-5}} = 123.4\times10^{-6}$ m/s.

40. (A)

(a) As liquid drop is in equilibrium.

So $F_{net} = 0$

Boyant force + surface tension = mg



$$\sigma\frac{V}{2}g + 2\pi RT = \rho Vg$$

$$\Rightarrow 2\pi RT = \frac{(2\rho-\sigma)}{2}\frac{4}{3}\pi R^3 g \quad \left[\because V = \frac{4}{3}\pi R^3\right]$$

$$\Rightarrow R^3 = \frac{3T}{(2\rho-\sigma)g} \Rightarrow R = \sqrt{\frac{3\times7.5\times10^{-2}\,N-m^{-1}}{(2\rho-\sigma)\times10}}$$

$$\Rightarrow R = \frac{3}{20\sqrt{(2\rho-\sigma)}}\,m = \frac{15}{\sqrt{2\rho-\sigma}}\,cm$$

41. (A)

(a) $E_i = 0^{-} A_i 0^{-}$ = surface energy per unit area

$= T a_i \; [\because 0^{-} = T] = T.4\pi r_i^2$

Now, $V_i = V_f \Rightarrow \frac{4}{3}\pi r_i^3 = 64\times\frac{4}{3}\pi r_f^3 \Rightarrow r_i^3 = 64 r_f^3$

$\Rightarrow r_i = 4r_F$

So, $E_f = 0^{-} A_f = T\times64\times\;4\pi r_f^2 = 256T\times\pi\frac{r_i^2}{16} = 16\pi T r_i^2$

So, $\Delta E = E_f - E_i = 12\pi + Tr_i^2 = 12\pi\times0.075\times0.1^2$

$= 2.82\times10^{-4}$ J

42. (101)

(101) Given : Radius of capillary tube,

$r = 0.015 \text{ cm} = 15 \times 10^{-5} \text{ mm}$

$h = 15 \text{ cm} = 15 \times 10^{-2} \text{ mm}$

Using, $h = \dfrac{2T\cos\theta}{\rho g r}$ $[\cos\theta = \cos 0° = 1]$

Surface tension,

$$T = \frac{rh\rho g}{2} = \frac{15\times10^{-5}\times15\times10^{-2}\times900\times10}{2} = 101$$

milli newton m^{-1}

43. (4)

(4) $T = Ml\omega^2$

$$\sigma = \frac{T}{A} = \frac{ml\omega^2}{A}$$

$$\frac{ml\omega^2}{A} \le 48\times10^7 \Rightarrow \omega^2 \le \frac{(48\times10^7)A}{ml}$$

$$\Rightarrow \omega^2 \le \frac{(48\times10^7)(10^{-6})}{10\times3} = 16 \Rightarrow \omega_{max} = 4 \text{ rad/s}$$

44. (40)

(40) Let m be mass that can be placed in the pan. For wire W_1

$$\text{Stress} = \frac{\text{Maximum weight}}{\text{Area}} = \frac{(m+30)g}{8\times10^{-7}}$$

$\Rightarrow \quad m + 30 = 1.25\times10^9\times8\times10^{-7}$

$\Rightarrow \quad m + 30 = 100 \Rightarrow m = 70 \text{ kg}$

For wire W_2

$$\text{Stress} = \frac{(m+10)g}{4\times10^{-7}} = 1.25\times10^9$$

$\Rightarrow \quad m + 10 = 50 \Rightarrow m = 40 \text{ kg}$

Maximum mass that can be placed = 40 kg.

45. (20)

Energy stored in stretched catapult is converted into kinetic energy of stone

$$\frac{1}{2}\cdot\frac{YA}{L}\cdot x^2=\frac{1}{2}mv^2$$

$$\frac{0.5\times10^9\times10^{-6}\times(0.04)^2}{0.1}=\frac{20}{1000}v^2\Rightarrow v^2=400$$

$\therefore\ v=20\,\text{m/s}$

46. (2)

As $\Delta l=\frac{F\cdot l}{Y\cdot\pi r^2}\Rightarrow\Delta l\propto\frac{l}{r^2}$

$$\Delta l_2=\left(\frac{l_2}{l_1}\right)\left(\frac{r_1}{r_2}\right)^2\Delta l_1=(2)\left(\frac{1}{2}\right)^2(0.04)\,\text{m}=2\,\text{cm}$$

47. (500)

(500) Bulk modulus, $B=-\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)}$

$$\Rightarrow B=-\frac{V}{dV}(\rho gh)\Rightarrow h=-\frac{B\frac{\Delta V}{V}}{\rho g}$$

$$\therefore\quad h=\frac{9.8\times10^8\times0.5}{100\times10^3\times9.8}=500\,\text{m}.$$

48. (2)

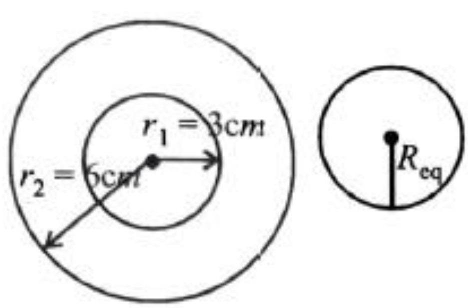
(2) Excess pressure inside bigger bubble $=\frac{4T}{r_2}$

So, Excess pressure inside the smaller soap bubble

$$\Delta P_1=\frac{4T}{r_1}+\frac{4T}{r_2}$$

The excess pressure inside the equivalent soap bubble.

$$\Delta P_2=\frac{4T}{R_{eq}}$$



$\Delta P_1=\Delta P_2$

$$\Rightarrow\quad\frac{4T}{R_{eq}}=\frac{4T}{r_1}+\frac{4T}{r_2}$$

$$\Rightarrow\quad R_{eq}=\frac{r_1 r_2}{r_1+r_2}$$

Putting $r_1=3\text{cm}$, $r_2=6$ cm $\Rightarrow R_{eq}=2$ cm

49. (50)

(50) Given, length of metal wire, $\ell = 0.5$ m

Cross-sectional area, $A = 10^{-4}\,m^2$

Breaking stress $= 5 \times 10^8\,Nm^{-2}$

Mass of block m = 10 kg

T_{max} = Breaking stress × Area

$$\frac{mv^2}{\ell} = 5 \times 10^8 \times 10^{-4} = 5 \times 10^4$$

$$\frac{10\,v^2}{0.5} = 5 \times 10^4 \Rightarrow v = \sqrt{\frac{0.5 \times 5 \times 10^4}{10}} = 50\text{ m/s}$$

50. (30)

$$\textbf{(30) Strain} = F/AY = \frac{mg + \frac{mv^2}{R}}{AY}$$

$$= \frac{20 + \frac{2(5)^2}{0.5}}{3 \times 10^{-6} \times 10^{11}} = 30 \times 10^{-5}$$

51. (2)

$$\textbf{(2)}\ \text{Slope} = \frac{\text{Extension/Load}}{\text{Length of wire}} = \frac{\Delta l/w}{L}$$

$$\text{Young's modulus, } Y = \frac{mg/A}{\Delta\ell/L} = \frac{wL}{\Delta\ell A}$$

$$\therefore\ Y = \frac{1}{(\text{slope})A} \Rightarrow Y = \frac{1}{2 \times 10^{-6}(0.25 \times 10^{-5})}$$

$$\Rightarrow Y = 2 \times 10^{11}\,N/m^2$$

52. (25)

Given mass of rod $m = 20$ kg

Cross-section area, $A = 0.4\ m^2$

Length $\ell = 20$ m

Let extension is dy in length dx.

$$Y = \frac{stress}{strain} = \frac{\frac{T}{A}}{\frac{dy}{dx}} = \frac{T}{A} \cdot \frac{dx}{dy} \Rightarrow dy = \frac{Tdx}{AY}$$

Tension at a distance x from lower end0020 $= \frac{mg}{\ell}x$

$$\int_0^{\Delta\ell} dy = \int_0^{\ell} \frac{mg}{\ell} x \frac{dx}{AY} \Rightarrow \Delta\ell = \frac{mg}{\ell AY}\left[\frac{x^2}{2}\right]_0^{\ell}$$

$$\Delta\ell = \frac{mg\ell}{2AY} = \frac{20\times10\times20}{2\times0.4\times2\times10^{11}} = 25\times10^{-9}\text{ m}$$

Compare with $x\times10^{-9}$ we have, $x = 25$

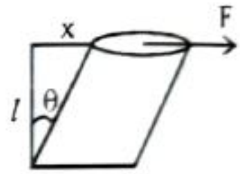
53. (5)

(5) $\Delta\ell = \frac{F\ell}{\gamma A} \Rightarrow \Delta\ell \propto \frac{F\ell}{r^2}$

$$\Rightarrow \Delta\ell_2 = \Delta\ell_1\left(\frac{F_2}{F_1}\right)\left(\frac{\ell_2}{\ell_1}\right)\left(\frac{r_1}{r_2}\right)^2 = 5\times4\times4\times\frac{1}{16} = 5\text{ cm}$$

54. (48)

(48) $\eta = \frac{F}{A\tan\theta} = \frac{F}{A\left(\frac{x}{\ell}\right)}$



$$\frac{F}{A} = \eta\frac{x}{\ell} \Rightarrow \frac{F\ell}{A\eta} = x$$

$$\Rightarrow x = \frac{18\times10^4\times60\times10^{-2}}{60\times10^{-2}\times15\times10^{-2}\times25\times10^2}$$

$= 48\times10^{-6}\text{ m} = 48\ \mu\text{m}$

55. (25)

(25) $y = \frac{\text{stress}}{\text{strain}} \Rightarrow \text{strain} = \frac{1}{y}\text{stress}$

$$\therefore \frac{1}{y} = \text{slope} = \frac{(2-1)\times10^{-10}}{40-20} = \frac{1}{20}\times10^{-10} \Rightarrow y = 20\times10^{10}$$

So, energy density $= \frac{1}{2}\times\text{stress}\times\text{strain}$

$= \frac{1}{2}\times y\times(\text{strain})^2 = \frac{1}{2}\times20\times10^{10}\times(5\times10^{-4})^2$

$= 10^{11}\times25\times10^{-8} = 25\times10^3 = 25\text{ kJ/m}^3$

56. (25)

(25) $F_V + F_B = mg$ (V = constant)

$F_V = mg - F_B = \rho_B Vg - \rho_L Vg = (\rho_B - \rho_L)Vg$

$$= (8-1.3)\times10^{+3}\times\frac{0.3\times10^{-3}}{8\times10^3}\times10 = 25\times10^{-4}\text{ N}$$

Hence, the value of viscous force acting on ball will be 25×10^{-4} N.

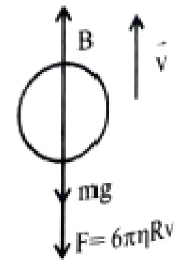
Compair with $x\times10^{-4}$, we get $x = 25$

57. (11)

(11) When bubble is rising steadily the net force acting on it will be zero.

So, buoyant force (B) = drag force (F)

$\Rightarrow \frac{4\pi}{3}R^3\rho g = 6\pi\eta Rv$



$\therefore \quad \eta = \frac{4}{18}\frac{R^2\rho g}{v}$

or, $\eta = \frac{4}{18}\frac{(10^{-3})^2 \times 1.75\times 10^3 \times 10}{0.35\times 10^{-2}}$

$\therefore \quad \eta = \frac{10}{9} = 1.11 = 11 \text{ poise}$

58. (20)

(20) Speed after falling '*h*' height = Terminal velocity

$\Rightarrow \sqrt{2gh} = \frac{2r^2 g}{g\eta}(\rho - \rho_f)$

$\Rightarrow \sqrt{2\times 10\times h} = \frac{2\times(0.1\times 10^{-3})^2 \times 10}{9\times 10^{-5}} \times (10^4 - 10^3)$

$\Rightarrow \sqrt{20h} = \frac{2\times 10^{-8}\times 10}{9\times 10^{-5}} \times 9\times 10^3$

$\Rightarrow \sqrt{20h} = 20 \Rightarrow 20h = 400 \Rightarrow h = 20 \text{ m}$

59. (500)

(500) $P = P_0 + \frac{2T}{R} \Rightarrow P - P_0 = \frac{2T}{R}$

$500 = \frac{2\times T}{2\times 10^{-3}}$

$T = 500 \times 10^{-3}$

So, x = 500

Properties of matter

In chapter – I

Q1 (B) Y is material property.

Q2 (B)

$$F = \frac{YA}{l}\Delta l$$

$$\frac{F_1}{F_2} = \frac{l_2}{l_1} \times \frac{A_1}{A_2} = \frac{3l}{l} \cdot \frac{\pi (3d)^2/4}{\pi d^2/4} = \frac{27}{1}$$

Q3 (A)

For wire, $V = Al$

$dV = A\,dl$ $A =$ constant

$$\Delta V = A\Delta l.$$

$$\therefore \frac{\Delta V}{V} = \frac{\Delta l}{l}.$$

Q4. (B)

$$F = \frac{YA}{l}(\Delta l)$$

$\Delta l = 2\pi(R - r)$ $l = 2\pi r$

$$\frac{\Delta l}{l} = \frac{R-r}{r} \Rightarrow F = YA\left(\frac{R-r}{r}\right)$$

Q5 (C) $\frac{\Delta V}{V} = \frac{0.01}{100} = 10^{-4}$

$B = -\frac{P}{\Delta V/V} = -\frac{100 \times 10^{6} \text{ dynes}}{10^{-4}}$

$B = 1 \times 10^{12}$ dynes/cm^2

Q6 (A) stress $= \frac{\text{wt.}}{\text{Area}} = \frac{Al\rho g}{A} = l\rho g$

$\sigma = l\rho g$.

Q7 (A) $B_{iso} = P$

$P =$ constant so, $B_{iso} =$ constant.

~~In chapter~~

In-chapter-2

Q1 (C) $U = \frac{1}{2} \times \text{stress} \times \text{strain} = \frac{(\text{stress})^2}{2Y}$

Q2 (B) Work performed $= \frac{1}{2} \times \text{stress} \times \text{strain} \times \text{volume}$

$=$ Energy stored

$= \frac{1}{2} \times (Y\alpha t) \times (\alpha t) \times (A \times l)$

$= \frac{1}{2} \times (YA\alpha t) \times (l\alpha t)$

Q.3 (B)

$$F = \frac{YA\,\Delta l}{l}$$

$$\frac{1\times10^{10}\times \pi R_B^2 \times (1\times10^{-3})}{L} = \frac{2\times10^{10}\times \pi R_S^2 \times (1\times10^{-3})}{l}$$

$$R_S = \frac{R_B}{\sqrt{2}}$$

Q.4. (A)

$$K = +\frac{PV}{\Delta V} \Rightarrow \sout{V = \frac{K(\Delta V)}{P}}$$

$$\Delta V = +\frac{PV}{K}$$

$$\Delta V = V(\alpha\,\Delta T) = +\frac{PV}{K}$$

$$\Delta T = +\frac{P}{\alpha K}$$

Q.5 (B)

$$Y = \frac{W/A}{l_a/L} = \frac{V\sigma g/A}{l_a/L}$$

$$Y = \frac{V(\sigma-\rho)g/A}{l_\omega/L}$$

$$\frac{\sigma}{l_a} = \frac{\sigma-\rho}{l_\omega}$$

$$\frac{\sigma}{\rho} = \frac{l_a}{l_a-l_\omega} = \sigma_r$$

Q.6 (B)

$B_{iso} = P.$

Q7. (B)

$$T = 2\pi\sqrt{\frac{m}{K_{eq}}}$$

$$K_{eq} = \frac{K_1K_2}{K_1+K_2} \qquad K_1 = K \qquad K_2 = \frac{YA}{L}$$

$$T = 2\pi\sqrt{\frac{m(YA+KL)}{YAK}}$$

Inchapter - 13

Q1 (a) (b) (c).

$$\Delta P_{drop} = \frac{2T}{R}$$

$$\Delta P_{bubble} = \frac{4T}{R}$$

$$(\Delta P_{cy})(2\pi R)(R) = T(2\pi R) \qquad \text{height of drop} = R$$

$$\Delta P_{cy} = \frac{T}{R}$$

Q2 (b)(c)(d)

force of cohesion > force of adhesion

so, convex meniscus. Angle of contact is obtuse and liquid will not wet solid.

Q3 (b) (c)

$$h = \frac{2T\cos\theta}{R\rho g}$$

Q4 (a)

Viscous force opposes motion.

Q5 (c) (d) If capillary is broken, radius of meniscus will be adjusted such that

$$hr = h'r'$$

Flow rate will be constant.

Surface energy = 2TS.

~~surface energy, also~~

surface energy increases by = $T(\Delta S)$

ΔS = increase in surface energy

$$\frac{4\pi}{3} R^3 = 10^6 \frac{4\pi}{3} r^3$$

$$r = \frac{R}{100}$$

$$\Delta S = 4\pi r^2 \times 10^6 - 4\pi R^2$$

$$= 4\pi R^2 10^4 - 4\pi R^2 \approx 4\pi R^2 \times 10^4$$

$$\therefore T\Delta S = 4\pi R^2 \times T \times 10^4$$

Q6 (c)

$$F = -\eta A \frac{dv}{dy}, \quad \frac{F}{A} = -\eta \frac{dv}{dy}$$

$$\frac{F}{A} = -1 \times 10^{-3} \times \frac{5}{10} = 0.5 \times 10^{-3}\ N/m^2$$

Q7 (d) surface tension of liquid decreases.

Q8 (b)

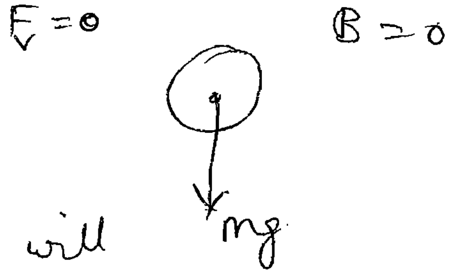
Work done = Increase in surface energy

$= 2S \times \left(4\pi \left(\frac{D}{2}\right)^2 - 4\pi \left(\frac{d}{2}\right)^2\right)$

$= 2S \, \pi (D^2 - d^2)$

$= 2\pi (D^2 - d^2) S.$

Q9. (d) In vacuum, no medium is present. So, no viscous force. Hence, ~~ball~~ body will continue to accelerate downward with acceleration 'g'.



$\therefore a = g$

So, terminal velocity will never be attained.

Q10 (a)

Energy needed = Increase in surface energy

$= T \times (\Delta S)$

$= T \times (n \times 4\pi r^2 - 4\pi R^2)$

$= 4\pi T (nr^2 - R^2)$

Q11. (c)

$$\Delta P = \frac{4T}{R} = P_{in} - P_{out}$$

$$P_{in} = P_{out} + \frac{4T}{R}$$

$P_{out} = P_0$ = atm. pressure

$$P_{in} = P_0 + \frac{4T}{R}$$

$$(P_{in})_{smaller} > (P_{in})_{bigger}$$

Air will flow from ~~the~~ smaller bubble to larger.

JP Sir **Solutions to Exercise - 1** Objective (Single choice) 1.

Page 1 to 5

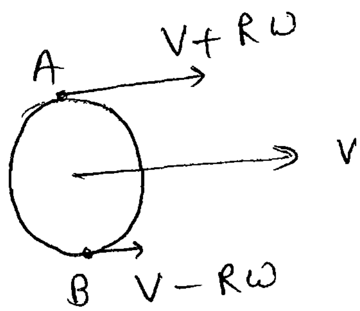
Q1. (A) Radius of bubble increases under Isothermal conditions, <u>Elasticity</u>

$PV =$ constant

$\therefore$ At surface $P_{surface} < P_{bottom}$

$V_{surface} > V_{bottom}$

~~(B)~~
Q2. (A) delete



speed of air at A > speed of air at B.

$P_A < P_B$ (From Bernoulli's Theorem)

$\therefore$ Net force due to pressure difference is upwards.

$$g_{effective} = g - \frac{F}{m}$$

$\therefore$ Time of flight increases.

Q3. (D) Hooke's law is obeyed in region where, stress ∝ strain

It is oa region

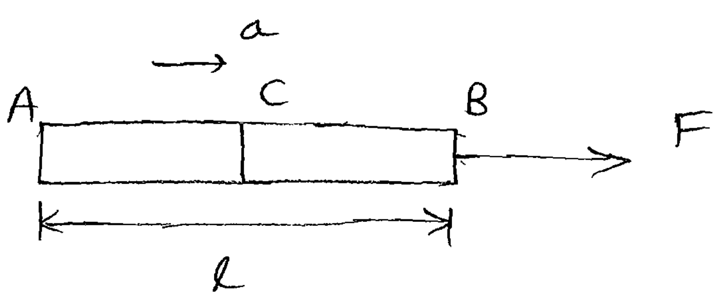
~~bc~~ b is ~~ela~~ elastic limit which comes just after proportional limit.

bc is region in which materials yields, which means material continuously ~~deform~~ deforms ~~with~~ with little or no increase in strain.

bc ~~as~~ is also called region in which material behaves as viscous liquid.

Q4. (C)
If material ~~deforms be~~ deforms beyond elastic limit then, on unloading there remains a net permanent deformation.

5. (A)



A

$F = (\rho A l) a$

Solution to Exercise - 1

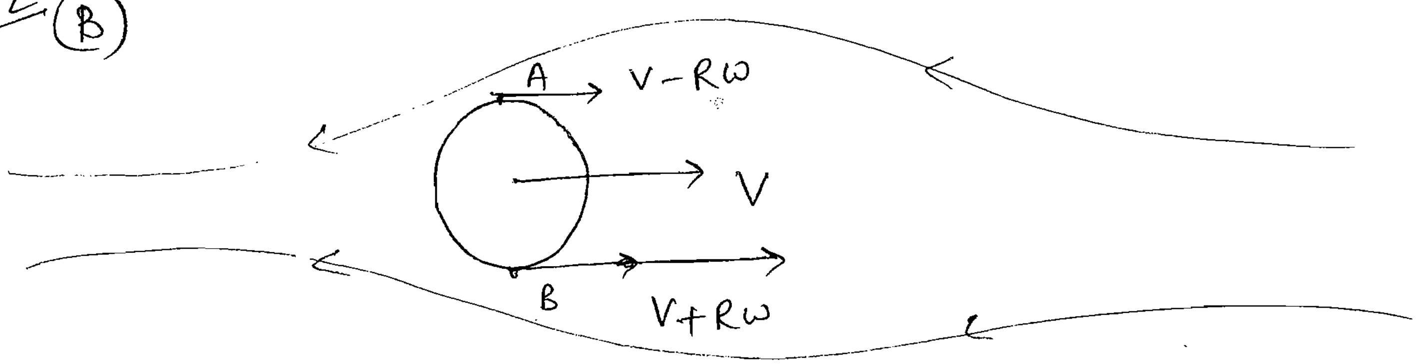
Properties of Matter

Q1 (A) ~~Ka~~ $PV =$ constant if $T =$ constant

At surface $P_{surface} < P_{bottom}$

$\therefore V_{surface} > V_{bottom}$

Q2 (B)



Velocity of air ~~at~~ at A > Velocity of air at B.

From Bernoulli's theorem,

$P_A < P_B$

$\therefore$ Net force acts upwards due to pressure difference.

$$g_{eff} = g - \frac{F}{m}$$

Q3 (D) Hooke's law, stress $\propto$ strain

It is region Oa. a = Proportional limit

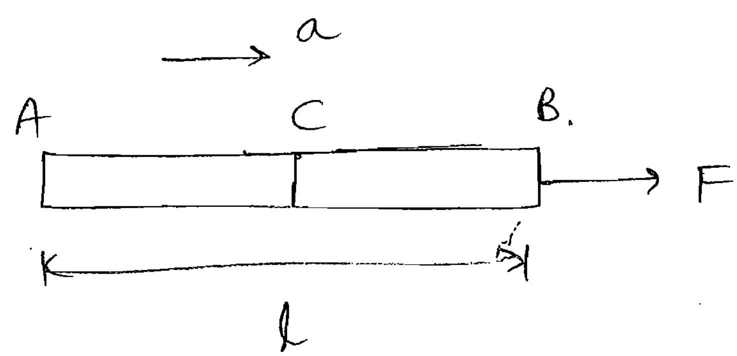
b is elastic limit which comes after proportional limit.

bc $\rightarrow$ region in which material yields. Material behaves as viscous liquid.

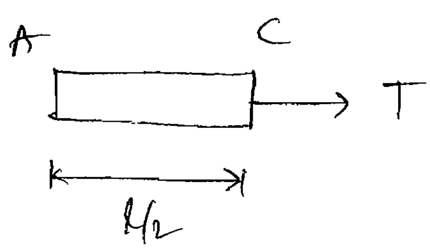
Q4 (C)

If material deforms beyond elastic limit then, on unloading there remains a net permanent deformation.

Q5 (A)

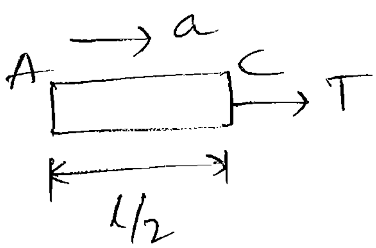


$$F = (\rho A l)\, a$$



$$T = ma = \rho A \frac{l}{2} a$$

$$\frac{T}{A} = \frac{\rho l A}{2} = \text{stress at midpoint}$$



$$T = ma$$

$$T = \rho A \frac{l}{2} a$$

$$\frac{T}{A} = \frac{\rho l a}{2} \qquad \frac{T}{A} = \text{stress}$$

$\therefore$ stress at midpoint $= \frac{1}{2} \rho l a$

06. (C)

$$\frac{\Delta l}{l} = \frac{\text{stress}}{Y} = \frac{F}{AY}.$$

$$\frac{l_1 - l}{l} = \frac{F_1}{AY} \qquad \frac{l_2 - l}{l} = \frac{F_2}{AY}.$$

$$\frac{l_1 - l}{l_2 - l} = \frac{F_1}{F_2}$$

solving, $l = \dfrac{l_1 F_2 - l_2 F_1}{F_2 - F_1}$

07. (B)

Viscous force $= 6\pi\eta R V$. depends only on viscosity of medium surrounding.

08. (A) Force $= F = \dfrac{Y A l}{\Delta l}$

and hence deforming force is lower at high temperature for given elongation.

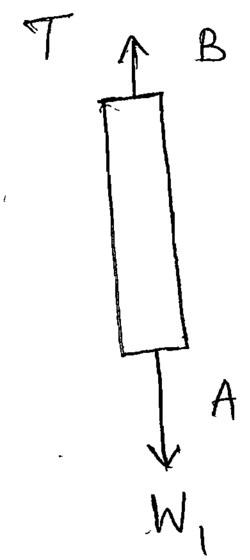
99. (C)

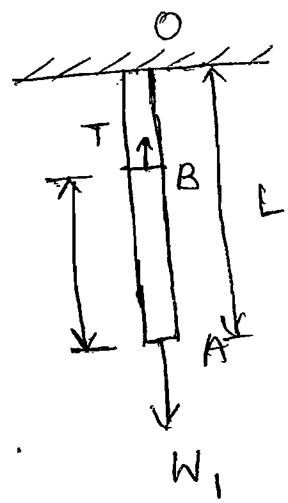
Breaking stress is a material property.

$$\therefore \quad \frac{20 \text{ kg-wt}}{\pi d^2/4} = \frac{F}{\pi (2d)^2/4}$$

$$\therefore \quad F = 80 \text{ kg-wt}$$

Q10. (C)





$$T = W_1 + W_{AB} = W_1 + \frac{W}{l} \times \frac{3l}{4}$$

$$T = W_1 + \frac{3W}{4}$$

$$\therefore \text{ stress at } B = \frac{T}{A} = \frac{(W_1 + 3/4\,W)}{S}$$

Q11. (B)

$$Y = \frac{\text{stress}}{\text{strain}} \qquad \text{strain} = \frac{2l - l}{l} = \frac{l}{l} = 1$$

$$Y = \frac{\text{stress}}{1} = \frac{F}{A} = \frac{2 \times 10^5 \text{ dyne}}{2 \text{ cm}^2} = 1 \times 10^5 \frac{\text{dyne}}{\text{cm}^2}$$

Q12 (C)

$$\text{stress} = Y \times \text{strain}$$

$$\text{strain} = \frac{\Delta l}{l} = \frac{\alpha l \Delta\theta}{l} = \alpha \Delta\theta.$$

$$(\sigma) \text{ stress} = Y(\alpha \Delta\theta)$$

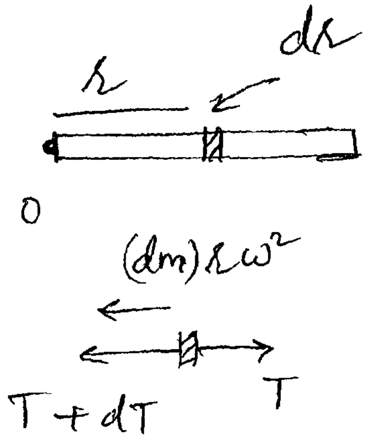
$$\sigma_1 = \sigma_2 \implies Y_1 \alpha_1 \Delta\theta = Y_2 \alpha_2 \Delta\theta$$

$$\therefore \frac{Y_1}{Y_2} = \frac{\alpha_2}{\alpha_1} = \frac{3}{2} = 3:2$$

Q13. (C)

$$(T + dT) - T = (dm) r \omega^2$$

$$dT = (dm) r \omega^2$$



$$(\sigma) \text{ stress} = \frac{dT}{dA} \quad \frac{dT}{A}$$

$$\text{strain} = \frac{\text{stress}}{Y} = \frac{dT}{YA} =$$

dl dT

$$dT = dm\, x\, \omega^2 \qquad dm = \frac{m}{L}\, dx$$

$$\int_0^T dT = \int_{x=L}^{x} \frac{m}{L}\, dx\, x\omega^2$$

$$\left[T \right]_0^T = \frac{m\omega^2}{L} \left[\frac{x^2}{2} \right]_L^x$$

$$T = \frac{m\omega^2}{2L} \left(x^2 - L^2 \right)$$

$\therefore$ stress at element $= \frac{T}{A}$

$$\text{strain at element} = \frac{dl}{dx} = \frac{\text{stress}}{Y} = \frac{T}{YA}$$

$$dl = \frac{m\omega^2}{2YLA} \left(x^2 - L^2 \right) dx$$

$$\int_0^l dl = \int_{x=0}^{L} \frac{m\omega^2}{2YLA} \left(x^2 - L^2 \right) dx$$

$$l = \frac{m\omega^2}{2YLA} \left[\frac{x^3}{3} - xL^2 \right]_0^L$$

$$= \frac{m\omega^2}{2YLA} \left[\frac{L^3}{3} - L^3 \right] = -\frac{m\omega^2 L^2}{3YA}.$$

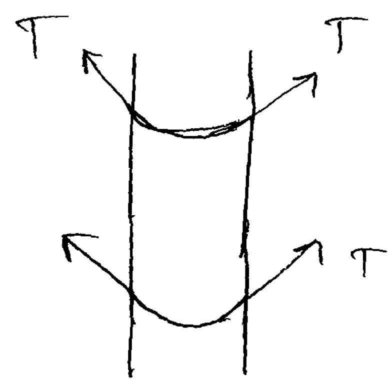
$$\therefore \quad \frac{2T}{R} = h\rho_w g$$

$$R = \frac{2T}{h\rho_w g} = \frac{2(0.07)}{10^{-2} \times 10^3 \times 9.8}$$

$R = 1.4285$ mm.

R = radius of meniscus

Q19 (A)



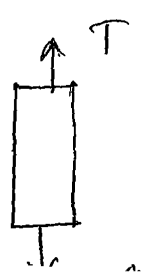
$T(2\pi r) + T(2\pi r)$

$= mg$

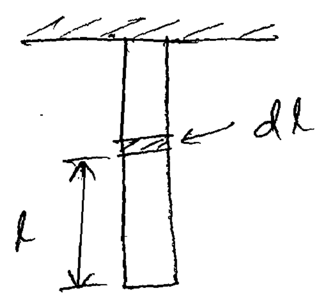
$2T(2\pi r) = \pi r^2 L \rho g$ (Two surfaces)

$L = \frac{4T}{r\rho g}$

Q20 (B)



$T = mg$



$$T = \frac{m}{L} l g$$

$$\frac{T}{A} = \frac{mlg}{AL}$$

strain in length (dl) $= \frac{\text{stress}}{Y} = \frac{mlg}{YAL} =$

$= \frac{dy}{dl}$

where dy = small increase in length of dl.

$$\therefore \int_0^y dy = \int_0^L \frac{mlg}{YAL}\, dl = \frac{mg}{YAL}\left[\frac{l^2}{2}\right]_0^L$$

$$y = \frac{mg L^2}{2YAL} = \frac{mgL}{2AY}.$$

Fluid (sub)

4, 5, 6, 11, 13, 14(b),

15, 16, 19.

(obj) → (due)

Elasticity

(obj) 2, 18.

(sub)

Q14 (B)

~~$V_t = \frac{2}{g\eta}$~~

At terminal velocity

$$6\pi\eta r V_t = mg.$$

$$V_t = \left(\frac{m}{r}\right)\frac{g}{6\pi\eta}$$

$$\therefore \quad V_t \propto \frac{m}{r}$$

Q15 (C)

Initially velocity increases and then, becomes constant.

Q16 (C) volume of big drop = volume of smaller drops

$$\frac{4\pi}{3}R^3 = 8 \times \frac{4\pi}{3}r^3$$

$$R^3 = 8r^3$$

$$r = \frac{R}{2}$$

Work done = change in surface energy

$$= T \times \Delta S$$

$$W = T \times (4\pi r^2 \times 8 - 4\pi R^2)$$

$$= T(4\pi R^2) = 4\pi R^2 T$$

Q17 (B)

Pressure at depth (h) $= P_0 + h\rho_w g$

$$P_h = P + hdg$$

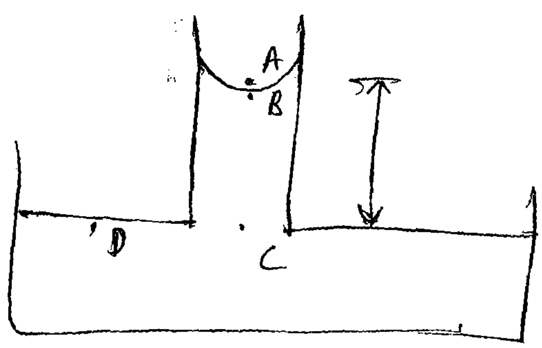
$P_{inside} - P_h$ = Excess pressure $= \frac{2T}{R}$

($\because$ Bubble in water has only one surface).

$$\therefore P_{inside} = P_h + \frac{2T}{R}$$

$$= P + hdg + \frac{2T}{R}$$

18 (D)



$$P_{atm} = P_D = P_C = P_A$$

$$P_B + h\rho_w g = P_C$$

$$P_B + h\rho_w g = P_A$$

$$P_A - P_B = h\rho_w g$$

Ex - 2 ~~PoM~~

Properties of Matter

Q1. (b)(c)

When rod falls vertically & slides on rough surface, tension in it is zero. So, stress is zero and hence, strain.

Q2 (a) (b)(c)

~~Avel~~ At any point $T_A = \frac{4}{3} mg$.

$T_B = \frac{mg}{3}$.

(a) ∴ A will reach breaking stress before B, if $r_A = r_B$.

(b) if $r_A < 2r_B$.

$$\sigma_A = \frac{4/3\, mg}{\pi R_A^2} \qquad \sigma_B = \frac{mg/3}{r_B^2}.$$

$$\left(\frac{4}{\sigma_A}\right)^2 < 2\left(\frac{1}{\sigma_B}\right)^2$$

$$\sigma_A > 4\sqrt{2}\,\sigma_B$$

ii) $r_A = 2r_B$.

$$\sigma_A = \frac{4/3\, mg}{\pi r_A^2} \qquad \sigma_B = \frac{mg/3}{\pi r_B^2}$$

(c)

$$\sigma_A = \sigma_B = \text{Breaking stress}$$

Q3. (a) (c) (d)

$$\Delta U_G = -Mgl$$

~~From energy conservation~~

~~$\Delta U_G + \Delta U_E = 0$~~

$\Rightarrow$ ~~$\Delta U_E = -\Delta U_G = Mgl$~~

$$\Delta U_E = \frac{1}{2} \times \left(\frac{Mg}{A}\right) \times \left(\frac{l}{L}\right) \times (L \times A)$$

$$\Delta U_E = \frac{1}{2} Mgl.$$

From conservation of Energy.

$\Delta H = \frac{1}{2} Mgl$ = Heat produced.

Q4 (a) (c) (d)

(a) $F = -F_{internal}$

$\therefore \quad W_F = -W_{internal} = -(-\Delta U_E)$

$$W_F = \Delta U_E = \frac{YAl^2}{2L}$$

$$\Delta U_E = \frac{YAl^2}{2L}$$

$\Delta H = 0$

Q5 (b) (d)

Elastic forces are conservative upto elastic ~~lim~~ limit only.

~~Q6.~~ (a) (b) (c)

~~$F = YA\alpha t$~~

~~$ma = YA\alpha t$~~

~~$a = \frac{YA\alpha t}{Al\rho}$~~

$$F = \frac{YA}{l}(\Delta l)$$

$$a = \frac{YA}{ml}(\Delta l)$$

$m = \rho A l$

$$\therefore \quad a = \frac{Y}{\rho l^2}(\Delta l) = \frac{Y}{\rho l^2}x.$$

$\omega = \sqrt{\frac{Y}{\rho l^2}} = \frac{1}{l}\sqrt{\frac{Y}{\rho}}$

~~Q6~~ (a) (b) $\theta < 90°$ → Concave meniscus → (wetting liquid),

Cohesive < adhesive

$\theta > 90°$ → convex → cohesive > adhesive

(Non-wetting).

$\theta = 0°$ → hemi-spherical ~~Plane meniscus~~ → cohesive < Adhesive.

(water).

$$h = \frac{2T\cos\theta}{R\rho g} \qquad R' = \frac{R}{2} \Rightarrow h' = 2h.$$

~~Q7~~ (c) (d)

~~h~~ Radius of meniscus will be changed such that

$$hr = h'r' = \frac{2T}{\rho g}$$

h' = new height of tube

r' = new radius of meniscus.

Q8 (a)(b)(d)

Surface tension is independent of area

$$\Delta P = \frac{4T}{R}$$

$T_m > T_w$

Q9. (b) (c) (d).

θ = obtuse, liquid depresses in tube, ~~conve~~ ~~meni~~

θ = acute, liquid ~~ris~~ rises and wets tube.

~~θ~~ θ = obtuse, convex meniscus

T decreases with temp.

Q10 (a)(d).

$P_{in} > P_{out}$

$$\Delta P = \frac{2T}{R} = P_{in} - P_{out}$$

$$P_{in} = P_{out} + \frac{2T}{R}$$

As bubble comes up Pout decreases

Properties of Matter - Ex-3

Passage 1

Q1 (A)

$$F = -\eta A \frac{dV}{dr}$$

$$A = 2\pi r l \qquad V = V_0\left(1 - \frac{r^2}{R^2}\right)$$

$$\frac{dV}{dr} = -\frac{2V_0 r}{R}.$$

$$\therefore \quad F \propto r^2$$

Q2 (A)

$$V = V_0\left(1 - \frac{r^2}{R^2}\right) \qquad V_0 = \frac{PR^2}{4\eta l}$$

$$r = R/2$$

$$V = \frac{3PR^2}{16\eta l}$$

Q3 (B)

$$V = \frac{\pi P R^4}{8\eta l} \qquad V_0 = \frac{PR^2}{4\eta l}$$

$$\therefore \quad V = \frac{(\pi R^2 V_0)}{2}$$

Q4 (C)

$$V_0 = \frac{PR^2}{4\eta l} \Rightarrow P = \frac{4\eta V_0 l}{R^2}$$

Q5 (C)

$F = P \times \pi R^2$. $P = \frac{4\eta V_0 l}{R^2}$

$F = 4\pi\eta l V_0$

Passage 2

Q6 (A) Range of force = 10 A° = 1 nm.

Q7 (C) Work done against intermolecular forces (Cohesive Force).

Q8 (A)

Concave meniscus → adhesive force > cohesive force

Angle of contact 90° → Plane meniscus

~~Cohes~~ Adhesive forces = Cohesive forces

Pressure below meniscus is greater → Convex meniscus (Adhesive force < ~~cohesive force~~ Cohesive force).

Q9 (C) Depends on both cohesive and adhesive
forces

Q10 (B) Net force on liquid is ~~at~~ normal to liquid surface.

Matrix Match type

Q11. Terminal velocity → Resultant of upthrust and viscous force.
(A → S)

Objects of high density can also float → When average density becomes less than liquid
eg. Iron built ship.
(B → P)

$\rho_{iron} > \rho_{water}$

(C → q) $B = V \rho g_{eff}$

under free fall $g_{eff} = 0$

(D → r) $F_v = 6\pi\eta R V$

$F_v \propto V$

JP - Sir (1 to 5) Elasticity - Ex - IV - solutions (subjective)

Q1 Let Y_s, Y_c be their Young's modulii.

$$Y_s = \frac{\text{stress}}{\text{strain}} = \frac{F/A_s}{\Delta l_s / l_s} = \frac{l_s}{\Delta l_s A_s}$$

$$Y_c = \frac{F/A_c}{\Delta l_c / l_c}$$

$$\frac{Y_s}{Y_c} = \frac{l_s}{\Delta l_s A_s} \cdot \frac{\Delta l_c A_c}{l_c} \qquad \Delta l_s = \Delta l_c \text{ (given)}$$

$$= \frac{(4.7)m}{(\Delta l_s)(3 \times 10^{-5} m^2)} \cdot \frac{(\Delta l_c)(4 \times 10^{-5})}{(3.5 m)}$$

$$\frac{Y_s}{Y_c} = \frac{47}{35} \cdot \frac{4}{3} = \frac{188}{105} = 1.79 = 1.79$$

Q2. For steel wire

Load = 10 kg $\qquad$ stress $= \frac{10g}{A_{steel}} = \frac{10g}{\pi\left(\frac{0.25 \times 10^{-2}}{2}\right)^2}$

$\therefore$ ~~$\frac{\Delta l_s}{l_s} = \frac{Y}{st}$~~ $\qquad Y = \frac{\text{stress}}{\text{strain}}$

~~sta~~ strain $= \frac{\Delta l_s}{l_s} = \frac{\text{stress}}{Y}$

$$\Delta l_c = l_s \times \text{stress} = 1.5m \times \frac{10g}{\ldots}$$

$\Delta l_{steel} = 1.5 \times 10^{-4}$ m

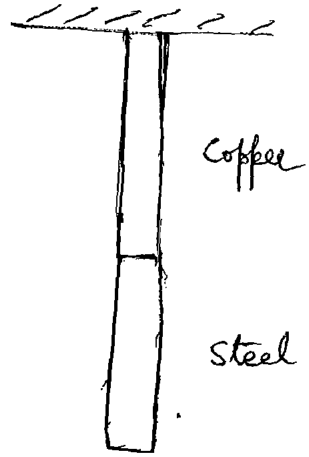
For Brass, load = 6 kg stress = $\frac{6g}{\pi\left(\frac{0.25 \times 10^{-2}}{2}\right)^2}$

$\Delta l_{brass} = \frac{\text{stress}}{Y} \times l_{brass}$

$= \frac{6g}{0.91 \times 10^{11} \times \frac{22}{7}\left(\frac{0.75 \times 10^{-2}}{2}\right)^2} \times (1\,m)$

$= 1.3 \times 10^{-4}$ m

93. $\Delta l = \frac{\text{stress}}{Y} \times l = \frac{Fl}{AY}$



Δl = change in length

l = original length

$\Delta l_s + \Delta l_c = 0.7$ mm $= 0.7 \times 10^{-3}$ m.

$\frac{F l_s}{A_s Y_s} + \frac{F l_c}{A_c Y_c} = 0.7 \times 10^{-3}$

$F = \frac{0.7 \times 10^{-3}\ m}{\frac{l_s}{A_s Y_s} + \frac{l_c}{A_c Y_c}}$

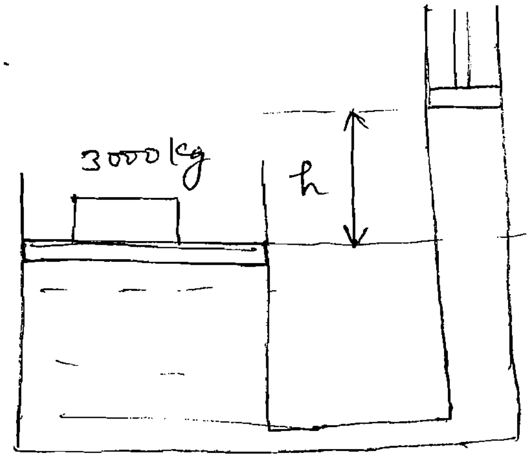
$F = \frac{0.7 \times 10^{-3}\ m}{\frac{1.6\ m}{\ldots} + \frac{2.2\ m}{\ldots}}$

$\therefore F = 1.77 \times 10^2 \ N$

74 From Pascal's law.

$P_s + h\rho g = P_L$

$P_s = P_L - h\rho g.$



Hydraulic lift

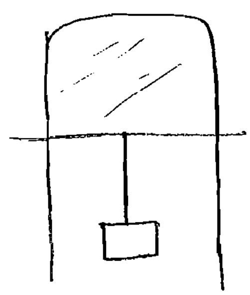
For maximum pressure,

$h = 0 \qquad P_s = P_L$

~~$P_s = P_L = \frac{3000 g}{(425 \times 10^{-4})^2}$~~

$$P_s = P_L = \frac{3000 g \ N}{425 \times 10^{-4} \ m^2} = 6.92 \times 10^5 \ N/m^2$$

75. There are two surfaces of soap film.



So, force due to surface tension is given by

~~$T(2l) = mg$ (weight)~~

~~$T = \frac{mg}{2l}$~~ since, two surfaces are there.

~~$F = 2T(2l)$~~

[illegible] where T = surface

This force supports weight (external + slider).

$\therefore \quad T(2l) = mg$

$$T = \frac{mg}{2l} = \frac{1.5 \times 10^{-2}}{2 \times 0.3}$$

$$T = 2.5 \times 10^{-2} \text{ N/m}$$

76.

Excess pressure inside drop

$$= \frac{2T}{R} = \frac{2 \times 4.65 \times 10^{-1}}{3 \times 10^{-3}} = 310 \text{ Pa}$$

Excess pressure $= P_{inside} - P_{atm}$

$P_{inside} = P_{atm} +$ Excess pressure

$= 1.01 \times 10^{5} \text{ Pa} + 310 \text{ Pa}$

$P_{inside} = 1.0131 \times 10^{5} \text{ Pa}$

Q7. At terminal velocity (V_t)

Viscous force = weight of drop.

$$6\pi\eta R V_t = mg = \frac{4\pi}{3} R^3 \rho g$$

$$V_t = \frac{2 \rho R^2 g}{9\eta}$$

$$V_t = \frac{2 \times (1.2 \times 10^3) \times (2 \times 10^{-5})^2 \times 9.8}{9 \times 1.8 \times 10^{-5}}$$

$$V_t = 5.8 \times 10^{-2} \text{ m/s} = 5.8 \text{ cm/s}.$$

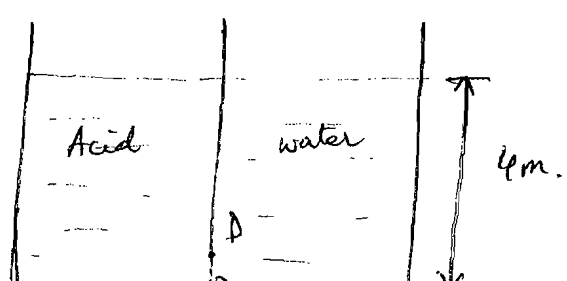
$$\text{Viscous Force} = 6\pi\eta R V_t = \frac{4\pi}{3} R^3 \rho g$$

$$= 6 \times \frac{22}{7} \times 1.8 \times 10^{-5} \times 2 \times 10^{-5} \times 5.8 \times 10^{-2}$$

$$= 3.93 \times 10^{-10} \text{ N}$$

Q8

Pressure difference at the door



$= h_{acid} \rho_{acid} g - h_{water} \rho_{water} g$

$= 4m \times 1.7 \times 10^3 \times 9.8 - (4m) \times (1 \times 10^3) \times 9.8$

$= 4 \times 0.7 \times 10^3 \times 9.8 \ N/m^2$

$= 2.744 \times 10^4 \ N/m^2$

$\therefore$ Force required $= \Delta P \times A$

$= 2.744 \times 10^4 \times 20 \times 10^{-4}$ N.

$= 54.88$ Newton

79. Pressure inside soap bubble is given by

$$P_{in} = P_{atm} + \frac{4T}{R}$$

P_{atm} = Atmospheric pressure = Pressure in cylinder = P_{cy}

T = Surface Tension

R = Radius of bubble

Initially, ~~$P_i = P_{atm} + \frac{4T}{R}$~~

$$P_i = P_{cy} + \frac{4T}{R}$$

$$P_{cy} = 10^5 \text{ N/m}^2$$

$$P_i = \left(10^5 + \frac{4}{3} \times 10^3\right) \text{ Pa}$$

$$P_f = P + \frac{4T}{R_2} = \left(P + \frac{8}{3} \times 10^3\right) \text{ Pa}$$

$$R_2 = \frac{R}{2}$$

Under Isothermal conditions

$$PV = \text{constant}$$

$$P_i V_i = P_f V_f$$

$$\left(P + \frac{8}{3} \times 10^3\right) \times \frac{4\pi}{3}\left(\frac{R}{2}\right)^3 = \left(10^5 + \frac{4}{3} \times 10^3\right) \times \frac{4\pi}{3} R^3$$

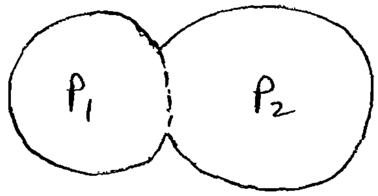
$$8\left(10^5 + \frac{4}{3}\times 10^3\right) = P + \frac{8}{3}\times 10^3$$

$$\therefore \quad P = 8.08 \times 10^5 \text{ Pa}$$

Q10

$$P_{in} = P_{atm} + \frac{4T}{R}$$

$$P_1 = P_0 + \frac{4T}{R_1}$$



$$P_2 = P_0 + \frac{4T}{R_2}$$

$$P_1 - P_2 = 4T\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{—— (1)}$$

$P_1 - P_2$ = Excess pressure for common surface

$$P_1 - P_2 = \frac{4T}{R_{common}} \quad \text{—— (2)}$$

from (1) & (2).

$$\frac{1}{R_{common}} = \frac{1}{R_1} - \frac{1}{R_2}$$

$R_{common} = 0.004\ m$

Q11 Let a & ~~be~~ b be radii of two soap bubbles and P_a, P_b be pressure inside them before they coalesce. If c be radius and P_c be pressure inside combined bubbles. Then

$$\therefore\ P_a = P_0 + \frac{4T}{a} \qquad P_b = P_0 + \frac{4T}{b}$$

$$P_c = P_0 + \frac{4T}{c}$$

If V_a, V_b, & V_c are corresponding ~~soap~~ volumes then, from Boyle's law under isothermal condition.

$$P_a V_a + P_b V_b = P_c V_c$$

$$\left(P_0 + \frac{4T}{a}\right)\frac{4\pi}{3}a^3 + \left(P_0 + \frac{4T}{b}\right)\frac{4\pi}{3}b^3 = \left(P_0 + \frac{4T}{c}\right)\frac{4\pi}{3}c^3$$

$$P\left(\frac{4\pi}{3}a^3+\frac{4\pi}{3}b^3-\frac{4\pi}{3}c^3\right)+\frac{4T}{3}\left(4\pi a^2+4\pi b^2-4\pi c^2\right)=0$$

Now, $\frac{4\pi}{3}a^3+\frac{4\pi}{3}b^3-\frac{4\pi}{3}c^3 = V$

= change in volume

$4\pi a^2+4\pi b^2-4\pi c^2 = S$ = change in surface area

$\therefore \quad PV+\frac{4T}{3}S=0$

~~$3PV+4TS=0$~~

$3PV+4ST=0$

Only One Option Correct

1. (C)

Using, $Y = \dfrac{F/A}{\Delta\ell/\ell_0}$

$$y = \frac{F/\pi(2R)^2}{\Delta\ell_1/2L} = \frac{F/\pi R^2}{\Delta\ell_2/L}$$

$$\therefore \frac{\Delta\ell_2}{\Delta\ell_1} = 2$$

2. (D)

3. (A)

Given: $\dfrac{1}{\rho}\dfrac{d\rho}{dt} = \text{constant}$

$$\therefore \frac{4\pi R^3}{3m} = \frac{d}{dt}\left[\frac{m}{\frac{4}{3}\pi R^3}\right] = \text{constant}$$

$$\Rightarrow R^3 \frac{d}{dt}\left(R^{-3}\right) = \text{constant}$$

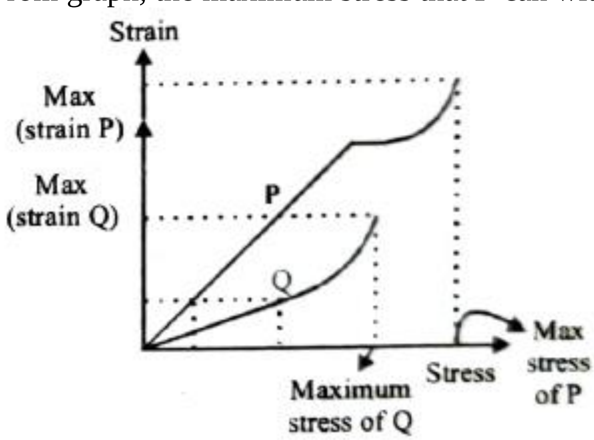
$$\Rightarrow R^3\left(-3R^4\right)\frac{dR}{dt} = \text{constant}$$

$$\therefore \left|\frac{dR}{dt}\right| \propto R$$

One or More than One Option Correct

1. (A, B)

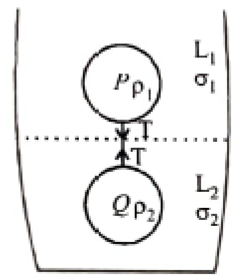
From graph, the maximum stress that P can withstand before breaking is greater than Q.



The strain of P is more than Q therefore P is more ductile.

$\because\ Y = \dfrac{\text{stress}}{\text{strain}}$

So, a given strain, straign is more for Q.
$\therefore\ Y_Q > Y_P$.

2. (A, D)
Since string is taut,
$\rho_1 < \sigma_1$ and $\rho_2 < \sigma_2$
For floating, net weight of system = net upthrust
$(\rho_1+\rho_2)V_g = (\sigma_1+\sigma_2)V_g$
Upwards terminal velocity
$$V_P = \frac{2r^2(\sigma_2-\rho_1)g}{9\eta_2}$$


Where r is radius of sphere.
Downwards terminal velocity
$$V_Q = \frac{2r^2(\rho_2-\sigma_1)g}{9\eta_1}$$
$$\therefore\ \left|\frac{V_P}{V_Q}\right| = \frac{\eta_1}{\eta_2} \qquad (\because \rho_1-\sigma_2 = \sigma_1-\rho_2)$$
Again $V_P.V_Q < 0$ i.e., negative as V_P and V_Q are opposite to each other.

3. (A, C)
As we know $h = \dfrac{2\sigma\cos\theta}{r\rho g_{\text{eff}}}$
As 'r' increases, h decreases $h \propto \dfrac{1}{r}$ [all other parameters remaining constant]
Also $k \propto \sigma$
Further if lift is going up with an acceleration 'a' then $g_{\text{eff}} = g + a$.
As g_{eff} increases, 'h' decreases.
Also $h \propto \cos\theta$ not $h \propto \theta$

4. (A, C, D)
$F = -\eta A\left(\dfrac{dv}{dx}\right)$ or $|F| = \eta A\dfrac{u_0}{h}$
Where $\dfrac{u_0}{h}$ = velocity gradient $\left(\dfrac{dv}{dx}\right)$
$F \propto \eta;\ F \propto A;\ F \propto \mu_0$ and $F \propto \dfrac{1}{h}$

5. (A, B, C)
For case I
$$h_1 = \frac{2T\cos\theta_1}{r\rho g} = \frac{2\times 0.75\times\cos 0^\circ}{2\times 10^{-4}\times 1000\times 10} = 3.75\text{ cm}$$
For case II

$$h_2 = \frac{2T\cos\theta_2}{r\rho g} = \frac{2\times 0.75\times 60°}{2\times 10^{-4}\times 1000\times 10} = 3.75\text{ cm}$$

The correction in the height of water column raised in the tube, due to weight of water contained in the meniscus will be different for both cases.

In case II, if the capillary joint is 5 cm above the water surface then water in capillary will not reach the interface.

Water will reach till 3.75 cm.

Comprehensions Type

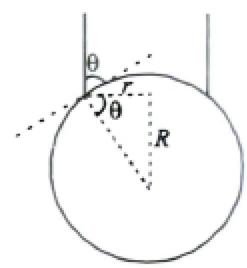
1. (D)
2. (B)
3. (B)

4. (C)

Vertical force due to surface tension

$$F_V = (T\cos\theta)\times 2\pi r$$

$$= T\left(\frac{r}{R}\right)\times 2\pi r = \frac{2\pi r^2 T}{R} \qquad \left[\because \cos\theta = \frac{r}{R}\right]$$



5. (A)

When the drop is about to detach from the dropper weight = vertical force due to surface tension $(mg = F_V)$

$$\therefore \frac{4}{3}\pi R^3 \rho g = \frac{2\pi r^2 T}{R}$$

$$\Rightarrow R^4 = \left(\frac{3}{2}\frac{r^2 T}{\rho g}\right) = \frac{3}{2}\times\frac{(5\times 10^{-4})^2\times 0.11}{1000\times 10} = 4.12\times 10^{-12}$$

or, $R = 1.42\times 10^{-3}$ m

6. (B)

Surface energy $= T\times 4\pi R^2$

$$= 0.11\times 4\times\frac{22}{7}\times(1.42\times 10^{-3})^2 = 2.27\times 10^{-6}\text{ J}$$

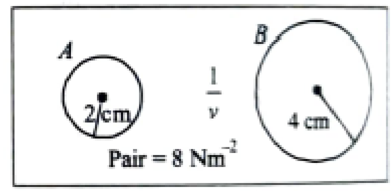
Integer / Numerical Answer Type

1. (6)

Pressure, $P_A + P_{air} + \frac{4T}{R_A}$ ($\because$ Excess pressure due to surface tension (T) in soap bubble = $4T/r$)

$$P_A = 8 + \frac{4T}{R_A} = \frac{4\times 0.04}{0.02} + 8 \quad \Rightarrow \quad P_A = 16\text{ N/m}^2$$

Similarly, $P_B = 8 + \frac{4T}{R_B} = 8 + \frac{4 \times 0.04}{0.04} = 12 \text{ N/m}^2$



According to ideal gas equation, $P_V = nRT$

$$P_A V_A = n_A R T_B \Rightarrow 16 \times \frac{4}{3}\pi(0.02)^3 = n_A R T_A \qquad ...(i)$$

$$P_B V_B = n_B R T_B \Rightarrow 12 \times \frac{4}{3}\pi(0.04)^3 = n_B R T_B \qquad ...(ii)$$

Dividing eq. (ii) by (i)

$$\frac{12 \times \frac{4}{3}\pi(0.4)^3}{16 \times \frac{4}{3}\pi(0.02)^3} = \frac{n_B}{n_A} \quad [\because T_A = T_B]$$

$$\therefore \frac{n_B}{n_A} = 6$$

2. (3)

As we know, terminal velocity

$$V_T = \frac{2r^2}{9\eta}(\rho - \sigma) g$$

$$\frac{V_P}{V_Q} = \frac{\frac{2r_1^2(\sigma - \rho_1) g}{9\eta_1}}{\frac{2r_2^2(\sigma - \rho_2) g}{9\eta_2}} = \frac{r_1^2(\sigma - \rho_1)}{r_2^2(\sigma - \rho_2)} \times \frac{\eta_2}{\eta_1}$$

$$= \frac{1^2}{(0.5)^2} \frac{[8 - 0.8]}{[8 - 1.6]} \times \frac{2}{3} = 3$$

3. (6)

$$\frac{4}{3}\pi R^3 = k \times \frac{4}{3}\pi r^3 \qquad \therefore R = K^{1/3} r$$

$$\Delta U = S\left[k \times 4\pi r^2 - 4\pi r^2\right]$$

$$\therefore \Delta U = 4\pi S R^2 \left[10^{\alpha/3} - 1\right] \qquad \left[\because K = 10^{\alpha}\right]$$

$$\therefore 10^{-3} = 4\pi \times \frac{0.1}{4\pi} \times \left(10^{-2}\right)^2 \left[10^{\alpha/3} - 1\right]$$

$\therefore 10^2 = 10^{\alpha/3} - 1$ Neglecting

$$10^2 = 10^{\alpha/3} \Rightarrow \frac{\alpha}{3} = 2$$

$$\therefore \alpha = 6$$

4. (2)

Given : $l_c = \sqrt{3}\,\text{m}; l_s = 1\text{m}; Y_c = 1 \times 10^{11} \text{ N/m}^2$ and $Y_s = 2 \times 10^{11} \text{ N/m}^2$.

At equilibrium, $T_s \cos 60 = T_c \cos 30°$

$$\Rightarrow \frac{T_s}{2} = \frac{T_c\sqrt{3}}{2}$$

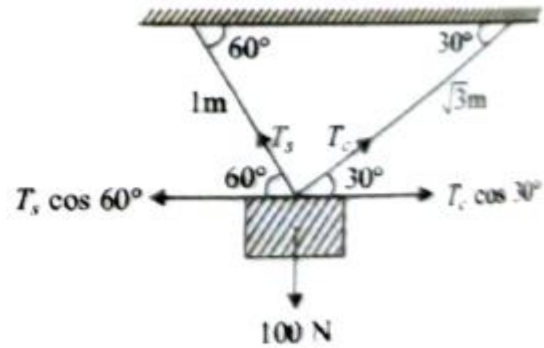
$$\Rightarrow T_s = \sqrt{3}T_c \Rightarrow \frac{T_c}{T_s} = \frac{1}{\sqrt{3}}$$

$$\therefore \frac{l_c}{l_s} = \frac{\sqrt{3}}{1} \text{ and } \frac{Y_c}{Y_s} = \frac{1\times10^{11}}{2\times10^{11}} = \frac{1}{2}$$

From, $Y = \frac{Fl}{A\Delta l} \Rightarrow \Delta l = \frac{Fl}{AY}$

Here, $A_s = A_c$

$$\therefore \frac{\Delta l_c}{\Delta l_s} = \left(\frac{T_c}{T_s}\right)\times\left(\frac{l_c}{l_s}\right)\times\left(\frac{Y_s}{Y_c}\right) = \left(\frac{1}{\sqrt{3}}\right)\times\left(\frac{\sqrt{3}}{1}\right)\times\left(\frac{2}{1}\right) = 2$$



5. (3.74)

According to question, the water above the rim as a disc of thickness ‘h’ having semi-circular edges.
r = h/2
pressure at the bottom of disc = pressure due to surface tension

$$\rho gh = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$R_1 >>> R_2 \quad \therefore \frac{1}{R_1} <<< \frac{1}{R_2} \text{ and } R_2 = \frac{h}{2}$$

$$\therefore \rho gh = T\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = T\left[0 + \frac{1}{h/2}\right] = \frac{2T}{h}$$

$$\Rightarrow h^2 = \frac{2T}{\rho g} \Rightarrow h = \sqrt{\frac{2T}{\rho g}} = \sqrt{\frac{20\times0.07}{10^3\times10}} = \sqrt{\frac{10\times100}{10^4\times100}}$$

$$\therefore h = \sqrt{14} \text{ mm} = 3.741$$