1. (B)
Magnitude of velocity at centre of oscillation
$$= \omega A = \frac{2\pi}{T} A = \frac{2\pi}{0.01} \times 0.2 = 40\,\pi \text{ m/s}$$

2. (A)
$F = -10x \quad \Rightarrow \quad k = 10 \text{ N/m}$
$\Rightarrow \; \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{10}{0.1}} = 10 \,\text{rad/s}^2$
Speed, at mean position, $v_{max} = A\omega = 6 \text{ m/s}$
$\therefore \; A = \frac{6}{\omega} = \frac{6}{10} = 0.6 \text{ m}$

3. (C)
$A = 10 \text{ mm}$ and $\omega = 2\pi / T = 2\pi / 2 = \pi \text{ rad/s}$
Let $x = A\sin(\omega t + \phi) = 10\sin(\pi t + \theta)$
At $t = 0, x = 5 \text{ mm}$
$\Rightarrow \; 5 = 10\sin\phi \qquad \Rightarrow \; \phi = \pi/6$
$\therefore \; x = 10\sin(\pi t + \pi/6)$

4. (D)
$v_{max} = 0.04 \text{ ms}^{-1}$
At $x = 0.02$ m, $a = 0.06 \text{ ms}^{-2}$
$\Rightarrow \; 0.06 = 0.02\,\omega^2 \qquad \Rightarrow \; \omega = \sqrt{3} \text{ rad s}^{-1}$
$\therefore \; T = 2\pi/\omega = 3.63 \text{ s}$.
$v_{max} = \omega A$
$\therefore \; A = v_{max}/\omega = \frac{0.04}{\sqrt{3}} = 2.31 \times 10^{-2} \text{ m}$

5. (A)
$v = \omega\sqrt{A^2 - x^2}$
$\Rightarrow \; v_1^2 = \omega^2\left(A^2 - x_1^2\right)$ and $v_2^2 = \omega^2\left(A^2 - x_2^2\right)$
$\Rightarrow \; v_1^2 - v_2^2 = \omega^2\left(x_2^2 - x_1^2\right)$
$\therefore \; T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}} = 2\pi\sqrt{\frac{5^2 - 4^2}{10^2 - 8^2}} = \pi \sec$

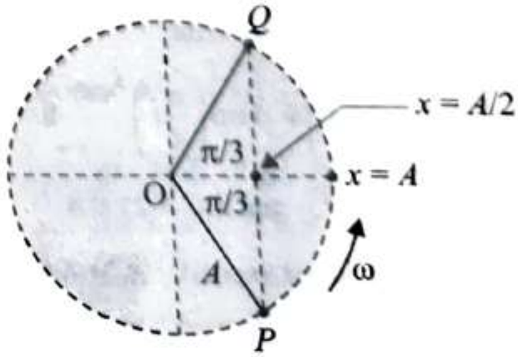
6. (B)
In SHM, $a = -kx$ and $v = \omega\sqrt{A^2 - x^2} = \omega\sqrt{A^2 - a^2/k^2}$

$\Rightarrow\ v^2 = -(\omega^2/k^2)a^2 + \omega^2 A^2$

Graph of v^2 vs a^2 is a straight line with negative slope and positive y-intercept.

7. (D)

From the phasor diagram, it is clear that moving from point P to Q, the vector OP traces an angle of $\pi/3 + \pi/3 = 2\pi/3$ at the centre.



8. (D)

Let the particle start from $x = 0$.

Then, $x = A\sin\omega t$. At $x = \frac{A}{2}, \omega t = \frac{\pi}{6}$

$\Rightarrow \frac{2\pi}{T}t = \frac{\pi}{6}$ $\therefore\ t = \frac{T}{12}$

9. (A)

Let the particle start from $x = A$.

Then, $x = A\cos\omega t$ At $x = \frac{A}{2}, \omega t = \frac{\pi}{3}$

$\Rightarrow \frac{2\pi}{T}t = \frac{\pi}{3}$ $\therefore\ t = \frac{T}{6}$

10. (A)

At $t = 0, x = 0$ and $v = A\omega$

At $t = T/2, x = 0$ and $v = -A\omega$

$$|a_{av}| = \frac{|\Delta v|}{\Delta t} = \frac{|-A\omega - A\omega|}{T/2} = \frac{2A\omega}{\pi/\omega} = \frac{2A\omega^2}{\pi}$$

11. (D)

$$E = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}m\left(\frac{2\pi}{T}\right)^2 A^2 \qquad \Rightarrow E \propto \left(\frac{A}{T}\right)^2$$

If A and T are doubled, E remains same.

$\therefore\ E' = E$

12. (B)

$$F = -kx \text{ and } U = \frac{1}{2}kx^2 = \frac{1}{2}\left(\frac{-F}{x}\right)x^2 = -\frac{F}{2}x$$

$\therefore \ \dfrac{2U}{F} + x = 0$

13. (D)

$U = U_0(1-\cos ax) = U_0 \times 2\sin^2\left(\dfrac{ax}{2}\right)$

For small x, $U \approx 2U_0\left(\dfrac{ax}{2}\right)^2 = \left(\dfrac{ax}{2}\right)^2 = \dfrac{a^2U_0}{2}x^2 \Rightarrow k = a^2U_0$

$\therefore \ T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{m}{a^2U_0}}$

14. (C)

$E = \dfrac{1}{2}m\omega^2A^2 = \dfrac{1}{2}m\left(\dfrac{2\pi}{T}\right)^2 A^2$

$\therefore \ A = \dfrac{T}{2\pi}\sqrt{\dfrac{2E}{m}} = \dfrac{\pi}{2\pi}\sqrt{\dfrac{2\times 0.04}{0.5}}$

$= 0.2\,\text{m} = 20\,\text{cm}$

15. (C)

At $x = \dfrac{A}{2}, U = \dfrac{1}{2}k\left(\dfrac{A}{2}\right)^2 = \dfrac{1}{4}\left(\dfrac{1}{2}kA^2\right) = \dfrac{E}{4}$

$\therefore \ K = E - U = \dfrac{3E}{4}$

16. (C)

$K = U$ and $K + U = E \Rightarrow U = \dfrac{E}{2}$

$\Rightarrow \dfrac{1}{2}kx^2 = \dfrac{1}{2}\left(\dfrac{1}{2}kA^2\right)$

$\therefore \ k = \dfrac{A}{\sqrt{2}}$

17. (C)

$K_{av} = K_{\max}/2 \qquad \Rightarrow K_{\max} = 2\,K_{av} = 2\times 5 = 10\text{ J}$

In equilibrium position, $K = K_{\max} = 10\text{ J}$ and $U = 15\text{ J}$

$\therefore \quad E = K + U = 10 + 15 = 25\text{ J}$.

Not that in equilibrium position, potential energy is minimum. Generally, we take it as zero but can be taken as any other constant value.

18. (C)

$U = 2 - 20x + 5x^2$

$\Rightarrow F = -(dU/dx) = -10x + 20 = -10(x-2)$

Since, $F \propto (x-2)$ with negative sign, the particle oscillates in SHM with $x = 2$ as mean position.

Here, $x = -3$ is one extreme position.
The other extreme position is $x = 2 + \left[2 - (-3)\right] = 7$.

19. (A)
From the given conditions, we can write
$$x = A\sin\omega t \text{ and } y = A\sin(\omega t + \pi/2)$$
$\Rightarrow$ $x^2 + y^2 = A^2$ which is the equation of a circle.

20. (B)
$$4\cos^2 0.5t \sin 1000t = 2(1 + \cos t)\sin 1000t$$
$$= 2\sin 1000t + \sin 1001t + \sin 999t$$
Therefore, the resultant is superposition of 3 independent harmonic motions.

21. (A)
The ratio of time period of two pendulums is 1:5/4 = 4:5. So, ratio of frequency is 5:4.
When small pendulum has completed 5 oscillations, the larger has completed 4 oscillations and they will be again in same phase.

22. (A)
The ratio of time periods is $\sqrt{100} : \sqrt{121} = 10 : 11$ and hence, ratio of frequency is 11:10.
The two pendulums will be in same phase at mean position again after the larger pendulum has completed 10 oscillations.

23. (C)
The net force on the bob in liquid is
$F = mg - \rho_f Vg = mg - mg/2 = mg/2$.
The effective acceleration is $g' = g/2$. It's time period is
$$T' = 2\pi\sqrt{\frac{l}{g'}} = 2\pi\sqrt{\frac{l}{g/2}} = \sqrt{2}\,T = 2\sqrt{2}\text{ sec}.$$

24. (D)
Time taken to complete first 1/4 oscillation from $x = 0$ to $x = A$ is $T/4$ and the second 1/8 oscillation from $x = A$ to $x = A/2$ is $T/6$.
Hence, time taken to complete
$\frac{1}{4} + \frac{1}{8} = \frac{3}{8}$ oscillation is $\frac{T}{4} + \frac{T}{6} = \frac{5T}{12}$

25. (D)
$$T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{L} + \frac{1}{R}\right)}} = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{2R} + \frac{1}{R}\right)}} = 2\pi\sqrt{\frac{2R}{3g}}$$

26. (A)
In equilibrium, $kx_0 = mg$ or $\frac{m}{k} = \frac{x_0}{g}$

$\therefore\ T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{x_0}{g}} = 2\pi\sqrt{\frac{9.8\times10^{-2}}{9.8}} = \frac{2\pi}{10}$ sec

27. (B)

Let k' be the spring constant of the longer piece. Then spring constant of shorter piece is $2k'$. The two together in series has spring constant *k*.

$\Rightarrow\ \frac{1}{k} = \frac{1}{k'} + \frac{1}{2k'} = \frac{3}{2k'}$

$\therefore\ k' = \frac{3k}{2}$

28. (A)

Effective spring constant is

$k_{eff} = \frac{k\times 2k}{k+2k} = \frac{2}{3}k$

$\therefore\ T = 2\pi\sqrt{\frac{m}{k_{eff}}} = 2\pi\sqrt{\frac{3m}{2k}}$

29. (A)

k_{eq} = Parallel [Series (2*k*, 2*k*), Parallel (*k*, *k*)]

= Parallel [*k*, 2*k*] = 3*K*

$\therefore\quad T = 2\pi\sqrt{\frac{m}{k_{eq}}} = 2\pi\sqrt{\frac{m}{2K}}$

30. (C)

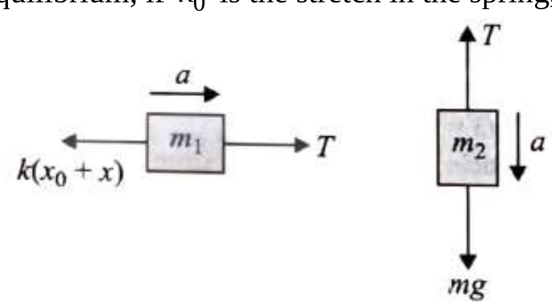
For the two systems, the equivalent spring constants are respectively

$k_1 = \frac{k\times k}{k+k} = \frac{k}{2}$ and $k_2 = \frac{(k+k)\times k}{(k+k)+k} = \frac{2k}{3}$

As $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}, \frac{f_1}{f_2} = \sqrt{\frac{k_1}{k_2}} = \sqrt{\frac{k/2}{2k/3}} = \frac{\sqrt{3}}{2}$

31. (D)

In equilibrium, if x_0 is the stretch in the spring, then $kx_0 = m_2 g$



When the system is displaced by *x* from equilibrium position, we have

$m_1 a = T - k(x_0 + x)$ and $m_2 a = m_2 g - T$

$\Rightarrow\ (m_1 + m_2)a = m_2 g - kx_0 - kx = -kx$

$$\therefore \quad \omega = \sqrt{\frac{k}{m_1 + m_2}}$$

32. (A)

Let the maximum downward displacement of the block be x.
The additional extension in the spring will be $2x$.
Applying energy conservation, we have

$$\Delta K + \Delta U_{\text{spring}} + \Delta U_{\text{gravity}} = 0$$

$$\Rightarrow \quad 0 + \frac{1}{2}k\left[\left(2x + x_0\right)^2 - x_0^2\right] - Mgx = 0$$

$$\Rightarrow \quad k\left(4x^2 + 4x_0\right) = 2Mgx$$

$$\therefore \quad x = \frac{Mg}{2k} - x_0$$

Note that the amplitude of oscillation will be $x/2$.

33. (A)

$T = 2\pi\sqrt{\frac{I}{mg\,d}}$ Here, $m = M + M = 2M$,

$d = \frac{3L}{2}$ and $I = \frac{ML^2}{3} + ML^2 = \frac{4}{3}ML^2$

$$\therefore \ T = 2\pi\sqrt{\frac{4ML^2/3}{2Mg3L/4}} = 2\pi\sqrt{\frac{8L}{9g}}$$

34. (B)

$$T = 2\pi\sqrt{\frac{I}{Mg\,d}}$$

Here, $d = \frac{R}{2}$ and $I = \frac{MR^2}{2} + M\left(\frac{R}{2}\right)^2 = \frac{3MR^2}{4}$

$$\therefore \quad T = 2\pi\sqrt{\frac{3ML^2/4}{MgR/2}} = 2\pi\sqrt{\frac{3R}{2g}}$$

35. (C)

$T = 2\pi\sqrt{\frac{\mu}{k}}$ where, $\mu = \frac{mm}{m+m} = \frac{m}{2}$

$$\therefore \ T = 2\pi\sqrt{\frac{m}{2k}}$$

36. (6.28)

37. (2.4)

38. (59.26)

$$T = 2\pi\sqrt{\frac{m}{k_{eff.}}} = 2\pi\sqrt{\frac{m}{2k}}$$

or $T^2 = \frac{4\pi^2 m}{2k}$ or $k = \frac{2\pi^2 m}{T^2}$

$$\therefore \quad k = \frac{2\times\left(\frac{22}{7}\right)^2\times 12}{4} = 6\times\left(\frac{22}{7}\right)^2 = 59.26\ \text{Nm}^{-1}$$

39. (50)

In SHM,

Total energy, $E = \frac{1}{2}m\omega^1 A^2$

Kinetic energy, $K = \frac{1}{2}m\omega^2\left(A^2 - x^2\right)$

Where x is the distance from the mean position.

At $x = 0.707\ A$

$$K = \frac{1}{2}m\omega^2\left[A^2 - (0.707A)^2\right] = \frac{1}{2}m\omega^2\left(0.5A^2\right)$$

As per question, $E = 100\ \text{J}$

$$\therefore \quad K = 0.5\left(\frac{1}{2}m\omega^2 A^2\right) = 0.5\times 100\ \text{J} = 50\ \text{J}$$

40. (20)

Here, $m = 4$ kg, $k = 800\ \text{Nm}^{-1}$; $E = 4$ J

In SHM, total energy is $E = \frac{1}{2}kA^2$, where A is the amplitude of oscillation.

$$\therefore \quad 4 = \frac{1}{2}\times 800\times A^2$$

or $A = \frac{1}{10}\text{m} = 0.1\text{m}$

Maximum acceleration,

$$a_{max.} = \omega^2 A = \frac{k}{m}A \qquad \left(\because \omega = \sqrt{\frac{k}{m}}\right)$$

$$= \frac{800\ \text{Nm}^{-1}}{4\ \text{kg}}\times 0.1\text{m} = 20\ \text{ms}^{-2}.$$

41. (2)

$$T = 2\pi\sqrt{\frac{L}{g}}$$

i.e., time period of a simple pendulum depends upon effective length and acceleration due to gravity, not no mass.

So, $T = 2\,\text{sec}$.

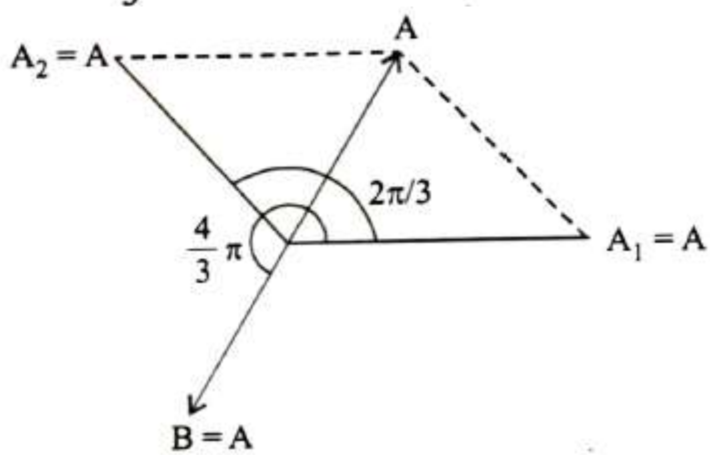
42. (4)

$$x = 4(\cos \pi t + \sin \pi t)$$
$$= 4\sqrt{2}\left(\frac{1}{\sqrt{2}}\cos \pi t + \frac{1}{\sqrt{2}}\sin \pi t\right)$$
$$= 4\sqrt{2}\left[\sin\frac{\pi}{4}\cos \pi t + \cos\frac{\pi}{4}\sin \pi t\right]$$
$$= 4\sqrt{2}\left[\sin \pi t\cos\frac{\pi}{4} + \cos \pi t\sin\frac{\pi}{4}\right]$$
$$= 4\sqrt{2}\sin\left(\pi t + \frac{\pi}{4}\right)$$

Hence, the amplitude of particle is $4\sqrt{2}$.

1. (B)

(b) Two sinusoidal displacements $x_1(t) = A \sin \omega t$ and $x_2(t) = A \sin\left(\omega t + \frac{2}{3}\pi\right)$ have amplitude A each, with a phase difference of $2\frac{\pi}{3}$. It is given that sinusoidal displacement $x_3(t) = B(\sin \omega t + \phi)$ brings the mass to a complete rest. This is possible when the amplitude of third $B = A$ and is having a phase difference of $\phi = 4\frac{\pi}{3}$ with respect to $x_1(t)$ as shown in the figure.



2. (A)

(a) $\omega_1 = \sqrt{\frac{g}{1}} = \omega_0$ (let)

$$\omega_2 = \sqrt{\frac{g}{4}} = \frac{\omega_0}{2}$$

$$|\omega_{rel}| = |\omega_1 - \omega_2| = \frac{\omega_0}{2}$$

So, to come in same phase $\theta_{rel} = 2\pi$

So, $t = \frac{\theta_{rel}}{\omega_{rel}} = \frac{2\pi}{\frac{\omega_0}{2}} = 2T$

So, no. of oscillation = 2

3. (D)

(d) In linear S.H.M., the restoring force acting on particle should always be proportional to the displacement of the particle and directed towards the equilibrium position.

i.e., $F \propto x$

or $F = -bx$ where b is a positive constant.

4. (B)

(b) $T^2 = \left(\frac{4\pi^2}{g}\right)l$ $\therefore$ Slope $= m = \frac{4\pi^2}{g}$

From graph, slope is $m = \frac{6}{1.5} = 4$

$\therefore 4 = \frac{4\pi^2}{g}$

$g = \pi^2 = 9.87 \text{ m/s}^2$

5. (C)

(c) At $t = 0$, $x(t) = 0$; $y(t) = 0$

$x(t)$ is a sinusoidal function

At $t = \frac{\pi}{2\omega}$; $x(t) = a$ and $y(t) = 0$

Hence trajectory of particle will look like as (c).

6. (D)

(d) For particle at $x = A$, equation of SHM is given as

$x_1 = A\cos\omega t$

For particle at $x = -\frac{A}{2}$, equation of SHM is given as

$x_2 = A\sin\left(\omega t - \frac{\pi}{6}\right)$ $\left[\because \frac{-A}{2} = A\sin(\omega.0 - \phi) \Rightarrow -\frac{1}{2} = \sin\phi \Rightarrow \phi = \frac{-\pi}{6}\right]$

When they will meet

$x_1 = x_2$

$\Rightarrow A\cos\omega t = A\sin\left(\omega t - \frac{\pi}{6}\right)$

$\Rightarrow \cos\omega t = \sin\omega t\sin\frac{\pi}{6} - \cos\omega t\cos\frac{\pi}{6}$

Solving further, we get

$\tan\omega t = \sqrt{3}$

$\Rightarrow \omega t = \frac{\pi}{6} \Rightarrow t = \frac{\pi}{6\omega}$

$\Rightarrow t = \frac{\pi \times T}{6 \times 2\pi} \Rightarrow t = \frac{T}{12}$

7. (B)

(b) Washer contact with piston is lost when, N = 0

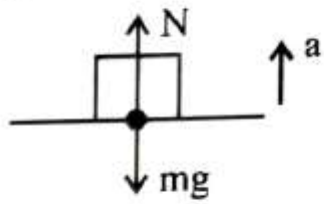
$N - mg = ma$

when $N = 0$

$|a_{max}| = g$

$a_{max} = g = \omega^2 A$

The frequency of piston



$$f = \frac{\omega}{2\pi} = \sqrt{\frac{g}{A}}\frac{1}{2\pi} = \sqrt{\frac{1000}{7}}\frac{1}{2\pi} = 1.9 \text{ Hz.}$$

8. (B)

(b) Maximum velocity in SHM, $v_{max} = a\omega$

Maximum acceleration in SHM, $A_{max} = a\omega^2$

where a and ω are maximum amplitude and angular frequency.

Given that, $\frac{A_{max}}{v_{max}} = 10$

i.e., $\omega = 10\ s^{-1}$

Displacement is given by

$x = a \sin(\omega t + \pi/4)$

at $t = 0$, $x = 5$

$5 = a \sin \pi/4$

$5 = a \sin 45° \Rightarrow a = 5\sqrt{2}$

Maximum acceleration $A_{max} = a\omega^2 = 500\sqrt{2}\ m/s^2$

9. (D)

(d) Since system dissipates its energy gradually, and hence amplitude will also decreases with time according to

$a = a_0 e^{-bt/m}$ .... (i)

∵ Energy of vibration drop to half of its initial value (E_0), as $E \propto a^2 \Rightarrow a \propto \sqrt{E}$

$$a = \frac{a_0}{\sqrt{2}} \Rightarrow \frac{bt}{2m} = \frac{10^{-2}t}{2 \times 0.1} = \frac{t}{20}$$

From eq^n (i),

$$\frac{a_0}{\sqrt{2}} = a_0 e^{-t/20}$$

$$\frac{1}{\sqrt{2}} = e^{-t/20} \text{ or } \sqrt{2} = e^{t/20}$$

$\ln\sqrt{2} = \frac{t}{20}$ $\therefore$ $t = 6.93$ sec

10. (B)

(b) As we know, Time-period of simple pendulum,

$T \propto \sqrt{l}$

differentiating both side, $\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l}$

$\because$ change in length $\Delta l = r_1 - r_2$

$5 \times 10^{-4} = \frac{1}{2}\frac{r_1 - r_2}{1} \Rightarrow r_1 - r_2 = 10 \times 10^{-4}$

10^{-3} m $= 10^{-1}$ cm $= 0.1$ cm

11. (C)

From the two mutually perpendicular S.H.M.'s, the general equation of Lissajous figure,

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} - \frac{2xy}{AB}\cos\delta = \sin^2\delta$$

$x = A\sin(at + \delta)$

$y = B\sin(bt + r)$

Clearly $A \neq B$ hence ellipse.

12. (D)

(d) Using $y = A\sin\omega t$

$a = A\sin\omega t_0$

$b = A\sin 2\omega t_0$

$c = A\sin 3\omega t_0$

$a + c = A[\sin\omega t_0 + A\sin 3\omega t_0] = 2A\sin 2\omega t_0\cos\omega t_0$

$$\frac{a+c}{b} = 2\cos\omega t_0$$

$$\Rightarrow \omega = \frac{1}{t_0}\cos^{-1}\left(\frac{a+c}{2b}\right) \Rightarrow f = \frac{1}{2\pi t_0}\cos^{-1}\left(\frac{a+c}{2b}\right)$$

13. (D)

(d) Kinetic energy, $k = \frac{1}{2}m\omega^2A^2\cos^2\omega t$

Potential energy, $U = \frac{1}{2}m\omega^2A^2\sin^2\omega t$

$$\frac{k}{U} = \cot^2\omega t = \cot^2\frac{\pi}{90}(210) = \frac{1}{3}$$

14. (C)

(c) $T = 2\pi\sqrt{\frac{l}{g}}$

When immersed non viscous liquid

$$\omega_{eff} = \rho v g - \rho_l v g = vg(\rho - \rho_l) = vg\left(\rho - \frac{\rho}{16}\right) = \frac{15}{16}\rho v g$$

$$g_{eff} = \frac{15g}{16}$$

Now $T' = 2\pi\sqrt{\frac{l}{g_{eff}}} = 2\pi\sqrt{\frac{l}{\sqrt{\frac{15g}{16}}}} = \frac{4}{\sqrt{15}}T$

15. (A)

(a) Angular frequency of pendulum $\omega = \sqrt{\frac{g}{\ell}}$

$\therefore$ relative change in angular frequency

$\frac{\Delta\omega}{\omega} = \frac{1}{2}\frac{\Delta g}{g}$ [as length remains constant]

Now, $g_{max} = g - \omega_s^2 A$

$g_{min} = g + \omega_s^2 A$

So, $\Delta g = 2A\omega_s^2$ [ω_s = angular frequency of support and, A = amplitude]

$$\frac{\Delta\omega}{\omega} = \frac{1}{2} \times \frac{2A\omega_s^2}{g}$$

$$\Rightarrow \quad \Delta\omega = \frac{1}{2} \times \frac{2 \times 1^2 \times 10^{-2}}{10}$$

$= 10^{-3}$ rad/sec.

16. (None)

(None) $B_0 + B = mg + ma$

$\therefore B = ma = \rho A x g$

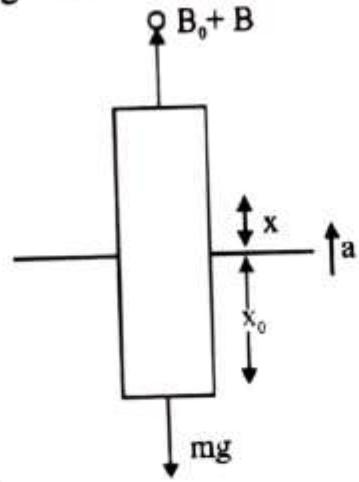
$= (\pi r^2 \rho g)x$

$$a = \frac{(\pi r^2 \rho g)x}{m}$$

using, $a = \omega^2 x$

$$\Rightarrow \omega = \sqrt{\frac{\pi r^2 \rho g}{m}}$$

$W \simeq 7.95 \text{ rads}^{-1}$



at equilibrium

$B_0 = mg$

17. (C)

(c) $f = \frac{1}{2\pi}\sqrt{\frac{K}{I}} \Rightarrow f \propto \frac{1}{\sqrt{I}}$

So, $\frac{f_1}{f_2} = \sqrt{\frac{I_2}{I_1}}$

$$\Rightarrow \frac{1}{0.8} = \sqrt{\frac{\frac{M(2l)^2}{12} + m\left(\frac{l}{2}\right)^2 \times 2}{\frac{M(2l)^2}{12}}} \Rightarrow \frac{5}{4} = \sqrt{1 + \frac{3m}{2M}}$$

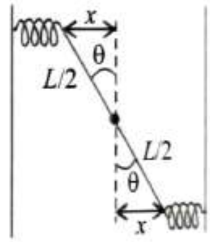
Solving, we get $\frac{m}{M} = 0.375$

18. (C)

(c) From figure, compression of the spring, $x = \frac{L}{2}\theta$.

Torque $= \left\{(k).\frac{L}{2}\theta\right\}L$

Also torque $= I\alpha = -\frac{kL^2}{2}$



$\frac{ML^2}{12}.\alpha = -\left(\frac{kL^2}{2}\right)\theta$

$\left(\because\ I = \frac{1}{2}ML^2\right)$

$\therefore\quad \alpha = -\left(\frac{6k}{M}\right)\theta = -\omega^2\theta$

$\therefore\quad f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{6k}{M}}$

19. (B)

(b) An elastic wire can be treated as a spring and its spring constant.

$k = \frac{YA}{L}$ $\left[\because Y = \frac{F}{A}\Big/\frac{\Delta l}{l_0}\right]$

Frequency of oscillation,

$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{YA}{mL}}$

20. (C)

(c) $y = y_0 \sin^2 \omega t$

$\Rightarrow y = \frac{y_0}{2}(1 - \cos 2\omega t) \quad \left(\because \sin^2 \omega t = \frac{1 - \cos 2\omega t}{2}\right)$

$\Rightarrow y - \frac{y_0}{2} = \frac{-y_0}{2} \cos 2\omega t$. So from this equation we can say mean position is shifted by $\frac{y_0}{2}$ distance and frequency of this SHM is 2ω.

So, at equilibrium $\frac{ky_0}{2} = mg \Rightarrow \frac{k}{m} = \frac{2g}{y_0}$

Also, spring constant $k = m(2\omega)^2$

$\Rightarrow 2\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2g}{y_0}} \Rightarrow \omega = \frac{1}{2}\sqrt{\frac{2g}{y_0}} = \sqrt{\frac{g}{2y_0}}$

21. (D)

(d) As $P_i = P_f$

So, $mV_0 = \frac{m}{2}V_0' \Rightarrow V_0 = \frac{V_0'}{2} \Rightarrow V_0' = 2V_0$

$\Rightarrow \quad \omega' A' = 2\omega A \Rightarrow \left(\frac{\omega'}{\omega}\right)\left(\frac{A'}{A}\right) = 2$

$\Rightarrow \quad \sqrt{\frac{m}{m'}} f = 2 \quad \left[\because \omega \propto \frac{1}{\sqrt{m}}\right]$

$\Rightarrow \quad \sqrt{2} f = 2 \Rightarrow f = \sqrt{2}$

22. (A)

(a) Restoration force, $F = m\omega^2 A$

$\therefore \frac{F}{m} = \omega^2 A = \left(\frac{2\pi}{T}\right)^2 A$

Point A covers $30° = \frac{\pi}{6}$ in 0.1 s

$\therefore$ 2π covered in $\frac{2\pi \times 0.1}{\pi/6} = 1.2$ s or $T = 1.2$ s

$\therefore$ Restoration force per unit mass,

$\frac{F}{m} = \frac{4\pi^2}{(1.2)^2} \times 0.36 = \pi^2 = (3.14)^2 \simeq 9.87$ N

23. (A)

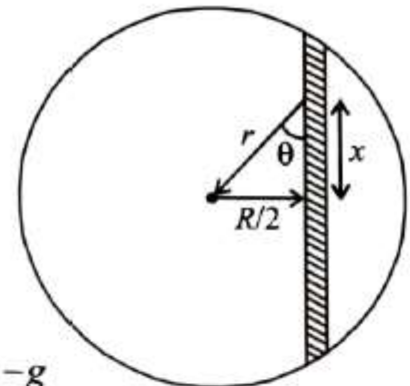
(a) P.E. $=\frac{1}{2}m\omega^2x^2=\frac{1}{2}m\left(\frac{2\pi}{T}\right)^2\times A^2\sin^2\left(\frac{2\pi}{T}\right)t$

Putting value of m, A and T, we get P.E. = 0.62 J

24. (C)

(c)

Force along the tunnel



$$F=\left(\frac{GMmr}{R^3}\right)\cos\theta$$

$$\Rightarrow F=\frac{GMr}{R^3}\times m\times\left(\frac{x}{r}\right)$$

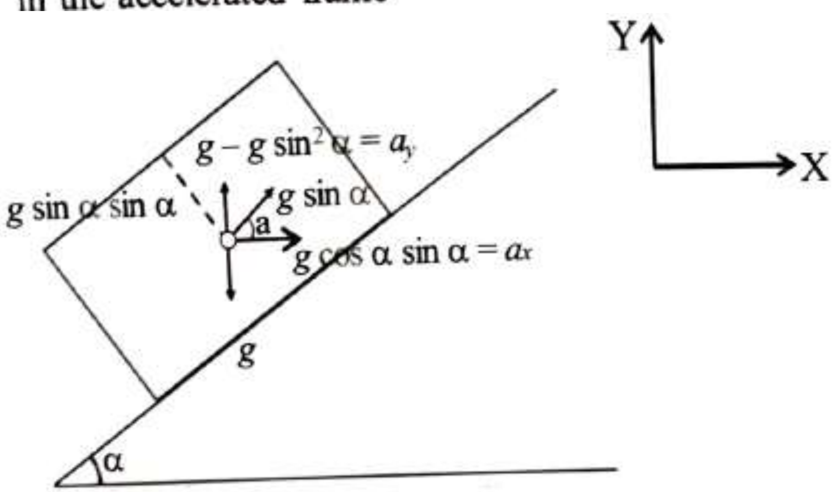
$$\Rightarrow a=\frac{d^2x}{dt^2}=-\frac{GM}{R^3}x=\frac{-g}{R}x$$

Comparing with $a=-\omega^2x$, we get $\omega^2=\frac{g}{R}$

$$\therefore\quad T=2\pi\sqrt{\frac{R^3}{GM}}=2\pi\sqrt{\frac{R}{g}}\qquad\left[\because g=\frac{GM}{R^2}\right]$$

25. (A)

(a) As shown in the figure, acceleration down the plane $a=g\sin\alpha$ is the pseudo acceleration applied by the observer in the accelerated frame



$a_x = g \sin \alpha \cos \alpha = a \cos \alpha$

$a_y = g - g \sin^2 \alpha = g(1 - \sin^2 \alpha) = g \cos^2 \alpha$

The effective acceleration due to gravity acting on the bob

$$g_{eff} = \sqrt{a_x^2 + a_y^2}$$

$$= \sqrt{g^2 \sin^2 \alpha \cos^2 \alpha + g^2 \cos^4 \alpha}$$

$$= g \cos\alpha \sqrt{\sin^2 \alpha + \cos^2 \alpha} = g \cos \alpha$$

$$\therefore \quad T = 2\pi \sqrt{\frac{L}{g_{eff}}} = 2\pi \sqrt{\frac{L}{g \cos \alpha}}$$

26. (A)

(a) $x = 4 \sin \left(\frac{\pi}{2} - \omega t\right) = 4 \cos \omega t$

$Y = 4 \sin \omega t$

So, $x^2 + y^2 = 4^2$, which is equation of circle.

27. (B)

(b) At $t = 1$ sec

$$x = \sin \pi \left(1 + \frac{1}{3}\right) = \sin \frac{4\pi}{3} = -\frac{\sqrt{3}}{2}$$

So, $V = \omega\sqrt{A^2 - x^2} = \pi\sqrt{1 - \frac{3}{4}} = \frac{\pi}{2} m/s = 157$ cm/s

28. (C)

(c) $T \propto \frac{1}{\sqrt{g}}$ and $g \propto \frac{1}{(R + h)^2}$

$$\frac{T_1}{T_2} = \sqrt{\frac{g'}{g}} = \sqrt{\frac{R^2}{(R + h)^2}}$$

$$\frac{T_1}{T_2} = \frac{4}{6} = \frac{R}{(R + h)} \Rightarrow h = 3200 \text{ km}$$

29. (D)

(d) Time period of simple pendulum is given as, T = $2\pi\sqrt{\frac{L}{g}}$,

$$g' = \frac{GM}{9R^2} = \frac{g}{9} = \frac{\pi^2}{9} \quad \left[\because g' = \frac{GM}{(R+h)^2}\right]$$

and $2 = 2\pi\sqrt{\frac{L}{\pi^2} \times 9}$

[$\because$ Time period of second pendulum = 2 sec]

$$\Rightarrow 1 = \pi\sqrt{L} \times \frac{3}{\pi} \Rightarrow L = \frac{1}{9}\text{ m}$$

30. (C)

(b) In SHM, $V_{max} = \omega A$

ωA = constant

$$\frac{A_1}{A_2} = \frac{\omega_2}{\omega_1} = \sqrt{\frac{k_2}{m_2} \times \frac{m_1}{k_1}} = \sqrt{\frac{9k}{100} \times \frac{50}{2k}} = \frac{3}{2}$$

31. (B)

(b) In SHM, $V_{max} = \omega A$

ωA = constant

$$\frac{A_1}{A_2} = \frac{\omega_2}{\omega_1} = \sqrt{\frac{k_2}{m_2} \times \frac{m_1}{k_1}} = \sqrt{\frac{9k}{100} \times \frac{50}{2k}} = \frac{3}{2}$$

32. (2)

(2) Given:

$x(t) = A\sin(\omega t + \phi)$

$v_{(t)} = \frac{dx}{dt} \qquad \therefore \quad v(t) = A\omega\cos(\omega t + \phi)$

$2 = A\sin\phi \qquad$ ... (i)

$2\omega = A\omega\cos\phi \qquad$ ... (ii)

From eq (i) and (ii)

$\tan\phi = 1$

$\phi = 45°$

Putting value of ϕ in equation (i)

$$2 = A\left\{\frac{1}{\sqrt{2}}\right\} \Rightarrow A = 2\sqrt{2}$$

$\therefore$ Value of x = 2

33. (8)

(8) K.E. = P.E.

$$\Rightarrow \frac{1}{2}m(V_0^2 - \omega^2 x^2) = \frac{1}{2}m\omega^2 x^2$$

$$\Rightarrow V_0^2 = 2\omega^2 x^2 \Rightarrow \omega^2 A^2 = 2\omega^2 x^2 \Rightarrow x = \frac{A}{\sqrt{2}}$$

$$\Rightarrow A\sin\omega t = \frac{A}{\sqrt{2}} \Rightarrow \omega t = \frac{\pi}{4} \Rightarrow \frac{2\pi}{T}t = \frac{\pi}{4} \Rightarrow t = \frac{T}{8}$$

So, $x = 8$

34. (7)

(7) $\frac{5}{8}$oscillation $= \frac{1}{2}$oscillation $+ \frac{1}{8}$oscillation

Displacement in $\frac{1}{2}$ oscillation = $4A \times \frac{1}{2} = 2A$

Displacement in $\frac{1}{8}$ oscillation = $4A \times \frac{1}{8} = \frac{A}{2}$

Time for $\frac{1}{2}$ oscillation = $\frac{T}{2}$

Time for $\frac{1}{8}$ oscillation (or $\frac{A}{2}$ displacement)

$$\frac{A}{2} = A\sin\omega t$$

$$\Rightarrow \frac{1}{2} = \sin\omega t \Rightarrow \frac{\pi}{6} = \left(\frac{2\pi}{T}\right)t \Rightarrow t = \frac{T}{12}$$

$$\therefore T_{net} = \frac{T}{2} + \frac{T}{12} = \frac{7T}{12}$$

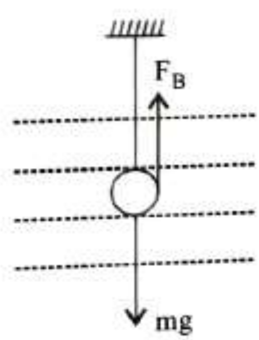
35. (5)

(5) $mg' = mg - F_B$

$$g' = \frac{mg - F_B}{m} = \frac{mg - m_w g}{m}$$

$$= \frac{\rho_B Vg - \rho_w Vg}{\rho_B V} \quad (\because m = \rho v)$$

$$= \left(\frac{\rho_B - \rho_w}{\rho_B}\right)g$$



$$= \frac{5-1}{5} \times g = \frac{4}{5} g$$

$$\frac{T'}{T} = \sqrt{\frac{g}{g'}} = \sqrt{\frac{\frac{g}{4}}{\frac{4}{5}g}} = \sqrt{\frac{5}{4}} \quad T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\Rightarrow \quad T' = T\sqrt{\frac{5}{4}} = \frac{10}{2}\sqrt{5} = 5\sqrt{5}$$

Compair with $5\sqrt{x}$, we have $x = 5$

36. (2)

(2) $U = 4(1 - \cos 4x)$

$$F = -\frac{dU}{dx} = -4(+\sin 4x)\,4 = -16\sin(4x)$$

For small θ

$\sin\theta \approx \theta$

$\Rightarrow \sin 4x \approx 4x$

So, $F = -64x$

$a = -64\,x/m = -16x$

Comparing with $a = -\omega^2 x$

we get, $\omega^2 = 16$

$\Rightarrow \omega = 4$

$$\text{So, } T = \frac{2\pi}{\omega} = \frac{\pi}{2}$$

37. (2)

(2) For given figure (a):

$$k_{eq} = \frac{k \times 2k}{k + 2k} = \frac{2k}{3}$$

$$T = 2\pi\sqrt{\frac{m}{k_{eq}}} = 2\pi\sqrt{\frac{m}{2k/3}} = 2\pi\sqrt{\frac{3m}{2k}}$$

For given figure (b):

$$k_{eq} = k + 2k = 3k,\ T' = 2\pi\sqrt{\frac{m}{3K}}$$

$$\frac{T'}{T} = \sqrt{\frac{m \times 2k}{3k \times 3m}} = \frac{\sqrt{2}}{3}\ (\because T = 3s)$$

$$T' = \sqrt{2}s$$

Compare with $\sqrt{x}$, we have

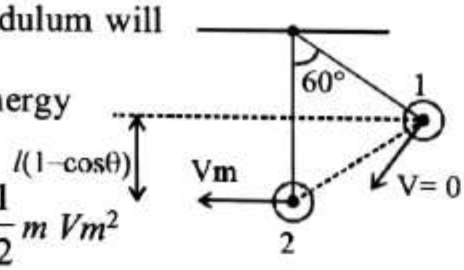
$\Rightarrow x = 2$

38. (5)

(5) At mean position, pendulum will have maximum velocity

So, By conservation of energy

$P_i + K_1 = P_2 + K_2$



$$mgl\,(1 - \cos\theta) + 0 = 0 + \frac{1}{2} m\, Vm^2$$

$$V_m = \sqrt{2gl(1 - \cos\theta)}$$

$$= \sqrt{2 \times 10 \times 2.5 \times (1 - \cos 60°)} = \sqrt{25} = 5 \text{ m/s.}$$

39. (10)

(10) To complete the entire vertical circle, the minimum speed of bob should be $\sqrt{5Rg}$

By law of conservation of momentum $\vec{P_i} = \vec{P_f}$

$$75V + 0 = 50\sqrt{5Rg} + 75 V/3$$

$$\Rightarrow 75 \times \frac{2V}{3} = 50\sqrt{5Rg}$$

$$\Rightarrow 50V = 50\sqrt{5Rg}$$

$$\Rightarrow V = \sqrt{5Rg} = \sqrt{5 \times 2 \times 10} = 10 \text{ m/s}$$

40. (700)

(700) Initially, at $x = 5$ am, Let $v = v_0$

by applying law of conservation of mechanical energy

$$M.E_{x=5m} = ME_{x=10cm}$$

$$\frac{1}{2}k(5)^2 + \frac{1}{2}mv_0^2 = \frac{1}{2}k(10)^2$$

$$Mv_0^2 = 75k \Rightarrow v_0 = \sqrt{\frac{75k}{m}}$$

Finally, at $x = 5$ cm. We have $v = 3v_0$

So amplitude will increase but mechanical energy is still conserved.

So, $\frac{1}{2}k(5)^2 + \frac{1}{2}m\left(\sqrt{\frac{75k}{m}}\right)^2 = \frac{1}{2}kA^2$

$$\Rightarrow 25k + 675k = kA^2 \Rightarrow 700 = A^2 \quad \therefore A = \sqrt{700} \text{ cm}$$

41. (16)

(16) $\frac{1}{2}kA^2 = \frac{1}{2}mv_{max}^2$

$$\Rightarrow \frac{1}{2}kA^2 = \frac{(mv_{max})^2}{2m} \Rightarrow \frac{1}{2}kA^2 = \frac{p^2}{2m}$$

$$\Rightarrow \left(\frac{A_1}{A_2}\right)^2 = \frac{m_2}{m_1} = \frac{1024}{900}$$

$$\Rightarrow \frac{A_1}{A_2} = \frac{32}{30} = \frac{16}{15} = \frac{16}{16-1} \quad \therefore \alpha = 16$$

42. (1)

(1) We have

$A = 8$ cm, $T = 6$ sec

Let us suppose at $t = 0$, particle is at maximum amplitude

and at $t = t, x = \frac{A}{2}$

So, $\frac{A}{2} = \text{A} \cos \omega t \Rightarrow \cos \omega t = \frac{1}{2}$

So, $\omega t = \frac{\pi}{3}$

$\frac{2\pi}{T} \times t = \frac{\pi}{3}$

$t = \frac{T}{6} = \frac{6}{6} = 1$ sec

Note: We have use $x = A \cos \omega t$ as equation of SHM because at $t = 0$, particle is at $x = A$.