IIT – JEE: 2025 **TW TEST (ADV)** **DATE: 23/03/24**

TOPIC: FLUID MECHANICS

Answer Key & Solution

1. (C)
   Let h = height to of water column then
   $\rho_w gh + \rho_{Hg} g(10-h) = \rho_{Cu} g\,10$
   $\Rightarrow\ h + 13.6(10-h) = 73$
   $\Rightarrow\ 63 = 12.6\,h\ \Rightarrow\ h = 5\,cm$

2. (A)
   $\rho = \frac{\text{Total mass}}{\text{Total volume}} = \frac{m_1 + m_2}{2V} = \frac{V(\rho_1 + \rho_2)}{2V} = \frac{\rho_1 + \rho_2}{2}$

3. (D)
   Force = mg, same in all vessels

4. (B)
   $v = \sqrt{2gh} \times \sqrt{2 \times 10 \times 20} = 20\,m/s$

5. (C)
   Reading will be 40 + 4 = 44 N

6. (A,B,C)
   By Archimedes principle

7. (A,C)
   Variation of pressure in horizontal direction also.

8. (A,C)
   By buoyancy force

9. (D)
   $F_b = v\rho_{liq} g$
   'g' is different on moon and on the earth.
   Hence only (iii) is a correct statement.

10. (D)
based on Bernoulli principle

11. (AB)
The maximum horizontal distance from the vessel comes from hole number 3 and 4
$v=\sqrt{2gh} \rightarrow h$ is height of hole from top.
Horizontal distance $x = vt = \sqrt{2gh}\sqrt{\frac{2(H-h)}{g}}$ $x = 2\sqrt{h(H-h)}$

12. (C)
conservation of energy

13. (C,D)
by buoyant force and Archimedes principle

14. (C,D)

15. (B)
air is pumped out. So pressure decreases

16. (12)
$a_1v_1 = a_2v_2 \Rightarrow \frac{v_2}{v_1} = \frac{a_1}{a_2} = \left(\frac{r_1}{r_2}\right)^2$
$\Rightarrow v_2 = 3\times(2)^2 = 12\ m/s$

17. (2)
The required to emptied the tank $t = \frac{A}{A_0}\sqrt{\frac{2H}{g}}$
$\therefore \frac{t_2}{t_1} = \sqrt{\frac{H_2}{H_1}} = \sqrt{\frac{4h}{h}} = 2$
$\therefore t_2 = 2t$

18. (1)
If the liquid is incompressible then mass of liquid entering through left end, should be equal to mass of liquid coming out from the right end.
$\therefore M = m_1 + m_2 \Rightarrow Av_1 = Av_2 + 1.5\ Av$
$\Rightarrow A\times3 = A\times1.5 + 1.5Av \Rightarrow v = 1\ m/s$

19. (3)
Upthrrust = weight of body
For A, $\frac{V_A}{2}\times\rho_w\times g = V_A\times\rho_A\times g \Rightarrow \rho_A = \frac{\rho_w}{2}$

For B, $\frac{3}{4}V_B \times \rho_w \times g = V_B \times \rho_B \times g \Rightarrow \rho_B = \frac{3}{4}\rho_w$

(Since 1/4 of volume of B is above the water surface)

$$\therefore \frac{\rho_A}{\rho_B} = \frac{\rho_w/2}{3/4\,\rho_w} = \frac{2}{3}$$

20. (2)

$$\rho = \frac{\text{Total mass}}{\text{Total volume}} = \frac{2m}{V_1 + V_2} = \frac{2m}{m\left(\frac{1}{\rho_1} + \frac{1}{\rho_2}\right)}$$

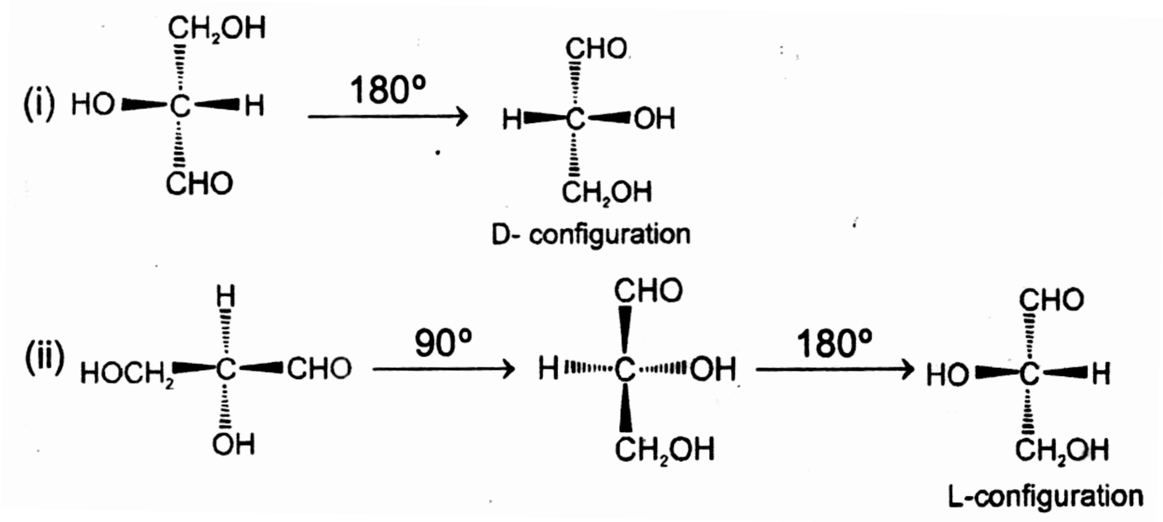
$$\therefore \rho = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}$$

SOLUTION

21. **(C)**
Both are enantiomer of each other. So optical rotation will be -12°

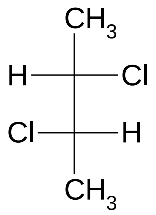
22. **(C)**

23. **(B)**



24. **(A)**
The configuration in a compound is independent of its physical properties(optical activity)

25. **(A)**



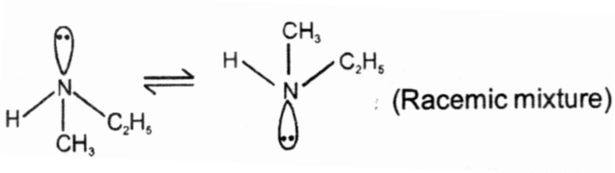
26. **(AB)**

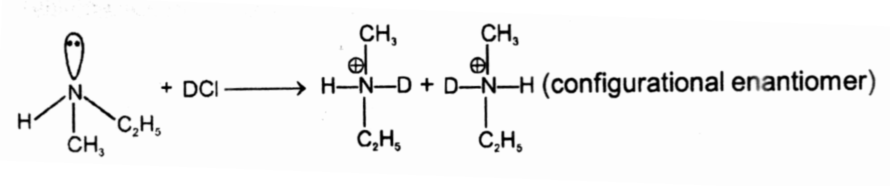
27. **(AD)**
For being functional isomers, functional group should not match.
(Phenol and aliphatic alcohol are considered as different functional groups)

28. **(ABC)**
I is trans isomer and II is cis isomers I and III are position isomers similarly II and III are position

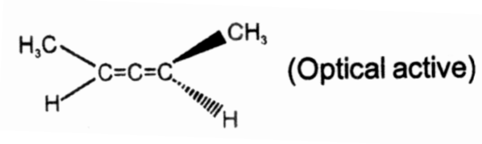
isomers.

29. **(ABCD)**

(A) 

(B) 

(C) Two gauche form of butane are chiral and have equal energy.

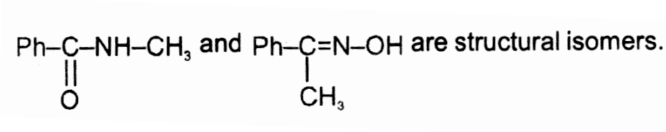
(D) 

30. **(ABD)**
Ortho substituted biphenyls are optically active (c) and exists as enantiomers.

31. **(BCD)**
It is a meso achiral compound and it is no centre of symmetry and no axis of symmetry but plane of symmetry.

32. **(BD)**

33. **(ACD)**

(a) 

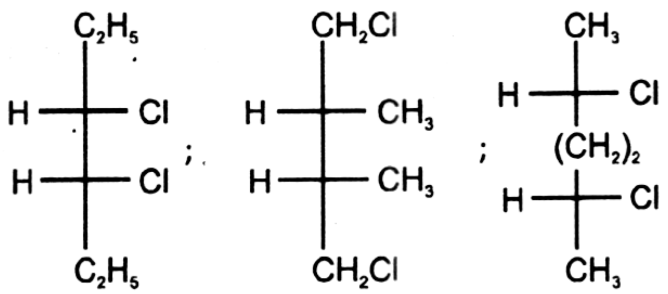
(b) These compounds are identicle (c) These are chain isomers

34. **(CD)**

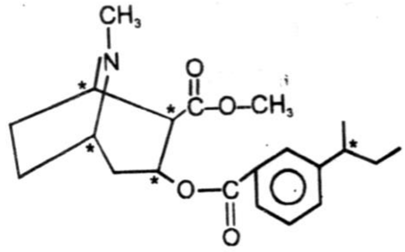
35. **(ABC)**
A meso compound has minimum two chiral centres and it has a plane of symmetry and it is inactive.

36. **(5)**

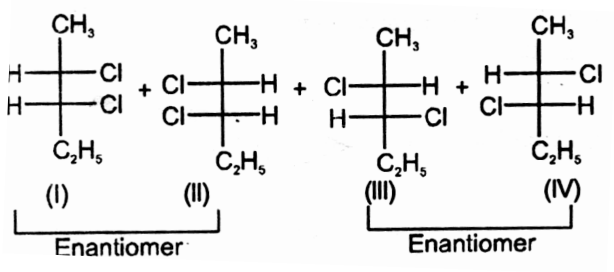
37. **(3)**



38. **(5)**

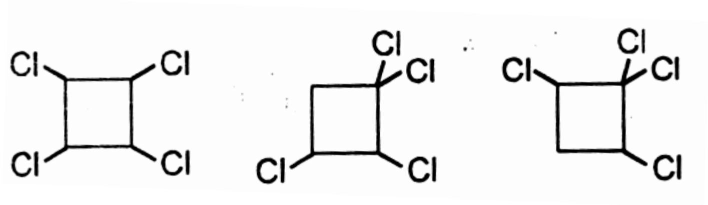


39. **(6)**



$\xrightarrow[\text{Distillation}]{\text{Fractional}} \text{Fraction}_1 (\text{I} + \text{II}) + \text{fraction}_2 (\text{III} + \text{IV})$

40. **(3)**



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TOPIC: METHOD OF DIFFERENTIAION

SOLUTIONS

41. (B)

We have $y=\cos^{-1}\cos x$ at $x=5\pi/4$ $y=\begin{cases} x & ,x\in(0,\pi) \\ \pi-\cos^{-1}\cos x & ,x\in\left(\pi,\dfrac{3\pi}{2}\right)\end{cases}$

$\Rightarrow y=\pi-\cos^{-1}\cos x$
$\Rightarrow y=\pi-x$
$\therefore \dfrac{dy}{dx}=-1$

42. (B)

We have $\sin(xy)+\cos(xy)=0$
$\Rightarrow \sin(xy)=-\cos(xy)$
$\Rightarrow \tan(xy)=-1$
$\Rightarrow xy=\tan^{-1}(-1)$
On differentiating w.r.t. x we get
$x\dfrac{dy}{dx}+y=0$
$\therefore \dfrac{dy}{dx}=\dfrac{-y}{x}$

43. (C)

We have $y=\sin^{-1}\left(\dfrac{x^2-1}{x^2+1}\right)+\sec^{-1}\left(\dfrac{x^2+1}{x^2-1}\right)$

$y=\cos ec^{-1}\left(\dfrac{x^2+1}{x^2-1}\right)+\sec^{-1}\left(\dfrac{x^2+1}{x^2-1}\right)$

$y=\dfrac{\pi}{2}$ $[\because \cos ec^{-1}x\sec^{-1}x=\pi/2]$

$\therefore \dfrac{dy}{dx}=0$

44. (B)

We have,
$y=x-x^2$
$\Rightarrow \dfrac{dy}{dx}=1-2x$

Let $u = y^2$ *and* $v = x^2$

$\frac{du}{dx} = 2y\frac{dy}{dx}$

And $\frac{dy}{dx} = 2x$

$\therefore \frac{du}{dv} = \frac{2ydy/dx}{2x} = \frac{(x-x^2)(1-2x)}{x}$ [From Equs. (i) and (iii)]

$= (1-x)(1-2x) = 2x^2 - 3x + 1$

45. (A)

We have $ax^2 + 2hxy + by^2 = 0$

On differentiating w.r.t. x we get

$2ax + 2h\left(x\frac{dy}{dx} + y\right) + 2by\frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-(ax+hy)}{(hx+by)}$

$$\left[\begin{array}{l} \because ax^2 + 2hxy + by^2 = 0 \\ ax^2 + hxy = -(by^2 + hxy) \\ \Rightarrow -\left(\frac{ax+hy}{by+hx}\right) = \frac{y}{x} \end{array}\right]$$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x}$

46. (A)

47. (AC)

48. (A, B, C)

We have $y + \log_e(1+x) = 0$

$\Rightarrow y = -\log_e(1+x) \Rightarrow y = \log_e(1+x)^{-1}$

$\Rightarrow e^y = \frac{1}{1+x}$

$\Rightarrow xe^y + e^y = 1$

On differentiating w.r.t. x we get

$xe^y y' + e^y + e^y y' = 0$

$xy' + y' + 1 = 0$ …(ii)

$\Rightarrow y' = -\frac{1}{x+1} \Rightarrow y' = -e^y \quad \left[\because e^y = \frac{1}{1+x}\right]$

$\Rightarrow y' + e^y = 0$

From Eq. (ii)

$xy' + y' + 1 = 0 \Rightarrow xy' + 1 = -y'$

$\Rightarrow xy' + 1 = e^y$

49. (B, C)

We have $y^2 + b^2 = 2xy$

$\Rightarrow 2xy - y^2 = b^2$

$$2x\frac{dy}{dx}+2y-2y\frac{dy}{dx}=0\Rightarrow\frac{dy}{dx}(x-y)=-y$$

$$\Rightarrow\frac{dy}{dx}=\frac{y}{y-x}$$

On multiping by (y – x) both numerator and denominator, we get

$$\Rightarrow\frac{dy}{dx}=\frac{y(y-x)}{(y-x)^2}=\frac{y^2-xy}{(y-x)^2}=\frac{2xy-b^2-xy}{(y-x)^2}=\frac{xy-b^2}{(y-x)^2}$$

50. (A, B)

Let $u=\sin^{-1}\frac{t}{\sqrt{1+t^2}}$

And $v=\cos-1\frac{1}{\sqrt{1+t^2}}$

$$\Rightarrow u=\begin{cases}\tan^{-1}t, & t\geq 0\\ -\tan^{-1}t & t<0\end{cases}$$

And $v=\begin{cases}\tan^{-1}t, & t\geq 0\\ -\tan^{-1}t & t<0\end{cases}$

$$\Rightarrow\frac{du}{dt}=\begin{cases}\frac{1}{1+t^2}, & t\geq 0\\ -\frac{1}{1+t}, & t<0\end{cases}$$

And $\frac{dv}{dt}=\begin{cases}\frac{1}{1+t^2}, & t\geq 0\\ \frac{1}{1+t^2}, & t>0\end{cases}$

$$\therefore\frac{du}{dv}=\begin{cases}1 & t\geq 0\\ -1 & t<0\end{cases}$$

51. (A, D)

We have $x=\cos t, y=\log_e t$

$$\frac{dx}{dt}=-\sin t,\frac{dy}{dt}=\frac{1}{t}$$

$$\frac{dy}{dx}=\frac{-1}{t\sin t}$$

$$\therefore\left(\frac{dy}{dx}\right)_{t=}=\frac{-2}{\pi}$$

And $\left(\frac{dy}{dx}\right)=\frac{-12}{\pi}$

52. (A, B, C, D)

We have $2^x+2^y=2^{x+y}$

On differentiating w.r.t. x we get

$$2^x\log 2+2^y\log 2\frac{dy}{dx}=2^{x+r}\left(1+\frac{dy}{dx}\right)\log 2$$

$$\Rightarrow 2^x + 2^y \frac{dy}{dx} = 2^{x+y}\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx}(2^x - 2^{x+y}) = 2^{x+y} - 2^x$$

$$\Rightarrow \frac{dy}{dx} = \frac{2^{x+y} - 2^x}{2^r - 2^{x+y}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x(2^x - 1)}{2^y(1-2^x)} = \frac{2^x(1-2^y)}{2^y(2^x - 1)}$$

Or $\frac{dy}{dx} = \frac{2^x + 2^y - 2^x}{2^y - 2^x - 2^y}$ $\qquad [\because 2^{x+y} = 2^x + 2^y]$

$$\frac{dy}{dx} = \frac{-2^y}{2^x} \Rightarrow \frac{dy}{dx} = \frac{2^{x+y} - 2^x}{2^y(1-2^x)}$$

$$\frac{dy}{dx} = -\frac{2^y}{2^x} \Rightarrow \frac{dy}{dx} = \frac{2^{x+y} - 2^x}{2^y(1-2^x)} \qquad [\because 2^x = 2^{x+y} - 2^y]$$

$$\Rightarrow \frac{dy}{dx} = \frac{2^x(2^y - 1)}{2^y - 2^{x+y}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2^x(2^y - 1)}{-2^x} \qquad \therefore \frac{dy}{dx} = 1 - 2^y$$

53. (A, B, D)

We have $x^p . x^q = \{x+y\}^{p+q}$

On taking log both sides, we get

$p\log x + q\log y = (p+q)\log(x+y)$

On differentiating w.r.t.x,we get

$$\frac{p}{x} + \frac{q}{y}\frac{dy}{dx} = \frac{p+q}{x+y}\left(1+\frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{q}{y} - \frac{p+q}{x+y}\right) = \frac{p+q}{x+y} - \frac{p}{x}$$

$$\therefore \frac{dy}{dx} = \frac{y}{x}$$

54. (A, C)

$y = e^{\sqrt{x}} + e^{-\sqrt{x}}$

$$\therefore \frac{dy}{dx} = e^{\sqrt{x}} \times \frac{1}{2\sqrt{x}} = \frac{e^{-\sqrt{x}}}{2\sqrt{x}}$$

$$= \frac{e^{\sqrt{x}} - e^{-\sqrt{x}}}{2\sqrt{x}} = \frac{\left[\left(e^{\sqrt{x}} - e^{-\sqrt{x}}\right)^2\right]^{1/2}}{2\sqrt{x}}$$

$$= \frac{\sqrt{\left(e^{\sqrt{x}}\right)^2 + \left(e^{-\sqrt{x}}\right)^2 - 2}}{2\sqrt{x}}$$

$$= \frac{\sqrt{\left(e^{\sqrt{x}}\right)^2 + \left(e^{-\sqrt{x}}\right)^2 + 2 - 4}}{2\sqrt{x}}$$

$$=\frac{\sqrt{\left(e^{\sqrt{x}}+e^{-\sqrt{x}}\right)^2-4}}{2\sqrt{x}}=\frac{\sqrt{y^2-4}}{2\sqrt{x}}$$

55. (A, B, C)

We have, $f(0)=\frac{2}{g(0)}\cdot f'(0)=2g'(0)=4g(0)$

$g''(0)=5f''(0)=6f(0)=3$

Now, on solving these equations, we get

$$f(0)=\frac{1}{2}\cdot g(0)=4,\ f'(0)=16,\ g'(0)=8$$

$$f''(0)=\frac{3}{5}\cdot g''(0)=3$$

(A) We have $h(x)=\frac{f(x)}{g(x)}$

$$h'(x)=\frac{g(x)f'(x)-g'(x)f'(x)}{g(x)^2}$$

$$\therefore\quad h'(0)=\frac{g(0)f'(0)-g'(0)f(0)}{g(0)^2}$$

$$=\frac{4\times16-8\times\frac{1}{2}}{(4)^2}=\frac{15}{4}$$

(B) $k(x)=f(x)\cdot g(x)\cdot\sin x$

$$k'(x)=f(x)g(x)\cos x+f(x)\sin x g'(x)+g(x)\sin x f'(x)$$

$$k'(0)=f(0)g(0)\cos\theta+f(0)\sin\theta g'(0)+g(0)\sin\theta f'(0)$$

$$k'(0)=\frac{1}{2}\times4\times1+0+0=2$$

(C) $\lim_{x\to0}\frac{g'(x)}{f'(x)}$

$$\Rightarrow\quad\frac{g'(0)}{f'(0)}=\frac{1}{2}$$

56. (5)

We have, $f(x)=2\log_e(x-2)-x^2+4x+1\qquad[x\neq2,\ x>2]$

$$\Rightarrow\quad f'(x)=\frac{2}{x-2}-2x+4$$

$$=\frac{2}{x-2}-2(x-2)$$

$$=2\left[\frac{1}{x-2}-x-2\right]$$

$$=2\left[\frac{1-\left(x^2-4x+4\right)}{x-2}\right]$$

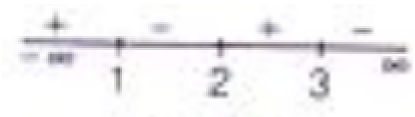
$$f'(x)=2\frac{\left(-x^2+4x-3\right)}{x-2}$$

$$f'(x)=\frac{-2(x-3)(x-1)}{x-2}$$

$\because\ f'(x)\ge 0$

$\therefore\ \frac{-2(x-3)(x-1)}{x-2}\ge 0$

$\frac{2(x-3)(x-1)}{x-2}\le 0$



$x\in(2,3]$ $\qquad [\because\ x>2]$

$\Rightarrow\quad a=2, b=3$

$\therefore\ a+b=2+3=50$

57. (1)

we have

$$f(x)=x+\cfrac{1}{2x+\cfrac{1}{2x+\cfrac{1}{2x+\cfrac{1}{2x+.....\infty}}}},$$

$\Rightarrow f(x)=x+\frac{1}{x+f(x)}$

$\Rightarrow f(x)-x=\frac{1}{x+f(x)}$

$\Rightarrow [f(x)]^2-x^2=1$

On differentiating w.r.t x we get

$2f(x).f'(x)-2x=0$

$f(x)f'(x)=x\Rightarrow \frac{f(x).f'(x)}{x}=1$

$\therefore \frac{f(100).f'(100)}{100}=1$

58. (1)

we have $y=\tan^{-1}\frac{u}{\sqrt{1-u^2}}, x=\sec^{-1}\frac{1}{2u^2-1}$

$y=\sin^{-1}u$

And $x=2\cos^{-1}u$

$\Rightarrow \frac{dy}{du}=\frac{1}{\sqrt{1-u^2}}$

And $\frac{dx}{du}=\frac{-2}{\sqrt{1-u^2}}$

$\Rightarrow \frac{dy}{dx}=-\frac{1}{2}$

$$\therefore \left|2\frac{dy}{dx}\right| = \left|2\times\frac{-1}{2}\right| = 1$$

59. (1)

We have $y = \tan^{-1}\left(\frac{x}{1+\sqrt{1-x^2}}\right) + \sin\left(2\tan^{-1}\sqrt{\frac{1-x}{1+x}}\right)$ …..(i)

On putting x = $\cos\theta$ in Eq. (i) we get

$$y = \tan^{-1}\frac{\cos\theta}{1+\sin\theta} + \sin\left[2\tan^{-1}\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right]$$

$$y = \tan^{-1}\left(\frac{\cos\theta/2 - \sin\theta/2}{1+\sin\theta+\sin\theta/2}\right) + \sin\left(2\tan^{-1}\tan\frac{\theta}{2}\right)$$

$$y = \tan^{-1}\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right) + \sin\theta$$

$$y = \frac{\pi}{4} - \frac{\theta}{2} + \sin\theta$$

$$y = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x + \sin(\cos^{-1}x)$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{1-x^2}} - \frac{\cos(\cos^{-1}x)}{\sqrt{1-x^2}}$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=0} = \frac{1}{2} - 1 = -\frac{1}{2}$$

$$\Rightarrow \left|2\frac{dy}{dx}\right| = \left|2\times\frac{-1}{2}\right| = 1$$

60. (1)

We have $xe^{xy} = y + \sin^2 x$

When x =0, then y =0

Now, on differentiating w.r.t. x we get

$$xe^{xy}\left(x\frac{dy}{dx} + y\right) + e^{xy} = \frac{dy}{dx} + 2\sin x\cos x$$

$$\Rightarrow \frac{dy}{dx}(x^2e^{xy} - 1) = 2\sin x\cos x - e^{xy} - xye^{xy}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin 2x - e^{xy}(1+xy)}{x^2e^{xy} - 1}$$

$$\therefore \left(\frac{dy}{dx}\right)_{x=0} = \frac{x - e^0}{0-1} = e^0 = 1 \qquad [\because x = 0 \text{ and } y = 0]$$