IIT – JEE: 2025 **TW TEST (MAIN)** **DATE: 22/03/24**

TOPIC: FLUID MECHANICS

SOLUTION

1. **(B)**
For a vertical accelerating elevator, variation of pressure will be only along vertical direction.
Hence, pressure will remain same at two points in the same horizontal plane.

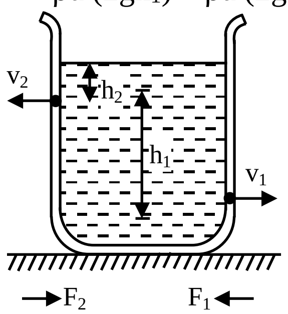
2. **(A)**
As the area of narrow cross section is less. its velocity will be greater.
Now, by using Bernoulli equation, pressure will be less where velocity is greater.
Hence, rise of liquid will be less in the narrower cross section area.

3. **(C)**
It is based on conservation of energy.

4. **(D)**
If water is flowing along horizontal, then we cannot use fluid static property of same pressure at same horizontal level. Although, pressure can still be equal at same horizontal level only if velocity will be equal and for that the cross-sectional area at A and B has to be equal.

5. **(C)**
Since, the area of cross section decreases, velocity will increase hence pressure will decrease by using Bernoulli equation.

6. **(C)**
Thrust force
$F = F_1 - F_2 = \rho a v_1^2 - \rho a v_2^2$
$= \rho a (2gh_1) - \rho a (2gh_2) = 2\rho a g (h_1 - h_2) = 2\rho a g h$



7. **(D)**
By using equation of continuity,
$A_1 v_1 = A_2 v_2$
$\pi(2R)^2 v = \pi(R)^2 v'$

$\nu' = 4\nu$

8. **(A)**
The rate of flow of water from the orifice is directly proportional to root of its speed and speed is directly proportional to height of orifice from top surface.

9. **(B)**
By using equation of continuity,

$A_1\nu_1 = A_2\nu_2$

$\pi(3R)^2\nu = \pi(2R)^2\nu'$

$\frac{\nu}{\nu'} = \frac{4}{9}$

10. **(A)**
Inside static fluid, pressure at same horizontal level will be equal.
Hence both membrane will fell the same pressure.

11. **(A)**

$N_1 = Mg$

$N_2 = Mg - F_b$

(Because liquid will also apply force on the block, net force of liquid will be downward)

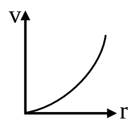
12. **(C)**
Using Bernoulli's equation, $h\rho g = 1/2\rho v^2$
$v = \sqrt{2gh}$ = horizontal speed of water jet from the hole.
Time of fall = $\sqrt{\frac{2(H-h)}{g}}$, x = vt

13. **(B)**
Conceptual question.

14. **(A)**

$v = \frac{2r^2(\rho-\sigma)g}{9\eta} \Rightarrow v \propto r^2$

$\therefore$



15. **(C)**
As the pseudo force on liquid will be towards left, the surface will be perpendicular to the net force on liquid.

16. **(D)**
The surface tension of a liquid depends on all above parameters.

17. **(C)**
Change in energy = T (final area – initial area)

And for finding final diameter of drop, conserve the volume.

18. **(D)**
First the ball will accelerate then its acceleration becomes zero and it will attain a constant terminal velocity.

19. **(B)**
$$v \propto r^2 \; ; \; \frac{r_1}{r_2} = \sqrt{\frac{v_1}{v_2}}$$

20. **(B)**
With increase of temperature, free flow of liquid increase, hence viscosity decrease.

21. **(1)**
$$A_1 v_1 = A_2 v_2 + A_3 v_3$$

22. **(4)**
According to Pascal's principle
$$\frac{f_1}{f_2} = \frac{A_1}{A_2} = \frac{r_1^2}{r_2^2} = \frac{1}{4}$$
$$f_1 = \frac{1}{4} Mg$$

23. **(20)**
$$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20} = 20 \text{ m/s}$$

24. **(4)**
Q = Av
$$v = \frac{Q}{A} = \frac{100 \times 10^{-6}}{0.25}$$
$v = 400 \times 10^{-3}$ mm/s = 0.4 mm/s $= 4 \times 10^{-4}$ m/s

25. **(5)**
F = Mg + 2TL
Extra force = 2TL = $2 \times 100 \times \frac{25}{100} = 5 \text{ N}$

26. **(2.00)**
From law of continuity $A_1 v_1 = A_2 v_2$
$$\Rightarrow \quad v_2 = \frac{\pi \times \left(4 \times 10^{-3}\right)^2 \times 0.25}{\pi \times \left(1 \times 10^{-3}\right)^2} = 4 \text{ m/s}$$
and $x = vt = v\sqrt{\frac{2h}{g}} = 2 \text{ m}$

27. **(3.00)**
From Bermoulli's theorem,

$p+p_0=\frac{1}{2}\rho v^2+p_0$

$\rho gh=\frac{1}{2}pv^2$

$\Rightarrow\ v=\sqrt{2gh}$

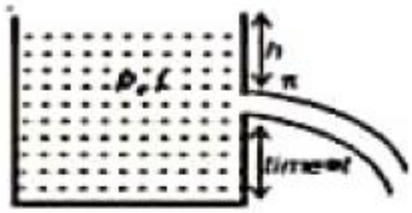
$R=vt$

When extra pressure applied,

$p+\rho gh+p_0=\frac{1}{2}\rho v_2^2+p_0$

$v_2^2=\sqrt{2\left(\frac{p}{\rho}+gh\right)}$

$R^1=v_2t$



$R^1=2R$

$v_1=2v_1$

$\sqrt{2\left(\frac{p}{\rho}+gh\right)}=2\sqrt{2gh}$

$\Rightarrow\ 2\left(\frac{p}{\rho}+gh\right)=4\times 2gh$

$\Rightarrow\ \frac{p}{\rho}+gh=4gh$

$\frac{p}{\rho}=3gh$

$p=3gh\rho=3\times10^3\times10\times10=3\times10^5=3$

28. **(3.00)**

Mass of water = Mass of ice melted

$\left(4\pi a^2-x^2\right)h=\left(a^3-x^3\right)\times0.9$ ...(1)

Weight of ice cube = Buoyant force

$x^3\times0.9\times g=x^2\times h\times1\times g$

$h=0.9x$ ...(2)

From (1) and (2),

$\left(4\pi a^2-x^2\right)0.9x=\left(a^3-x^3\right)\times0.9$

$4\pi a^2x-x^3=a^3-x^3$

$x=\frac{a}{4\pi}$

Now $a=12\pi$

$\therefore\ a=3\text{cm}.$

29. **(6.00)**

$\frac{v}{2}\rho g=\sigma vg$

$\rho=2\sigma$ ...(1)

When the container accelerated upward.

$$v'\rho\left(g+a\right)=mg+\frac{mg}{3}$$

$$\Rightarrow \quad v'\rho\left(\frac{4g}{3}\right)=\frac{4mg}{3}$$

$$\Rightarrow \quad \frac{v'}{v}=\frac{1}{2}$$

$$\Rightarrow \quad \frac{3}{x}=\frac{1}{2}$$

$$\Rightarrow \; x=6$$

30. **(6.00)**

$$P_0A\left(500-H\right)=\left(P_0-pgh\right)A\times 300$$

$$\Rightarrow \quad 500-H=\frac{\left(10^5-10^4\times 0.2\right)\times 300}{10^5}\Rightarrow H=206\,\text{mm}$$

Fall in height = 6 mm

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TOPIC: ISOMERISM

Answer Key & Solution

31. (C)
Non-superimposable mirror images.

32. (C)
Structural isomers.

33. (C)
14 sigma and one pi bonds.

34. (D)
Cis Form.

35. (B)
Gamma H will participate.

36. (D)
No plane of symmetry.

37. (D)

38. (D)
Degree of unsaturation different.

39. (A)
Geometrical isomers.

40. (C)
Double bond carbon.

41. (A)
$HCOOC_2H_5$ and CH_3COOH_3

42. (D)
Double bond in the ring having more than 7 C atoms.

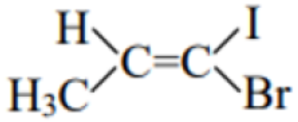
43. (D)
Identical

44. (A)
Neither superimposable nor mirror images.

45. (A)

46. (C)
Structural isomers.

47. (B)



48. (B)
Disubstituted ring.

49. (B)
Bottom most part decides D and L.

50. (B)
No alpha H.

51. (2)
Total 3 and one is meso.

52. (4)
Amines containing -NH2 group.

53. (4)
Two Chiral Carbon.

54. (8)
Three stereo-sites.

55. (7)
Four different groups.

56. (3)
Two terminal and one internal.

57. (5)
Benzene itself has 4 degree of unsaturation.

58. (3)
Structural and Cis trans.

59. (3)

60. (0)
It is meso compound.

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TOPIC: METHOD OF DIFFERENTIAON

SOLUTIONS

61. **(A)**

$y=\left(1+x^{1/2}\right)\left(1-x^{1/2}\right)=1-x$

$\therefore \quad \frac{dy}{dx}=-1$ **Ans.[A]**

62. **(C)**

$\frac{dx}{d\theta}=a\,(1+\cos\theta),\ \frac{dy}{d\theta}=a\sin\theta$

$\therefore\ \frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{a\sin\theta}{a(1+\cos\theta)}=\tan\frac{1}{2}\theta$

63. **(A)**

$y=\log e^x-\log(e^x+1)$

$=x-\log(e^x+1)$

$\therefore\ \frac{dy}{dx}=1-\frac{e^x}{e^x+1}=\frac{1}{e^x+1}$

64. **(C)**

$\frac{dy}{dx}=\frac{-2x}{(x^2-a^2)^2}\ \Rightarrow\ \frac{d^2y}{dx^2}$

$=-\frac{(x^2-a^2)^2.2-2x.2(x^2-a^2).2x}{(x^2-a^2)^4}$

$=\frac{2(3x^2+a^2)}{(x^2-a^2)^3}$

65. **(B)**

$y=\frac{\sec x-\tan x}{\sec x+\tan x}\cdot\frac{\sec x-\tan x}{\sec x-\tan x}$

$=\frac{\left(\sec x-\tan x\right)^2}{1}$

$\therefore\quad \frac{dy}{dx}=2\left(\sec x-\tan x\right)\left(\sec x\tan x-\sec^2 x\right)$

$=-2\sec x\,(\sec x-\tan x)^2$

66. **(B)**

Let us first express y in terms of *x* because all alternatives are in terms of *x*.

So, $x\sqrt{1+y} = -y\sqrt{1+x}$

$\Rightarrow \quad x^2(1+y) = y^2(1+x)$

$\Rightarrow \quad x^2 - y^2 + x^2y - y^2x = 0$

$\Rightarrow \quad (x-y)(x+y+xy) = 0$

$\Rightarrow \quad x + y + xy = 0 \qquad (\because x \neq y)$

$\Rightarrow \quad y = -\dfrac{x}{1+x}$

$\therefore \quad \dfrac{dy}{dx} = -\dfrac{(1+x)1 - x.1}{(1+x)^2} = -\dfrac{1}{(1+x)^2}$

67. (C)

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-\sin x}}.\frac{1}{2\sqrt{\sin x}}.\cos x$$

$$= \frac{\sqrt{1+\sin x}}{2\sqrt{\sin x}} = \frac{1}{2}\sqrt{1+\text{cosec } x}$$

68. (C)

$$y = \log_x 10 = \frac{\log_e 10}{\log_e x}$$

$$\therefore \quad \frac{dy}{dx} = \log_e 10 \left\{-\frac{1}{(\log_e x)^2}.\frac{1}{x}\right\}$$

$$= -\frac{1}{x\log_e 10}.\frac{(\log_e 10)^2}{(\log_e x)^2}$$

$$= -\frac{(\log_e 10)^2}{x\log_e 10}$$

69. (D)

$\because \ \cos(xy) - x = 0$

$$\therefore \ \frac{dy}{dx} = -\frac{-y\sin(xy) - 1}{-x\sin(xy)} = -\frac{y + \text{cosec}(xy)}{x}$$

70. (A)

Let $f(x, y) = x^2e^y + 2xye^x + 13$

$$\therefore \ \frac{dy}{dx} = -\frac{\partial f}{\partial x} / \frac{\partial f}{\partial y}$$

$$= -\frac{2xe^y + 2ye^x + 2xye^x}{x^2e^y + 2xe^x}$$

Dividing Num^r and Den^r by e^x

$$\frac{dy}{dx} = -\frac{2xe^{y-x} + 2y(x+1)}{x(xe^{y-x} + 2)}$$

71. (D)

Taking log on both sides, we have

$y \log x + x \log y = 0$

Now using partial derivatives, we have

$$\frac{dy}{dx} = -\frac{y/x + \log y}{\log x + x/y} = -\frac{y(y + x\log y)}{x(x + y\log x)}$$

72. **(B)**

$$x = e^{\tan^{-1}\left(\frac{y - x^2}{x^2}\right)}$$

Taking logarithm of both the sides, we get

$$\log x = \tan^{-1}\left(\frac{y - x^2}{x^2}\right)$$

$\Rightarrow \quad y = x^2 + x^2 \tan(\log x)$

$$\frac{dy}{dx} = 2x + 2x\tan(\log x) + x^2 \sec^2(\log x).\ \frac{1}{x}$$

$$= 2x[1 + \tan(\log x)] + x\sec^2(\log x).$$

73. **(C)**

$$y = \tan^{-1}\frac{3x - x^3}{1 - 3x^2} = 3\tan^{-1}x$$

$$\therefore \quad \frac{dy}{dx} = \frac{3}{1 + x^2}$$

74. **(B)**

$y = 2\tan^{-1} x$

$$\therefore \quad \frac{dy}{dx} = \frac{2}{1 + x^2}$$

75. **(C)**

$$y = \tan^{-1}\left(\frac{\sqrt{1 + \cos\theta} + \sqrt{1 - \cos\theta}}{\sqrt{1 + \cos\theta} - \sqrt{1 - \cos\theta}}\right), \text{ where}$$

$x^2 = \cos\theta$

$$= \tan^{-1}\left(\frac{\cos\theta/2 + \sin\theta/2}{\cos\theta/2 - \sin\theta/2}\right)$$

$$= \tan^{-1}\left(\frac{1 + \tan\theta/2}{1 - \tan\theta/2}\right)$$

$= \tan^{-1}[\tan(\pi/4 + \theta/2)] = \pi/4 + \theta/2$

$$= \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2$$

$$\therefore \quad \frac{dy}{dx} = -\frac{1}{2}\frac{1}{\sqrt{1 - x^4}}.2x = -\frac{x}{\sqrt{1 - x^4}}$$

76. **(B)**

Substituting $x = \sin\theta$ and $y = \sin\phi$ in the given equation, we get

$\cos\theta + \cos\phi = a(\sin\theta - \sin\phi)$

$$\Rightarrow 2\cos\frac{\theta + \varphi}{2}.\cos\frac{\theta - \varphi}{2} = 2a\cos\frac{\theta + \varphi}{2}.\sin\frac{\theta - \varphi}{2}$$

$$\Rightarrow \cot\frac{\theta - \varphi}{2} = a \Rightarrow \theta - \phi = 2\cot^{-1} a$$

$\Rightarrow \sin^{-1} x - \sin^{-1} y = 2\cot^{-1} a$

Differentiating with respect to x, we get

$$\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

77. **(C)**

$y = \sin^{-1}\frac{2x}{1+x^2} = 2\tan^{-1}x$

$\Rightarrow \frac{dy}{dx} = \frac{2x}{1+x^2}$ and $z = \tan^{-1}x \Rightarrow \frac{dz}{dx} = \frac{1}{1+x^2}$

$\therefore \quad \frac{dz}{dx} = \frac{dy/dx}{dz/dx} = \frac{2}{1+x^2}.\frac{1+x^2}{1} = 2$

78. **(B)**

Here $y = \sqrt{\sin x + y} \quad \Rightarrow y^2 = \sin x + y+$

$\therefore 2y\frac{dy}{dx} = \cos x + \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{\cos x}{2y-1}$

79. **(A)**

Here $y = \dfrac{x}{a+\dfrac{x}{b+y}} = \dfrac{x(b+y)}{a(b+y)+x}$

$\Rightarrow \quad aby + ay^2 + xy = bx + xy$

$\Rightarrow \quad ay^2 + aby = bx$

$\Rightarrow \quad 2ay\frac{dy}{dx} + ab\frac{dy}{dx} = b$

$\Rightarrow \quad \frac{dy}{dx} = \frac{b}{a(b+2y)}$

80. **(C)**

$y = e^{x+y}$

$\Rightarrow \quad \log y = x + y \quad \Rightarrow \quad \frac{1}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx}$

$\Rightarrow \quad \frac{dy}{dx} = \frac{y}{1-y}$

81. **(1)**

$\frac{dy}{dx} = \left(\frac{dy}{d\theta}\right)/\left(\frac{dx}{d\theta}\right)$

$= \frac{3a\sin^2\theta.\cos\theta}{-3a\cos^2\theta\sin\theta} = -\tan\theta$

$\therefore$ exp. $= \sqrt{1+\tan^2\theta} = \sec\theta$

82. **(6)**

From the given equation, we have

$y^2(1-x^2) = (\sin^{-1}x)^2$

$\Rightarrow$ $(1-x^2)\, 2y\, \dfrac{dy}{dx} - 2xy^2 = 2\, \dfrac{\sin^{-1} x}{\sqrt{1-x^2}}$

$\Rightarrow$ $2\,(1-x^2)\, y\, \dfrac{dy}{dx} - 2xy^2 = 2y$

$\Rightarrow$ $(1-x^2)\, \dfrac{dy}{dx} = 1 + xy = 6$

83. **(1)**

$\because\ g(x) = f\,[f\,(x)]$

$= f\,\{|x-2|\}$

$= ||\,x-2| - 2|$

But $x > 20 \Rightarrow |x-2| = x-2$

$\Rightarrow$ $g(x) = |x-2-2| = x-4$

$\therefore$ $g'(x) = 1$

84. **(11)**

$h'\,(x) = 2f\,(x)\, f\,'(x) + 2g(x)\, g'(x)$

$= 2f\,(x)\, g(x) + 2g\,(x)\, f\,''\,(x)$

$= 2f\,(x)\, g(x) - 2f\,(x)\, g(x)$

$= 0$ $\quad [\because\ f\,''(x) = -f\,(x)]$

$\Rightarrow$ $h(x) = c$

$\Rightarrow$ $h(10) = h\,(5) = 11$

85. **(1)**

$\because\ \lambda$ n $x = \log_e x$, so

$f\,(x) = \log_x (\log_e x) = \dfrac{\log(\log x)}{\log x}$

$\Rightarrow f\,'\,(x) = \dfrac{\log x\left(\dfrac{1}{x \log x}\right) - \log(\log x)\dfrac{1}{x}}{(\log x)^2}$

$\therefore$ $ef\,'\,(e) = e\dfrac{1/e - 0}{(1)^2} = \dfrac{1}{e}\ e = 1$

86. **(0)**

When $1 < x \le 3$,

$f\,(x) = (x-1) - (x-3) = 2$

$\Rightarrow$ $f\,'\,(2-0) = 0,\ f\,'\,(2+0) = 0$

$\therefore f\,'\,(2) = 0$

87. **(1)**

Let $y = \sin 2x \cos 2x \cos 3x + \log_2 2^{x+3}$

$= \dfrac{1}{2} \sin 4x \cos 3x + (x+3) \log_2 2$

$= \dfrac{1}{4} [\sin 7x + \sin x] + x + 3$

$\therefore$ $\dfrac{dy}{dx} = \dfrac{1}{4}$ $[7 \cos 7x + \cos x] + 1$

$\therefore$ $\left(\dfrac{dy}{dx}\right)_{x=\pi} = \dfrac{1}{4}$ $[7 \cos 7\,\pi + \cos \pi] + 1$

$$=\frac{1}{4}\ [-8]+1=-1$$

88. **(3)**

$$\frac{dy}{dx}=\frac{dy/dt}{dx/dt}=\frac{3}{2t}$$

$$\left.\frac{dy}{dx}\right|_{t=\frac{1}{2}}=3$$

89. **(1)**

$$y=e^{x^2\ln x}$$

$$\left.\frac{dy}{dx}\right|=e^{x^2\ln x}\left(2x\ln x+x^2\cdot\frac{1}{x}\right)$$

$$=1\ \text{When}\ x=1$$

90. **(3)**

$$y=\frac{\pi}{2}+3x$$

$$\frac{dy}{dx}=3\ \text{for all}\ x$$