IIT – JEE: 2025 **TW TEST (MAIN)** **DATE: 06/04/24**

TOPIC: GRAVITATION

Answer Key & Solution

1. (B)
P.E. becomes less negative, and KE become less positive

2. (C)
Escape speed $\upsilon_{min} = \sqrt{2gR}$ Since we talking about another planet with different radius, gravitational force also changes.

$$\upsilon_{min} = \sqrt{\frac{2GM}{R}} \Rightarrow \upsilon_{min} \propto \sqrt{\frac{1}{R}} \Rightarrow$$

Given $R_2 = \frac{R_1}{4}$

$$\therefore \frac{\upsilon_2}{\upsilon_1} = \sqrt{\frac{R_1}{\left(\frac{R_1}{4}\right)}} \Rightarrow \frac{\upsilon_2}{\upsilon_1} = \sqrt{4} \Rightarrow \upsilon_2 = 2\upsilon_1$$

3. (A)

$$m\upsilon'^2 = 2 \times \frac{1}{2} m\upsilon^2$$
$$= \sqrt{2}\,\upsilon_0$$
$$= \upsilon_e$$

So escape

4. (D)

$$P.E. = -\frac{Gm_1m_2}{r}$$
$$T.E. = -\frac{Gm_1m_2}{2r}$$
$$K.E. = +\frac{Gm_1m_2}{2r}$$

5. (B)

$$U_f - u_i = \frac{-GmM}{\left(R + \frac{R}{5}\right)} + \frac{GmM}{R}$$

$$= \frac{-5GmM}{6R} + \frac{GmM}{R} = \frac{GmM}{6R} = \frac{mgR}{6}$$

6. (B)

$$V_P = -\frac{GM}{R}$$

As $R \downarrow \frac{GM}{R} \uparrow$ but dur to -ve it decreases.

7. (A)

Gravitational field strength at m_1 is

$$I_1 = \frac{Ggm_2}{d^2}$$

Gravitational field strength of m_2 is

$$I_2 = \frac{Ggm_1}{d^2}$$

$\Rightarrow \frac{Gg}{d^2} = \frac{l_1}{m_2} = \frac{-l_1}{m_1}$ (There is a minus sign since $\bar{l}_1$ and $\bar{l}_2$ are in opposite direction)

$$\Rightarrow \bar{l}_1 m_1 + \bar{l}_2 m_2 = 0$$

$$\bar{F}_{net} = 0$$

8. (C)

Let the point masses be

$p_1, p_2, p_3 ......p_a ....(a > 0)$

Now potential at x =0 due to p_1 be

$$P_1 = \frac{-Gm}{1}$$

Similarly $P_2 = \frac{-Gm}{2}$

$$P_3 = \frac{-Gm}{4}$$

So, total potential is given by

$$P = P_1 + P_2 + P_3 + ...$$

$$P = \frac{-Gm}{1} + \frac{-GM}{2} + \frac{-Gm}{4} + \frac{-Gm}{8} +$$

$$= -Gm\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...\right)$$

This is a sum of infinite terms with common ratio $\frac{1}{2}$.

$$P = (-Gm)\frac{1}{\left(1 - \frac{1}{2}\right)} = -2GM$$

9. (B)

$$g = \frac{Gm}{R^2}$$

10. (D)

The value of gravity changes as we move away from or towards the centre of the Earth.

This is given by $9R = \frac{GM}{R^2}$, where M is the mass of a planet of radius R

So $M = \frac{4}{3}\pi R^3 \rho$; substituting in above equation

$$g_R = G\frac{4}{3}\pi\rho \times R$$

Since we want the value of g at depth (d) from the Earth's surface, we replace R by (R – d)

$$\Rightarrow g_d = G\frac{4}{3}\pi\rho \times (R - d)$$

$$\Rightarrow \frac{g_R}{g_d} = \frac{R}{(R - d)}$$

$$\Rightarrow \frac{g_d}{g_R} = \left(1 - \frac{d}{R}\right)$$

The given depth in the problem is d = 3200 km, substituting we get,

$$\frac{g_d}{g_R} = \left(1 - \frac{3200}{6400}\right)$$

$$g_d = 9.8 \times \left(1 - \frac{3200}{6400}\right)$$

$$g_d = 9.8 / 2$$

$$g_d = 4.9 ms^{-2}$$

11. (C)

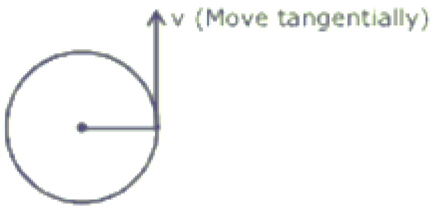
Escape speed is $\upsilon_e = \sqrt{2gR}$

$$\frac{\upsilon_1}{\upsilon_2} = \sqrt{\frac{g_1 R_1}{g_2 R_2}}$$

It is given $\frac{R_1}{R_2} = K_1$ and $\frac{g_1}{g_2} = K_2$

$$\frac{\upsilon_1}{\upsilon_2} = \sqrt{K_1 K_2}$$

12. (B)



13. (C)

$$T \propto r^{3/2} \left[\omega = \frac{2\pi}{T}\right]$$

$$\omega \propto \frac{1}{r^{3/2}}$$

$$\left(\frac{\omega}{2\omega}\right) = \left(\frac{R_1}{r}\right)^{3/2}$$

$$\frac{R_1}{r} = \frac{1}{(2)^{2/3}}$$

$$R_1 = \frac{r}{(4)^{1/3}}$$

14. (C)

$$\upsilon = \sqrt{\frac{Gm}{r}} = \sqrt{\frac{Gm}{R + h}}$$

$$\upsilon_1 = \sqrt{\frac{Gm}{R + \frac{R}{2}}} = \sqrt{\frac{2}{3}}\,\upsilon$$

15. (B)

$$\frac{GMm}{r^2} = \frac{m\upsilon^2}{r}$$

16. (C)

$$\frac{1}{2}m\upsilon^2 = \frac{GM_e m}{R_e + h} = \frac{gR_e^2 m}{R_e + 4R_e}$$

$$\frac{1}{2}m\upsilon^2 = \frac{Mg\,R_e}{5}$$

17. (B)

$$W = \frac{GmM}{R} - \frac{GmM}{nR + R}$$

$$= \frac{GmM}{R}\left[1 - \frac{1}{nH}\right] = \frac{GmM}{R}\left[\frac{n}{n+1}\right]$$

$$= mgR\left(\frac{n}{n+1}\right)$$

18. (A)

When the 2 particles are at rest, at a separation d

Potential energy of the system = $\frac{-Gm}{d}$ and kinetic energy = 0

Let V be the speed at half the separation

Potential energy at $\frac{d}{2}$ is $= \frac{-2Gm^2}{d}$ and

Kinetic energy $= 2 \times \frac{1}{2} mV^2$

From conservation of energy principle,

$$\frac{-Gm^2}{d} = \frac{-2Gm^2}{d} + 2 \times \frac{1}{2} mV^2$$

$$\frac{-Gm^2}{d} = \frac{-2Gm^2}{d} + V^2$$

Hence $V = \sqrt{\frac{Gm}{d}}$

19. (C)

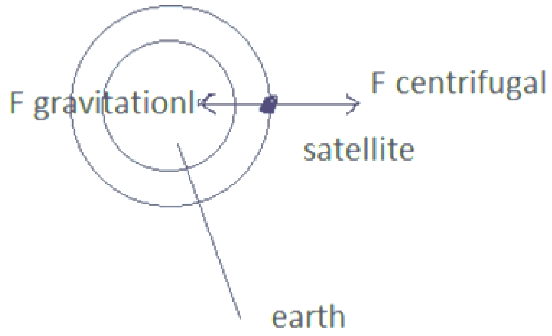
For a body revolving around the earth at any orbital radius, the gravitational force towards earth is countered by centrifugal force as shown in figure.

i.e. $F_g = F_\upsilon$

So, for a body hanging in the satellite

$W_1 + F_\upsilon = F_g \rightarrow W_1 = 0$

Similarly $W_2 = 0$. Thus $W_1 = W_2$



20. (A)

Here $G = 6.67 \times 10^{-11}$ N m^2/kg^2

Mass of the pulsar, $M = 1.98 \times 10^{30}$ kg

Radius of the pulsar, R = 12 km $= 12 \times 10^3$ m

Acceleration due to gravity on the surface of the pulsar is

$$g = \frac{GM}{R^2}$$

Substituting the given numerical values, we get

$$g = \frac{(6.67 \times 10^{-11} Nm^2/kg^2)(1.98 \times 10^{30} kg)}{(12 \times 10^3 m)^2}$$

$$= 0.092 \times 10^{13} \ m/s^2 = 9.2 \times 10^{11} m/s^2$$

21. (1.5)

Time period $T^2 = \frac{4\pi^2}{GM} R^3$

i.e. $T^2 \propto R^3$

$$\Rightarrow \left(\frac{T_n}{T_S}\right)^2 = \left(\frac{R_n}{R_S}\right)^3$$

Given $\frac{R_n}{R_S} = \frac{10^{13}}{10^{12}} = 10$

$$\left(\frac{T_n}{T_S}\right)^2 = 10^3$$

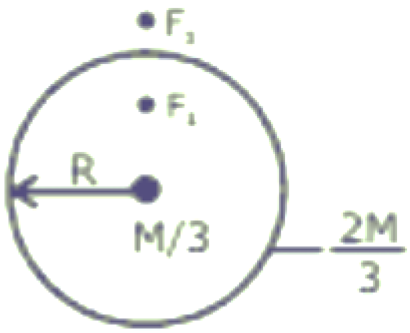
$$\Rightarrow \frac{T_n}{T_S} = 10^{3/2} = 10\sqrt{10}$$

22. (1.5)

$$g' = \frac{G(2M_e)}{(2R_e)^2} = \frac{g}{2}$$

$$T = 2\pi\sqrt{\frac{l}{g}} = 2$$

New time period $T' = 2\pi\sqrt{\frac{l}{g/2}} = 2\sqrt{2}$



23. (1)

As the stars will always be diametrically opposite each other they rotate with the same angular velocity.

24. (100)

$$\frac{1}{2}mv^2 = mgh = \frac{mGM}{R^2}\times 90 \qquad .....(1)$$

$$\frac{1}{2}mv^2 = \frac{mG\left(\frac{1}{10}M\right)}{\left(\frac{R}{3}\right)^2}\times G_1 \qquad .....(2)$$

From (1) and (2)

$$m\frac{GM}{R^2}\times 90 = \frac{9}{10}\frac{mGM}{R^2}\times h_1 \Rightarrow h_1 = 100 \text{ m}$$

25. (12)

$$T = \frac{2\pi}{\omega_{rel.}} = \frac{2\pi}{2\omega} = \frac{2\pi\times 24\times hr.}{2\times 2\pi}$$

$$T = 12\,hr.$$

26. (2)

$$v_0 = \sqrt{\frac{2GM_e}{R_e}} = \sqrt{\frac{2G\rho\frac{4}{3}\pi R_e^3}{R_e}}$$

$$= \sqrt{2G\rho \frac{4}{3}\pi R_e^2}$$

$$v' = \sqrt{2G\rho \frac{4}{3}\pi (2R_e)^2}$$

$$= 2v_0$$

27. (0.2)

$$g = \frac{GM_e}{R_e^2} = \frac{GM_e}{(5R_e)^2}$$

$$\frac{\frac{4}{3}\pi R_e^3 \rho}{R_e^2} = \frac{\frac{4}{3}\pi (5R_e)^3 \rho'}{(5R_e)^2}$$

$$\rho = 5\rho'$$

$$\rho' = \frac{\rho}{5}$$

28. (4)

Considering the origin of the coordinates system at 4*m*, we evaluate the positon of the centre of mass as

$$\frac{4m \times 0 + m \times r}{4m + m} = \frac{r}{5}$$

Thus the center of mass is $\frac{r}{5}$ from 4*m* and $\frac{4r}{5}$ from *m*.

The ratio of their kinetic energy is given as $\frac{\frac{1}{2}\left[I\omega^2\right]_{2m}}{\frac{1}{2}\left[I\omega^2\right]_m}$

As the angular velocity of the both the masses would be same we get the ratio of kinetic energy as

$$\frac{4m\left(\frac{r}{5}\right)^2}{m\left(\frac{4r}{5}\right)^2} = \frac{1}{4}$$

29. (1.4)

All point on the circumference of the ring are at distance $\sqrt{R^2 + x^2}$ from the center *O* of the ring.

Force on the mass *m* at *P* :

$$F = \frac{GMm}{R^2 + x^2} \times (2\cos\theta) \text{ where } \cos\theta = \frac{x}{\sqrt{R^2 + x^2}}$$

Due to symmetry, vertical components of the forces from two symmetrical elements cancel out, the horizontal components add up, hence we get a factor of 2.

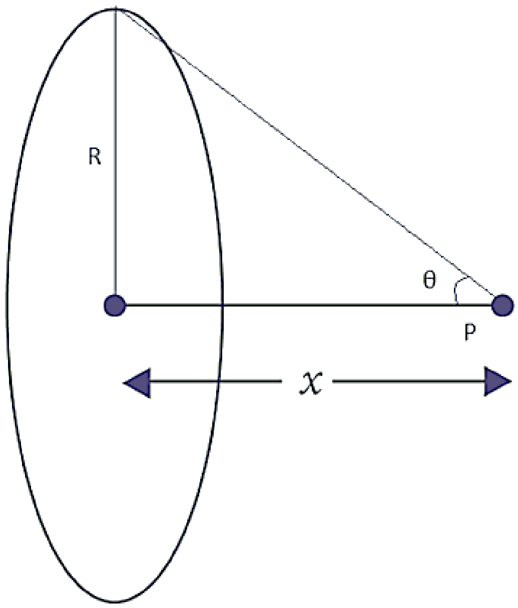
$$\therefore \quad F = \frac{GMm}{R^2 + x^2} \times 2\frac{x}{\sqrt{R^2 + x^2}}$$

When *F* is maximum, $\frac{dF}{dx} = 0$

$\Rightarrow \left(R^2+x^2\right)^{-\frac{3}{2}}+x\left(-\frac{3}{2}\left(R^2+x^2\right)^{-\frac{3}{2}-1}\times 2x\right.$

$\Rightarrow \left(R^2+x^2\right)=3x^2$

$\Rightarrow x=\frac{R}{\sqrt{2}}$



30. (0.5)

Gravitational force between 2 bodies is given by $F=\frac{Gm_1m_2}{R^2}$

Where, m_1 and m_2 are the masses of the bodes, G is the universal gravitational constant and R is the distance between them.

For a given distance, $F=\frac{Gm(1-X)m(X)}{R^2}$ is maximum when $X(1-X)$ is maximum.

By differentiation, we get,

$$\frac{dF}{dX}=\frac{Gm^2}{R^2}\frac{d}{dX}\left(X(1-X)\right)$$

$$\frac{dF}{dX}=\frac{Gm^2}{R^2}\frac{d}{dX}\left(X-X^2\right)$$

$$\frac{dF}{dX}=\frac{Gm^2}{R^2}(1-2X)=0$$

$$1-2X=0$$

$$X=\frac{1}{2}$$

The gravitational force of attraction has a maximum value at X = 1/2.

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TOPIC: CHEMICAL KINETICS

Answer Key & Solution

31. (D)

$$\frac{\Delta[NO_2]}{\Delta t} = \frac{2.4\times10^{-2}}{6} = 4\times10^{-3}$$

$$-\frac{1}{2}\frac{\Delta[N_2O_5]}{\Delta t} = \frac{1}{4}\frac{\Delta[NO_2]}{\Delta t}$$

$$-\frac{\Delta[N_2O_5]}{\Delta t} = \frac{1}{2}\frac{\Delta[NO_2]}{\Delta t} = 2\times10^{-3}$$

32. (C)

Rate constant depends upon temperature & catalyst.

33. (B)

$$r_1 = K(a)^2(b)^{\frac{1}{2}}$$

$$r_2 = K(2a)^2(4b)^{\frac{1}{2}}$$

$$\frac{r_2}{r_1} = 8$$

34. (B)

$$K = 10^{-2}\ \ell\ \text{mole}^{-1}\ \text{sec}^{-1}$$

$$= \frac{10^{-2}\times10^{3}}{6\times10^{23}}\times60\ \text{ml molecule}^{-1}\ \text{min}^{-1}$$

$$= 10^{-21}$$

35. (B)

$$A_2(g) \rightarrow B(g) + \frac{1}{2}C(g)$$

	$A_2(g)$	$B(g)$	$\frac{1}{2}C(g)$
t = 0	100	0	0
t = 5 min	100 – p	p	$\frac{p}{2}$

$$100 - p + p + \frac{p}{2} = 120$$

$$100 + \frac{p}{2} = 120 \Rightarrow p = 40\ \text{min}$$

$$\text{Rate} = -\frac{\Delta P_{A_2}}{\Delta t} = \frac{40}{5} = 8\ \text{mm/min}$$

36. (B)

$$N_2(g)+3H_2(g)\rightarrow 2NH_3(g)$$

$$\frac{\Delta[NH_3]}{\Delta t}=10^{-3}\ kg/h=\frac{10^{-3}\times 10^{3}}{17}mol/h$$

$$-\frac{\Delta[H_2]}{\Delta t}=\frac{3}{2}\frac{\Delta[NH_3]}{\Delta t}=\frac{3}{2}\times\frac{10^{-3}\times 10^{3}}{17}\ mol/h$$

$$=\frac{3}{2}\times\frac{1}{17}\times 2\ gm/h$$

$$=\frac{3}{17}\times 10^{-3}\ kg/h=1.76\times 10^{-4}\ kg/h$$

37. (C)

Rate = K

38. (B)

$$1.837=(1.5)^n$$

$$n=1.5$$

39. (A)

$$\text{rate }=K[A]^m[B]^n$$

$$0.1=K(0.012)^m(0.035)^n \quad ...(1)$$

$$0.1=K(0.024)^m(0.035)^n \quad ...(2)$$

$$(2)\div(1)$$

$$1=2^m \Rightarrow m=0$$

$$0.8=K(0.024)^m(0.070)^n \quad ...(3)$$

$$(3)\div(2)$$

$$8=2^n \Rightarrow n=3$$

$$\text{rate }= K[B]^3$$

40. (A)

$$[A]=[A]_0-Kt$$

$$\frac{[A]_0}{4}=[A]_0-K(10) \Rightarrow K=\frac{3[A]_0}{4\times 10}$$

$$\frac{[A]_0}{10}=[A]_0-\frac{3[A]_0}{4\times 10}t$$

$$\frac{3[A]_0}{4\times 10}t=\frac{9[A]_0}{10}$$

$$t=12\ hr$$

41. (C)

$$K = \frac{2.303}{t}\log\frac{a}{a-\frac{3a}{4}}$$

$$t = \frac{2.303}{K}\log 4$$

42. (B)

$$t_{\frac{1}{2}} \propto \frac{1}{[A]_0}$$

$$[A]_0 \xrightarrow{t_1} \frac{[A]_0}{2} \xrightarrow{2t_1} \frac{[A]_0}{4}$$

$$[A]_0 \xrightarrow{3t_1} \frac{[A]_0}{4}$$

43. (D)

[Reactant] decreases

[Product] increases

Rate of decreases in concentration of B is greater than A

44. (C)

$$t_{\frac{1}{2}} \propto \frac{1}{[A]_0^{n-1}}$$

$$\frac{60}{29} = \left(\frac{0.75}{1.55}\right)^{1-n}$$

$$1 - n = -1 \Rightarrow n = 2$$

45. (B)

Activation energy is different for different substance.

46. (C)

$$\log K = \log A - \frac{Ea}{2.303RT}$$

$\log K$ v/s $\frac{1}{T}$ graph is straight line.

47. (D)

$$\Delta H = y - x$$

$$\Delta H = E_{a,f} - E_{a,b}$$

$$y - x = E_{a,f} - z$$

$$E_{a,f} = y - x + z$$

48. (B)

$K = A$ if $T \to \infty$

49. (D)

$$r_1 = K(a)^n (b)^m$$

$$r_2 = K(2a)^n\left(\frac{b}{2}\right)^m$$

$$\frac{r_2}{r_1} = 2^n\left(\frac{1}{2}\right)^m = 2^{n-m}$$

50. (C)

$r \propto [NO]^2[O_2]$

If volume is tripled then concentration decreases to $\frac{1}{3}$rd .

51. (0.80)

$2.4\times10^{-5} = K\left(3\times10^{-5}\right)$

$K = 0.8$

52. (0.00)

rate = K for zero order

53. (0.33)

$r = K(a)^m$

$2r = K(8)^m$

$2 = 8^m \Rightarrow 2 = 2^{3m}$

$3m = 1 \Rightarrow m = \frac{1}{3} = 0.33$

54. (2.00)

Unit of K for 2nd order reaction is $\ell\, mol^{-1} sec^{-1}$.

55. (0.00)

$\frac{[A]_0}{4} = [A]_0 - K \Rightarrow K = \frac{3[A]_0}{4}$

Time taken for completion $= \frac{[A]_0}{K}$

$= \frac{4}{3} hr = 1.33\, hr$

i.e. after 1.33 hr, [reactant] = 0

56. (2.00)

$t_{99} = \frac{2.303}{K}\log\frac{100}{1} = \frac{2.303}{K}\times2$

$t_{90} = \frac{2.303}{K}\log\frac{100}{10} = \frac{2.303}{K}\times1$

$t_{99} = 2\times t_{90}$

57. (30.00)

$K = \frac{0.693}{10} min^{-1}$

Initial rate $= 6\times10^{21}$ molecules $ml^{-1}\ sec^{-1}$

Final rate $= 4.5\times10^{25}$ molecules $\ell^{-1}\ min^{-1}$

$$= \frac{4.5\times10^{25}}{10^3\times60} \text{ molecules } ml^{-1}\ sec^{-1}$$

$$= 0.75\times10^{21}$$

Ratio of rate is ratio of conc. for 1st order reaction.

$$t = \frac{2.303}{K}\log\frac{6\times10^{21}}{0.75\times10^{21}}$$

$$t = \frac{2.303}{0.693}\times10\times\log 8 = 30$$

58. (0.00)

For zero order, $t_{1/2} \propto [A]_0$

59. (3.00)

For 2nd order reaction, time taken for 75% reaction is 3 times of half life period.

60. (32.00)

2^5 times = 32 times

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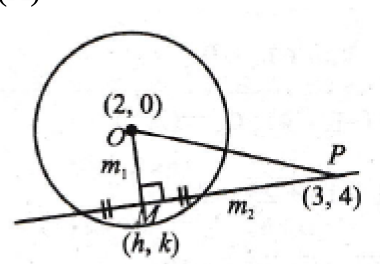
TOPIC: CIRCLE

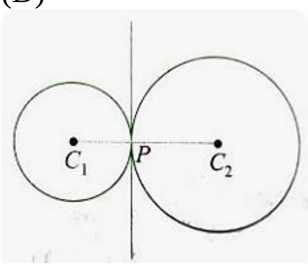
SOLUTIONS

61. (C)
If two circles intersect at two distinct points
$\Rightarrow |r_1 - r_2| < C_1C_2 < r_1 + r_2$
$|r-2| < \sqrt{9+16} < r+2$
$|r-2| < 5$ and $r+2 > 5$
$-5 < r-2 < 5 \quad r > 3 \quad .....(2)$
$-3 < r < 7 \quad .....(1)$
From (1) and (2)
$3 < r < 7$

62. (A)


$\Rightarrow m_1m_2 = -1$
$\Rightarrow \left(\frac{4-k}{3-h}\right)\left(\frac{k-0}{h-2}\right) = -1$
Hence, locus is $x^2 + y^2 - 5x - 4y + 6 = 0.$

63. (C)
Locus of the centre of the circle cutting $S_1 = 0$ and $S_2 = 0$ orthogonally is the is the radical axis between $S_1 = 0$ and $S_2 = 0$, i.e., $S_1 - S_2 = 0$ or $9x - 10y + 11 = 0.$

64. (D)


$C_1C_2 = r_1 + r_2$
$C_1 = (0,0); C_2 = (3\sqrt{3}, 3)$ and $r_1 = 2, r_2 = 4$

$\Rightarrow$ Circles touch each other externally.

Equation of common tangent is $\sqrt{3}x+y-4=0$ (1)

Comparing it with $x\cos\theta+y\sin\theta=2$, we get $\theta=\frac{\pi}{6}$

65. (B)

The centre of $x^2+y^2-4x-4y=0$ is $(2,2)$.

Given $ax+by=2$

$\therefore 2a+2b=2$ or $a+b=1$

$ax+by=2$ Touches $x^2+y^2=1$.

So, $1=\left|\frac{-2}{\sqrt{a^2+b^2}}\right|$

$\therefore a^2+b^2=4$ or $a^2+(1-a)^2=4$

Or $2a^2-2a-3=0$

$\therefore a=\frac{2\pm\sqrt{4+24}}{4}=\frac{1\pm\sqrt{7}}{2}$

$\therefore b=1-a=1-\frac{1\pm\sqrt{7}}{2}$

$=\frac{1\pm\sqrt{7}}{2}$

66. (B)

Let the tangent be of the form $\frac{x}{x_1}+\frac{y}{y_1}=1$ and area of Triangle formed by it with coordinate axes is

$\frac{1}{2}x_1y_1=a^2$ (1)

Again, $y_1x+x_1y-x_1y_1=0$

Applying conditions of tangency

$\frac{|-x_1y_1|}{\sqrt{x_1^2+y_1^2=a}}$ or $\left(x_1^2+y_1^2\right)=\frac{x_1^2y_1^2}{a^2}$ (2)

From Eqs. (1) and (2), we get x_1, y_1, which gives equation of tangent as $x\pm y=\pm a\sqrt{2}$.

67. (C)

The equation of the line $y=x$ in distance from is $\frac{x}{\cos\theta}=\frac{y}{\sin\theta}=r$, where $\theta=\frac{\pi}{4}$.

For point $p, r=6\sqrt{2}$. Therefore, coordinates of P are

Given by $\frac{x}{\cos\frac{\pi}{4}}=\frac{y}{\sin\frac{\pi}{4}}=6\sqrt{2}\Rightarrow x=6, y=6.$

Since $P(6,6)$ lies on $x^2+y^2+2gx+2fy+c=0$,

$72+12(g+f)+c=0$ (1)

Since $x+y$ touches the circle, the equation $2x^2+2x(g+f)+c=0$ has equal roots.

$\Rightarrow 4(g+f)^2=8c$

$\Rightarrow (g+f)^2=2c$ (2)

From (1), we get

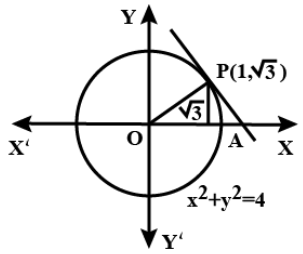
$[12(g+f)]^2 = [-(c+72)]^2$

$\Rightarrow 144(g+f)^2 = (c+72)^2$

$\Rightarrow 144(2c) = (c+72)^2$

$\Rightarrow (c-72)^2 = 0 \Rightarrow c = 72$

68. (A)



The equations of the tangent and normal to $x^2+y^2=4$ at $(1,\sqrt{3})$ are $x+\sqrt{3}y=4$ and $y=\sqrt{3}x$

The tangent meets the x –axis at (4,0)

Therefore, area of $\Delta OAP = \frac{1}{2}(4)\sqrt{3} = 2\sqrt{3}$ sq. Units

69. (A)

The midpoint is the intersection of the chord and perpendicular line ti it from the centre $(3,-1)$.

The equation of perpendicular line is $5x+2y-13=0$.

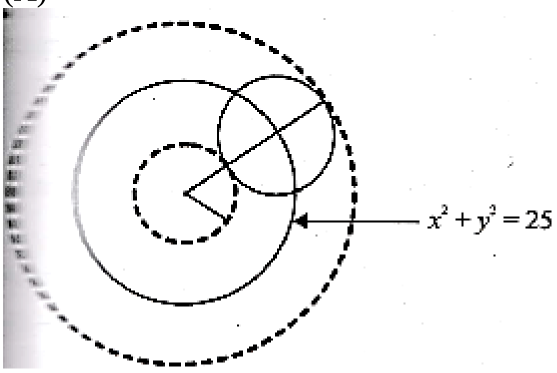
Solving this with the given line, we get the point $(1,4)$.

70. (B)

The line $2y = gx+\alpha$ should pass through $(-g,-g)$,

So $-2g = -g^2+\alpha \Rightarrow \alpha = g^2-2g = (g-1)^2-1 \geq -1$.

71. (A)



Let (h,k) ne any point in the set, then equation of circle is

$(x-h)^2+(y-k)^2 = 9$

But (h,k) lies on $x^2+y^2=25$,

Then $h^2+k^2=25$

$\therefore 2 \leq$ Distance between the centres of two circles ≤ 8

$\Rightarrow 4 \leq h^2+k^2 \leq 64$

Therefore, locus of (h,k) is $4 \leq x^2+y^2 \leq 64$.

72. (C)

Equation of radical axis (i.e. Common chord) of the two circles is

$10x + 4y - a - b = 0$ (1)

Centre of first circle is $H(-4,-4)$.

Since second circle bisects the circumference to the first circle, centre $H(-4,-4)$ of the first circle must le on the common chord Eq.(1).

$\therefore -40 - 16 - a - b = 0$

$\Rightarrow a + b = -56$

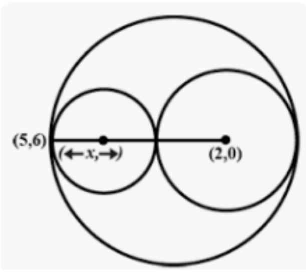
73. (A)

Centre of the circle $x^2 + y^2 = 2x$ is $(1,0)$. Common chord of the other two circles is

$8x - 15y + 26 = 0.$

Distance of $(1,.0)$ from $8x - 15y + 26 = 0$ is $\frac{|8+26|}{\sqrt{15^2 + 8^2}} = 2$

74. (B)



Given circle is $(x-2)^2 + y^2 = 4$

Centre is $(2,0)$ and radius = 2

Therefore, distance between $(2,0)$ and $(5,6)$ is

$\sqrt{9+36} = 3\sqrt{5}$

$\Rightarrow r_1 = \frac{3\sqrt{5-2}}{2}$

And $r_2 = \frac{3\sqrt{5}+2}{2}$

$= r_1 r_2 = \frac{41}{4}$

75. (C)

$x^2 + y^2 - 12x + 35 = 0$ (1)

$x^2 + y^2 + 4x + 3 = 0$ (2)

Equation of radical axis of circles (1) and (2) is $-16x + 32 = 0 \Rightarrow x = 2$

It interest the line joining the centres, i.e. = 0 at the point $(2,0)$

$\therefore$ Required radius $= \sqrt{4 - 24 + 35} = \sqrt{15}$ [length of tangent from $(2,0)$].

76. (B)

Let $(\alpha, 3-\alpha)$ be any point on $x + y = 3.$

Equation of chord of contact is $\alpha x + (3-\alpha)y = 9$

i.e., $\alpha(x-y) + 3y - 9 = 0$

The chords passes through the point $(3,3)$ for all values of α

77. (C)

$C_1=(-1,-4); C_2=(2,5);$

$r_1=\sqrt{1+16+23}=2\sqrt{10};$

$r_2=\sqrt{4+25+19}=\sqrt{10};$

$C_1C_2=\sqrt{9+18}=3\sqrt{10}$

$\Rightarrow C_1C_2=r_1+r_2.$

Hence, circles touch externally.

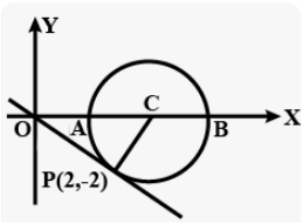
78. (C)

Centre of circle is $(1,0)$ and radius is 1.

Line will touch the circle if $|\cos\theta-2|=1 \Rightarrow \cos\theta=1.3$

Thus, $\cos\theta=1 \Rightarrow \theta=2n\pi, n\in I$

79. (C)



If $(a,0)$ is the centre C and P is $(2,-2)$, then $\angle COP=45^\circ$

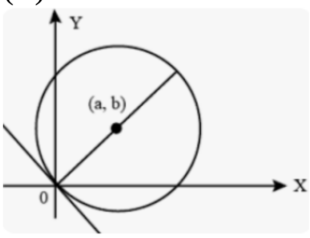
Since the equation of OP is $x+y=0$

$\therefore OP=2\sqrt{2}=CP.$ Hence, $OC=4$

The point on the greatest x- coordinate is B.

$\alpha=OB=OC+CB=4+2\sqrt{2}$

80. (B)



Obviously, the slope of the tangent will be $-\left(\frac{1}{b/a}\right)$, i.e., $-\frac{a}{b}.$

Hence, the equation of the tangent is $y=-\frac{a}{b}x$, i.e., $by+ax=0$

81. (6)

The slope of the chord is $m=-\frac{8}{y}$

$\Rightarrow y=\pm1,\pm2,\pm4,\pm8$

But $(8,y)$ must also lie inside the circle $x^2+y^2=125$

$\Rightarrow$ y can be equal to $\pm1,\pm2,\pm4$

$\Rightarrow$ 6 values

82. (9)

For $x^2+y^2=9$, the centre $=(0,0)$ and the radius = 3 For $x^2+y^2-8x-6y+n^2=0$,

the centre $=(4,3)$ and the radius $=\sqrt{4^2+3^2-n^2}$

$\therefore 4^2+3^2-n^2>0$ or $-5<n<5$

Circles should cut to have exactly two common tangents.

So, $r_1+r_2>d$ [distance between centres]

$\therefore 3+\sqrt{25-n^2}>\sqrt{4^2+3^2}$

or $\sqrt{25-n^2}>2$

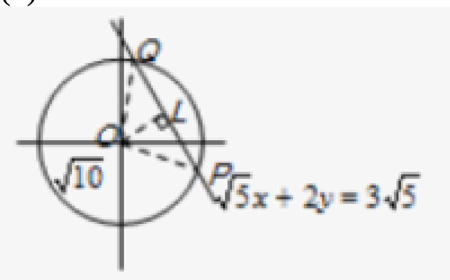
or $25-n^2>4$

$\therefore n^2<21$ or $-\sqrt{21}<n<\sqrt{21}$

Therefore, common values of n should satisfy $-\sqrt{21}<n<\sqrt{21}$.

But $n\in Z$. so, $n=-4,-3,\dots,4$

83. (5)



Length of perpendicular from origin to the line

$x\sqrt{5}+2y=3\sqrt{5}$ is

OL = $\dfrac{3\sqrt{5}}{\sqrt{\left(\sqrt{5}\right)^2+2^2}}=\dfrac{3\sqrt{5}}{\sqrt{9}}$

Radius of the given circle = $\sqrt{10}=OQ=OP$

$\Rightarrow PQ=2QL=2\sqrt{OQ^2-OL^2}$

$=2\sqrt{10-5}=2\sqrt{5}$

Thus, area of $\Delta OPQ=\frac{1}{2}\times PQ\times OL$

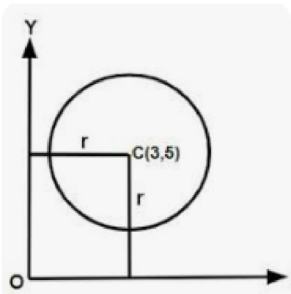
$\frac{1}{2}\times 2\sqrt{5}\times\sqrt{5}=5$

84. (3)

The equation of the circle is

$x^2+y^2-6x-10y+k=0$ (1)

Its centre is $C(3,5)$ and radius r is $\sqrt{34-k}$.



If the circle dose not touch or interest the x-axis, then radius $r < y$ - coordinate of centre C

Or $\sqrt{(34-k)} < 5$

$\Rightarrow 34-k < 25 \Rightarrow k > 9$ (2)

Also if the circle does not touch or intersect the y – axis, the radius $r < x$ - coordinate of centre

Or $\sqrt{(34-k)} < 3$

$\Rightarrow 34-k < 9 \Rightarrow k > 25$ (3)

If the point $(1,4)$ is inside the circle, then its distance from centre $C < r$ (radius),

Or $\sqrt{\left[(3-1)^2+(5-4)^2\right]} < \sqrt{(34-k)}$

$\Rightarrow 5 < 34-k \Rightarrow k < 29$ (4)

Now all the conditions $(2),(3)$, and (4) are satisfied if $25 < k < 29$.

Hence, the required range of the value of k is $25 < k < 29$.

85. (6)

The equation of the common chord is $S_1 - S_2 = 0$

$\Rightarrow 2x-2y=0, \text{i.e.}, x-y=0$

Since the length of perpendicular drawn from C_1 to

$x-y=0$ is $1\sqrt{2}$, length of common chord = $2\sqrt{\frac{19}{2}-\frac{1}{2}}$

$=6$

86. (2)

The equation of line PQ is $(y-1)=m(x-4)$ or $y-mx+4m-1=0$.

For the required m, we have to make sure that the line PQ meets the circle having diameter AB at real and distinct points.

The equation of the circle having AB as diameter is Thus we have to make sure that

$\frac{|0-m+4m-1|}{\sqrt{1+m^2}} < 2$

$\Rightarrow 5m^2-6m-3>0$

$\Rightarrow m \in \left(\frac{3-2\sqrt{6}}{5}, \frac{3+2\sqrt{6}}{5}\right)$

87. (5)

Let the equation of the required circle be $x^2+y^2+2gx+2fy+c=0$.

The condition for orthogonal intersection is

$2g\times 0+2f\times 0=-1+c \Rightarrow 1$

And $2\times 4\times g+2\times 4\times f$

$=-33+c$

$=-33+1$

$=-32$

$\Rightarrow g+f=-4.$

Hence, radius $=\sqrt{g^2+f^2-c}$

$\sqrt{g^2+(g+4)^2-1}$

$\text{i.e.}, r=\sqrt{2g^2+8g}+15$

$\sqrt{2(g+2)^2+7}$

For minimum r,

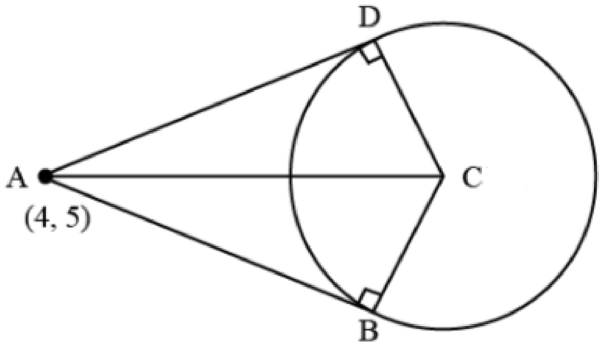
$g+2=0 \Rightarrow g=-2.$

Hence, the centre is $(2,2)$

88. (8)

The equation of the circle is $x^2+y^2-4x-2y-11=0.$

Its centre is $(2,1)$ and radius = $\sqrt{4+1+1}=4=BC.$



Length of the tangent from the point $(4,5)$ is

$\sqrt{16+25-16-10-11}=\sqrt{4}=2=AB.$

Area of quadrilateral ABCD

$=2(\text{Area of } \Delta ABC)$

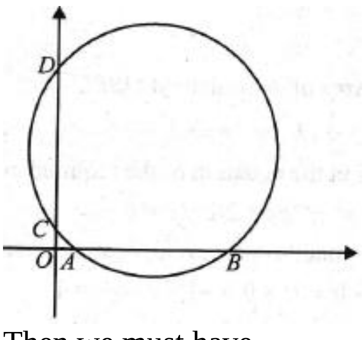
$=2\times\frac{1}{2}\times AB\times BC$

$=2\times\frac{1}{2}\times 2\times 4$

$=8$ sq.units

89. (2)

The given line are $\lambda x-y+1=0$ and $x-2y+3=0,$ which meet x- axis at $A(-1/\lambda,0)$ and $B(-3,0)$ and y-axis at $C(0,1)$ and $D(0,3/2),$ respectively.



Then we must have,

$OA\times OB=OC\times OD$

$\Rightarrow\left(-\frac{1}{\lambda}\right)(-3)=1\times\frac{3}{2}$

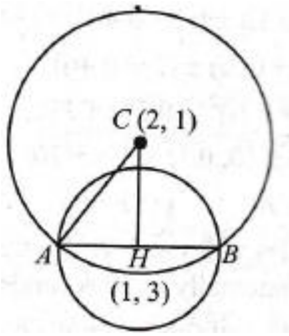
$\Rightarrow \lambda=2$

90. (3)

The given circle is $x^2+y^2-2x-6y+6=0.$

Its centre is $H(1,3)$ and radius is 2.

So $AH = 2$



Radius of the required circle

$= AC = \sqrt{AH^2 + CH^2}$

$= \sqrt{2^2 + 5} = 3$