IIT – JEE: 2025 ADVANCED | **MAJOR TEST - 4** | **DATE: 28/04/24**

ANSWER KEY

PAPER – 1 (Code – 11)

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	B	**19.**	B	**37.**	B
2.	B	**20.**	C	**38.**	B
3.	B	**21.**	A	**39.**	B
4.	A	**22.**	A	**40.**	C
5.	B	**23.**	B	**41.**	B
6.	A	**24.**	D	**42.**	A
7.	C	**25.**	ABC	**43.**	ABC
8.	AC	**26.**	ABC	**44.**	ABCD
9.	AC	**27.**	AC	**45.**	BD
10.	ABD	**28.**	ABC	**46.**	BCD
11.	BC	**29.**	AB	**47.**	AC
12.	BC	**30.**	ACD	**48.**	AB
13.	7.00	**31.**	6.98 - 7.01	**49.**	1.07 – 1.10
14.	1.58	**32.**	209.71 - 209.75	**50.**	0.50
15.	1.00	**33.**	0.05	**51.**	0.51 – 0.54
16.	4.00	**34.**	14.00	**52.**	25.30
17.	4.00	**35.**	5.00	**53.**	1.00
18.	3.00	**36.**	Bonus	**54.**	1.10

PART (A) : PHYSICS

1. (B)

Rate of cooling = Cooling by radiation per second + Cooling by conduction per second.

$$\therefore \quad -\frac{dT}{dt} = K_1(T - T_A) + \frac{KA}{LC}(T - T_A)$$

$$\Rightarrow \quad \int_{T_1}^{T_2} \frac{dT}{(T - T_A)} = -\left(K_1 + \frac{KA}{LC}\right)\int_0^t dt$$

$$\Rightarrow \quad \log\left(\frac{T_2 - T_A}{T_1 - T_A}\right) = -\left(K_1 + \frac{KA}{LC}\right)\cdot t$$

Note In conduction,

$$\frac{dQ}{dt} = \frac{KA(T - T_A)}{L} = C\left(-\frac{dT}{dt}\right)$$

$$\therefore \quad -\frac{dT}{dt} = \frac{KA}{Lc}(T - T_A)$$

2. (B)

Efficiencies of AB,AC and AD are

$$1 - \frac{T_B}{T_A}, 1 - \frac{T_C}{T_A} \text{ and } 1 - \frac{T_D}{T_A}$$

As, per given condition,

$$\eta_{AC} = \frac{1}{2}(\eta_{AB} + \eta_{AD})$$

$$1 - \frac{T_C}{T_A} = \frac{1}{2}\left\{\left(1 - \frac{T_B}{T_A}\right) + \left(1 - \frac{T_D}{T_A}\right)\right\}$$

$$1 - \frac{T_C}{T_A} = 1 - \frac{T_B + T_D}{2T_A}$$

$$\Rightarrow \quad T_C = \frac{T_B + T_D}{2}$$

3. (B)

Least count of screw gauge = $\frac{1\text{mm}}{100} = 0.01\text{mm}$

Diameter of wire = $(1 + 47 \times 0.01)\text{mm} = 1.47\text{mm}$

Curved surface area $(\text{in cm}^2) = (2\pi)\left(\frac{d}{2}\right)(L)$

Or $\quad S = \pi dL$

$$= (\pi)(1.47 \times 10^{-1})(5.6)\text{cm}^2$$

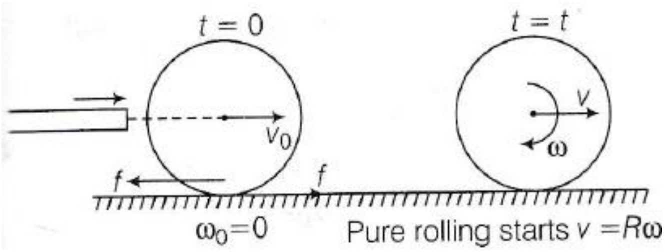
Rounding off to two significant digits,

$$S = 2.6\,\text{cm}^2$$

4. (A)

$f = -ma \quad \Rightarrow \quad \mu mg = -ma$

$\Rightarrow \quad a = -\mu g$



$fR = I\alpha$

$\Rightarrow \quad \mu mgR = \frac{2}{5}mR^2\alpha \quad \Rightarrow \quad \alpha = \frac{5\mu g}{2R}$

$v = u + at$

$\Rightarrow \quad v = v_0 - \mu gt$

$\omega = \omega_0 + \alpha t$

$\Rightarrow \quad \omega = 0 + \frac{5\mu g}{2R}t = \frac{5\mu gt}{2R}$

$v = R\omega$

$\Rightarrow \quad (v_0 - \mu gt) = R\left(\frac{5\mu gt}{2R}\right) \Rightarrow \mu gt\left(1 + \frac{5}{2}\right) = v_0$

$t = \frac{2v_0}{7\mu g}$

$s = ut + \frac{1}{2}at^2$

$\Rightarrow \quad s = v_0 \cdot \frac{2v_0}{7\mu g} - \frac{1}{2}\mu g \cdot \frac{4v_0^2}{49\mu^2 g^2}$

$s = \frac{2v_0^2}{7\mu g} - \frac{2v_0^2}{49\mu g} = \frac{12v_0^2}{49\mu g}$

5. (B)

Suppose the wire vibrates at 420 Hz in its nth harmonic and at 490 Hz in its (n + 1)th harmonic.

$420\,s^{-1} = \frac{n}{2L}\sqrt{\frac{F}{\mu}}$ ….. (i)

and $490\,s^{-1} = \frac{(n+1)}{2L}\sqrt{\frac{F}{\mu}}$ ….. (ii)

This gives $\frac{490}{420} = \frac{n+1}{n}$ or, n = 6.

Putting the value in (i),

$420\,s^{-1} = \frac{6}{2L}\sqrt{\frac{450\,N}{5.0\times10^{-3}\,kg/m}} = \frac{900}{L}\,m/s$

or, $L = \frac{900}{420}$ $\quad L = 2.1\,m$

6. (A)
From conservation of angular momentum,

$$mv_0 l_0 = mvr \Rightarrow v = \frac{v_0 l_0}{r}$$

From conservation of mechanical energy

$$\frac{1}{2}mv_0^2 = -\frac{GM_s m}{r} + \frac{1}{2}m\left(\frac{v_0 l_0}{r}\right)^2$$

$$v_0^2 = \frac{-2GM_s r + v_0^2 l_0^2}{r^2}$$

$$\Rightarrow \quad \left(v_0^2\right)r^2 + \left(2GM_s\right)r - \left(v_0^2 l_0^2\right) = 0$$

$$r = \frac{-2GM_s \pm \sqrt{4G^2M_s^2 + 4\left(v_0^2\right)\left(v_0^2 l_0^2\right)}}{2v_0^2}$$

Neglecting negative sign.

$$r_{min} = \frac{GM_s}{v_0^2}\left[-1 + \sqrt{1 + \left(\frac{v_0^2 l_0}{GM_s}\right)^2}\right]$$

Or $$r_{mim} = \frac{GM_s}{v_0^2}\left[\sqrt{1 + \left(\frac{v_0^2 l_0}{GM_s}\right)^2} - 1\right]$$

7. (C)
Electric field between the two charges is negative.
So, q_2 will be positive and q_1 will be negative.
Since, electric field is zero at a point closer to q_2, $|q_2| < |q_1|$.

8. (AC)

$$e = \left|\frac{\vec{v}\cdot\left(\vec{v}-\vec{u}\right)}{\vec{u}\cdot\left(\vec{v}-\vec{u}\right)}\right|$$

$$\hat{n} = \frac{\left(\vec{v}-\vec{u}\right)}{\left|\vec{v}-\vec{u}\right|}$$

9. (AC)

For B, $0.9 = \frac{u_B \sin 37^\circ}{g}$

$\therefore \quad u_B = \frac{0.9 \times 10}{3/5} = 15\ ms^{-1}$

The time of flight of u_A should be greater than 0.9 s.

$$\because \quad T_A = \frac{2u_A}{g}$$

But $\quad T_A > 0.9 \Rightarrow \frac{2u_A}{g} > 0.9$

$\therefore \quad u_A > \frac{0.9\times 10}{2}$

$\therefore \quad u_A > 4.5 m/s$

10. (ABD)

$m_{Ag} - t = m_A a \quad$ or $\quad 10 - T = a$

And $T + m_B g \sin 37^\circ = m_B a$ or $\quad T + 6 = a = 10 - T$

$\therefore \quad T = 2N$

Also, $\quad 10 - T = a$

$a = 8 m/s^2$

$a_{rel} = \sqrt{a_A^2 + a_B^2 - 2a_A a_B \cos 53^\circ}$

$= \sqrt{8^2 + 8^2 - 2.8.8 \times \frac{3}{5}} = \sqrt{51.2}$

11. (BC)

Here, $T = mg\cos\theta + \frac{mv^2}{l}$

$= mg\cos\theta + \frac{m}{l}\left(v_0^2 + 2gh\right)$

$mg\cos\theta + \frac{m}{l}\left(v_0^2 + 2gl\cos\theta\right)$

Or $\quad 20 = mg\cos\theta + \frac{mv_0^2}{l} + 2mg\cos\theta$

(at the time breaking)

Or $\quad 20 = 10\cos\theta + \frac{25}{2} + 20\cos\theta$

Or $\quad 30\cos\theta = \frac{15}{2}$

$\therefore \quad \theta = \cos^{-1}\left(\frac{1}{4}\right)$

12. (BC)

$pV = nRT$

$\left(p_{atm} + \frac{mg}{A}\right)Ah = nRT \Rightarrow n = \frac{\left(p_{atm} + \frac{mg}{A}\right)Ah}{RT}$

Also, gas on heating lifts piston against gravity, spring and atm pressure.

$\therefore \quad W_{gas} = p_{atm}\cdot A\cdot x + \frac{1}{2}kx^2 + mgx$

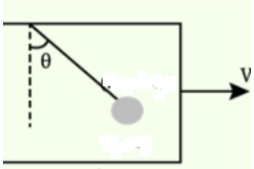
13. (7.00)

$$H=\frac{u^2\sin^2\theta}{2g}=\frac{(20)^2\times(3/4)}{2(10)}=15m$$

i.e. the shell strikes the ball at highest point of its trajectory.
Velocity of (ball + shell) just after Collison, using conservation of momentum is

$$v=\frac{u\cos 60^\circ}{2}=\frac{20(1/2)}{2}=5ms^{-1}$$

Let v' be the velocity of the trolley, when combined reaches highest point. Then,



$$2\times5=(2+18)v'$$

$$v'=\frac{1}{2}ms^{-1}$$

By conservation of mechanical energy, we get

$$\frac{1}{2}(2)(5)^2-\frac{1}{2}(2+18)\left(\frac{1}{2}\right)^2=(2)(10)(2)(1-\cos\theta)$$

$$25-2.5=40(1-\cos\theta)$$

$\Rightarrow$ $$22.5=40(1-\cos\theta)$$

$\Rightarrow$ $$\frac{9}{16}=1-\cos\theta=\cos\theta=\frac{7}{16}$$

14. (1.58)

On m (force equation)

$T-50\times10 = 50\times2 \Rightarrow T=600N$

On cylinder (Torque)

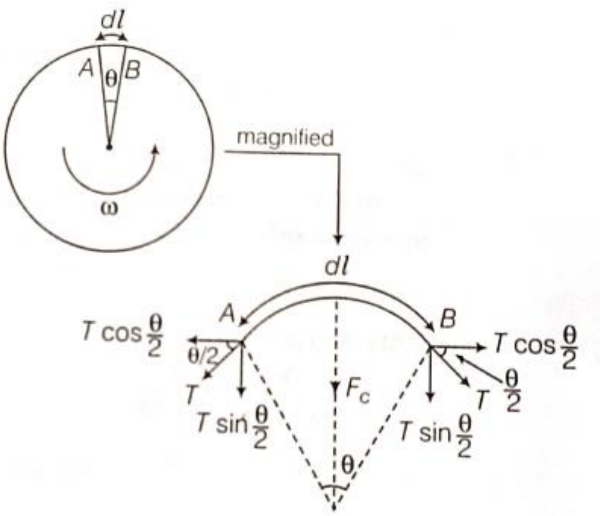
$$F\times0.1-T\times0.25=1\times\frac{2}{0.25} \quad \left(\therefore \propto=\frac{a}{R}\right)$$

$\Rightarrow$ $F=1580$

$\therefore$ $F=1.58\,kN$

15. (1.00)

Consider an element AB of length AB of length dl. Let T be the tension and a be the area of cross – section of the wire.



Mass of the element is dm = (volume)(Density) = $(a\,dl)\rho$

The component of T, i.e $\left(T\sin\frac{\theta}{2}+T\sin\frac{\theta}{2}\right)$ provides requires centripetal force to the element consider.

$\therefore \quad F_C = 2T\sin\frac{\theta}{2}$

$$(dm)R\omega^2 = 2T\left(\frac{\theta}{2}\right) \qquad \left(\because \theta \text{ is small, } \sin\frac{\theta}{2}\approx\frac{\theta}{2}\right)$$

$$T = \frac{(adl)\rho R\omega^2}{\theta}$$

$$\theta = \frac{dl}{R}$$

$\therefore \quad T = aR^2\rho\omega^2 \qquad$ (i)

Let ΔR be the increase in radius of the ring.

Longitudinal strain $= \frac{\Delta l}{l} = \frac{\Delta(2\pi R)}{2\pi R} = \frac{\Delta R}{R}$

$$Y = \frac{T/a}{\Delta R/R} \Rightarrow \Delta R = \frac{TR}{aY}$$

From Eq. (i) $\quad \Delta R = \frac{R}{aY}\left(aR^2\rho\omega^2\right)$

$\therefore \quad \Delta R = \frac{\rho R^3\omega^2}{Y}$

$\Rightarrow \quad x = 1$

16. (4.00)

As, $\quad \frac{95-90}{30} = K\left(\frac{95+90}{2}-\theta_0\right)$

And $\quad \frac{55-50}{70} = K\left(\frac{55+50}{2}-\theta_0\right)$

$\therefore \quad \frac{7}{3} = \frac{92.5-\theta_0}{52.5-\theta_0}$

So, $\theta_0 = 22.5$

If liquid cools from $50^\circ C$ to $45^\circ C$ in t second, then

$$\frac{50-45}{t} = K\left(\frac{50-45}{2} - \theta_0\right)$$

Using $\theta_0 = 22.5$ and dividing, we get

$$\frac{t}{30} = \frac{92.5-22.5}{47.5-22.5} \Rightarrow t = 84\,s = 4\times(21)s$$

$\therefore$ k = 4

17. (4.00)

E at AB $= \frac{\alpha}{\ell}(\ell+\ell) = 2\alpha$ $\quad\therefore$ F on AB $= 2\alpha\lambda\ell$.

E at CD $= \frac{\alpha}{\ell}(2\ell+\ell) = 3\alpha$ $\quad\therefore$ F on CD $= 3\alpha\lambda\ell$

on BC & AD electric field is non uniform since x is not constant. But on BC & AD electric field will have the same type of variation.

$$\therefore F_{AD} = F_{BC} = \int_{x=\ell}^{2\ell} (\lambda dx)\cdot\frac{\alpha}{\ell}(x+\ell)$$

$$= \frac{\alpha\lambda}{\ell}\left[\frac{x^2}{2} + \ell x\right]_{\ell}^{2\ell} = \frac{\alpha\lambda}{\ell}\left[\frac{3\ell^2}{2} + \ell^2\right] = \frac{5}{2}\alpha\lambda\ell$$

$$= 2\alpha\lambda\ell + 3\alpha\lambda\ell + 2\left(\frac{5}{2}\alpha\lambda\ell\right)$$

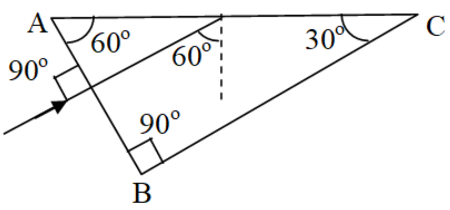
$\therefore$ Total force on the loop $F = 10\alpha\lambda\ell$

Using value $F = 4\times10^{-6}\,N$

18. (3.00)

Critical angle between glass and liquid is

$$\sin\theta_c = \frac{\mu}{(3/2)} = \frac{2\mu}{3}$$



Angle of incidence on AC is 60°

For TIR, $i > \theta_c \Rightarrow \sin 60^\circ = \frac{2}{3}\mu$

$$\mu < \frac{3\sqrt{3}}{4} = \frac{I\sqrt{3}}{4}$$ (given) so I = 3.

PART (B) : CHEMISTRY

19. (B)

$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$

$K_c = \frac{(0.12)^2}{(0.12)^2 \times 5} = 0.2$

Another vessel

	$2SO_2(g)$	$+ O_2(g)$	$\rightleftharpoons 2SO_3(g)$
Moles at eqm.	0.5-2x	y-x	2x

As per given $2x = \frac{20}{100} \times 0.5 = 0.1$

$K_c = \frac{(0.1)^2}{(0.4)^2 (y - 0.05)} = 0.20$

$y = 0.3625$ mole

$\therefore$ mass of O_2 added = 11.6 g

20. (C)

Non-superimposable mirror image.

21. (A)

A

$n = \frac{PV}{RT} = \frac{6 \times 2}{0.08 \times 300} = 0.5 \text{mol}$

$q = 0$

$\Delta U = q + W$

$n \times 3R \times (T_2 - T_1) = -P_{ext}(8 - 2)$

$0.5 \times 3 \times 0.08 (T_2 - 300) = -1\text{atm} \times 6\text{L}$

$(T_2 - 300) = -50$

$T_2 = 250$

Now,

$2L, 300\,K \longrightarrow 8L, 300\,K \longrightarrow 8L, 250\,K$

$\Delta s = nR \ln \frac{8}{2}$ | $\Delta s = nC_V \ln \frac{250}{300}$

$= 0.5 \times 8.0 \times \ln 4$ | $= 0.5 \times 3R \ln \frac{5}{6}$

$= 8 \ln 2$ | $= 12 \ln \frac{5}{6}$

$= 8 \times 2.3 \times 0.3$ | $= 12[\ln 10 - \ln 3 - \ln 4]$

$= 12 \times 2.3[\log 10 - \log 3 - \log 4]$

$= 2.4 \times 2.3$ | $= 12 \times 2.3 \times [-0.08]$

$= 5.52$ | $= -2.3 \times 0.96$

$= -2.208$ $\quad \Delta S_{Total} = -3.3$

22. (A)

oxidation half – cell reaction

$Ag(s) \rightarrow Ag^{+}(aq) + e^{-}$

Reduction half – cell reaction

$AgCl(s) + e^{-} \longrightarrow Ag(s) + Cl^{-}(aq)$

Net cell Reaction : –

$AgCl(s) \longrightarrow Ag^{+}(aq) + Cl^{-}(aq)$

At equilibrium : –

$$E_{cell} = 0 \Rightarrow E^{0}_{cell} = \frac{RT}{nF} \ell n\, K_{sp}$$

and $E^{0}_{cell} = E^{0}_{Cl^{-}/AgCl/Ag} - E^{0}_{Ag^{+}/Ag}$

$= -0.57V$

$K_{sp} = 10^{-10} \times \sqrt{10}$

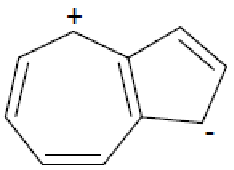
$= 3.16 \times 10^{-10}$

23. (B)
More resonance here implies more length.

24. (D)
More resonance of lone pair of N less is the basic strength.

25. (ABC)

ABC



26. (ABC)
Acid Base Reactions

27. (AC)
In BF_3 due to $p\pi - p\pi$ back bond and existence of pi-bond in CO_2.

28. (ABC)

0.005mole NaOH+0.00235mole Na_2CO_3(0.25/106)

so H+ required = 0.0073 mole

$0.00735 = x \times 15 \times 10^{-3}$

$x = 0.49$

0.5gm (0.0059 mol)$NaHCO_3$ is added. so total mole of $NaHCO_3$ =0.00825

$0.00825 = y \times 25.5 \times 10^{-3}$

$y = 0.323$

$\frac{x}{y} = 1.5$

$0.00736 = 0.32 \times V(L)$

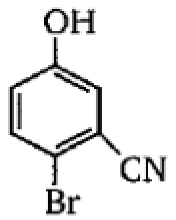
$y = 23ml$

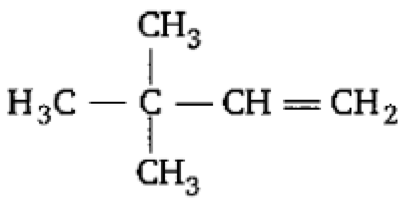
$0.03211 = 0.49 \times V \times 10^{-3}$

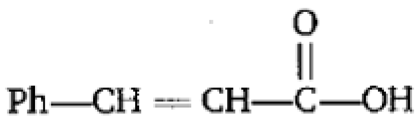
$V = 65.5ml$

29. (AB)

2-Hydroxypropane-1, 2, 3-tricarboxylic acid

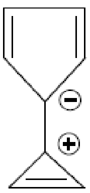




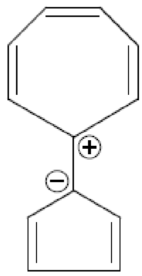


30. (ACD)

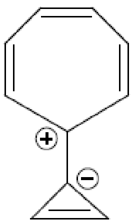
(B) After charge separation due to shifting of π electron

(1) 

2 aromatic ring stabilised by resonance.

(2) 

2 aromatic ring more stable than (1) due to extra resonance

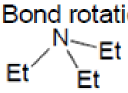
(3) 

1 aromatic ring

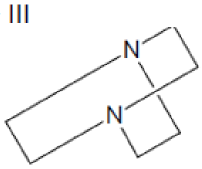
1 anti - aromatic ring required more energy for charge separation

Bond rotation energy is III > I > II

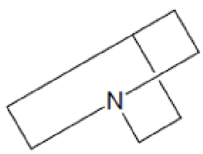
Bond rotation order II > I > III

(C) 

Least basic
due to amine inversion



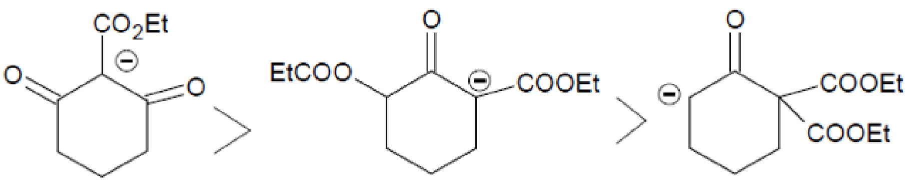
Less basic
due to - I of 2nd N



most basic

No amine inversion

(D) Stability of conjugate base α Acidic nature of conjugate acid



(stability order of C.B.)

31. (6.98 - 7.01)

$Cu^{2+} + e \rightarrow Cu^{+}$

$2I^{-} \rightarrow I_2 + 2e$

$2S_2O_3^{2-} \rightarrow S_4O_6^{-} + 2e$

Meq. of Cu^{2+} = Meq. of liberated I_2

$= Meq.of\ Na_2S_2O_3 = 12.12 \times 0.1 \times 1 = 1.212$

$\therefore w_{Cu^{2+}} = 0.077g = w_{Cu} \qquad \left(Cu \xrightarrow{H_2SO_4} CuSO_4\right)$

$\therefore \%Cu = \frac{0.077}{1.10} \times 100 = 7\%$

32. (209.71 – 209.75)

Given $\Delta E = +0.21 kJ\,mol^{-1} = 0.21 \times 10^3 J\,mol^{-1}$

$P = \bar{1} = 1.0 \times 10^5 Pa$

$= \left(\frac{100}{2.93} - \frac{100}{2.71}\right) cm^3\,mol^{-1}$ of $CaCO_3$

$= -2.77\,cm^3 = -2.77 \times 10^{-6}\,m^3$

$\therefore \Delta H = 0.21 \times 10^3 - 1 \times 10^5 \times 2.77 \times 10^{-6} = 209.72$

33. (0.05)

$$\left[Ag(NH_3)_2\right]^+(aq) \rightleftharpoons \underset{x+y}{Ag^+(aq)} + \underset{2x}{2NH_3(aq)}$$

$$AgCl(s) \rightleftharpoons \underset{x+y}{Ag^+(aq)} + \underset{y}{Cl^-(aq)}$$

In case of simultaneous solubility, Ag^+ remains same in both the equilibrium

$K_c = \frac{(x+y) \times 2^2}{\left[Ag(NH_3)_2\right]^+} \qquad ......(1)$

$K_{sp} = (x+y) \times y \qquad ......(2)$

$\therefore \frac{K_c}{K_{sp}} = \frac{(2x)^2}{\left[Ag(NH_3)_2\right]^+ \times y}$ Given, $[NH_3] = 2x = 1M$

$\left[Ag(NH_3)_2\right]^+ = \left[Cl^-\right] = y$ because Ag^+ obtained from $AgCl$ passes in $\left[Ag(NH_3)_2\right]^+$ state.

$\frac{K_c}{K_{sp}} = \frac{1}{y \times y} \Rightarrow y^2 = \frac{2.378 \times 10^{-10}}{8.2 \times 10^{-8}} = 0.29 \times 10^{-2} \therefore y = 0.539 \times 10^{-1} = 0.0539M$

That is, $\left[Ag(NH_3)_2\right]^+ = 0.0539M$

34. (14.00)
Total 64 stereo isomers and 4 chiral carbon atoms are there.

35. (5.00)

36. (Bonus)

$$\begin{array}{ccc} 1 & 0 & 0 \\ PCl_5 \rightleftharpoons & PCl_3 + & Cl_2 \\ 1-\alpha & \alpha & \alpha \end{array}$$

$$1+\alpha = \frac{D}{d}$$

$$1+\alpha = \frac{250.2}{208.5}$$

$\alpha = 0.2$

Mole of PCl_5 = 0.8, PCl_3 Cl_2 = 0.2

$$\therefore K = \frac{0.2 \times 0.2}{0.8} = \frac{1}{20}$$

Now, when it transferred to 2L vessel reaction proceeds in forward direction.

$$\begin{array}{ccc} PCl_5 \rightleftharpoons & PCl_3 + & Cl_2 \\ 0.8-x & 0.2+x & 0.2+x \end{array}$$

$$\frac{(0.2+x)^2}{(0.8-x)\times 2} = \frac{1}{20}$$

$\therefore x = 0.07$

Degree of dissociation $= \frac{0.07}{0.08} = 0.0875(\text{approx}) = 0.09$

PART (C) : MATHEMATICS

37. (B)

$f[g(x)] = x \Rightarrow f'[g(x)].[g'(x)] = 1$

$\Rightarrow f'(a).g'(2) = 1$ [putting x = 2]

Given, $f'(a) = \dfrac{a^{10}}{1+a^2} \Rightarrow g'(2) = \dfrac{1+a^2}{a^{10}}$

38. (B)

Let $f(x) = ax^2 + bx + c > 0, \forall x \in R$

$\Rightarrow a > 0$

And $b^2 - 4ac < 0$ ...(i)

$\therefore g(x) = f(x) + f'(x) + f''(x)$

$\Rightarrow g(x) = ax^2 + bx + c + 2ax + b + 2a$

$\Rightarrow g(x) = ax^2 + x(b+2a) + (c+b+2a)$

Whose discriminant

$= (b+2a)^2 - 4a(c+b+2a)$

$= b^2 + 4a^2 + 4ab - 4ac - 4ab - 8a^2$

$= b^2 - 4a^2 - 4ac = (b^2 - 4ac) - 4a^2 < 0$ [from Eq. (i)]

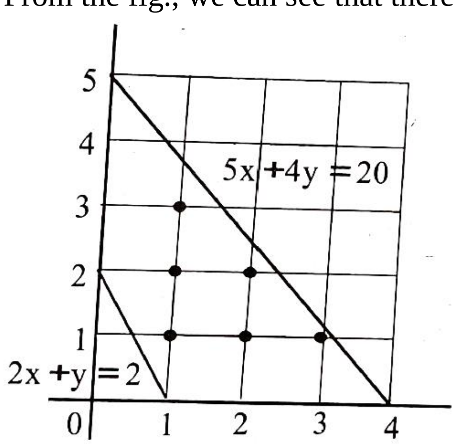
$\therefore g(x) > 0 \forall x$ as $a > 0$ and discriminant < 0.

Thus, $g(x) > 0, \forall x \in R$

39. (B)

40. (C)

From the fig., we can see that there are six points.



41. (B)

$3y^2y' - 3y' = -1 \Rightarrow y' = \dfrac{-1}{3\left(y^2-1\right)}$

$y'\left(-10\sqrt{2}\right) = -\dfrac{1}{21}\left\{\text{as } f\left(-10\sqrt{2}\right) = 2\sqrt{2}\right\}\ 3y^2y'' + 6y\,y'^2 - 3y'' = 0$

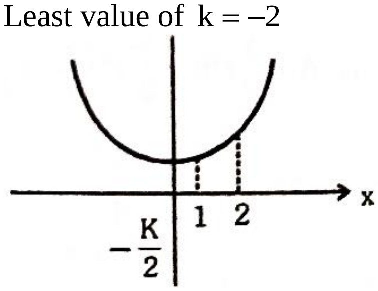
$y''\left(-10\sqrt{2}\right) = -\dfrac{4\sqrt{2}}{7^3 3^2}$

42. (A)

$f(x) = x^2 + kx + 1$ is increasing

$x > -\dfrac{k}{2}$ i.e. $-\dfrac{k}{2} \le 1 \Rightarrow k \ge -2$

Least value of $k = -2$



43. (ABC)

The number of terms $= N = 676$

44. (ABCD)

$-\left({}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots\ldots - {}^{20}C_{13}\right)$

$-\left({}^{19}C_0 - \left({}^{19}C_0 + {}^{19}C_1\right) + \left({}^{19}C_1 + {}^{19}C_2\right)\ldots\ldots\left({}^{19}C_{12} + {}^{19}C_{13}\right)\right) = {}^{19}C_{13} = {}^{19}C_6$

45. (BD)

$2[x+32] = 3[x-64]$

$2[x] + 64 = 3[x] - 3.64$

$[x] = 256;\ [\alpha] = 256$

$\beta = \prod_{r=1}^{9} \sin\left(\dfrac{2r-1}{18}\right)\pi$

$= \sin\dfrac{\pi}{18}\cdot\sin\dfrac{3\pi}{18}\cdot\sin\dfrac{5\pi}{18}\cdot\sin\dfrac{7\pi}{18}\cdot\sin\dfrac{9\pi}{18}\ldots\left(\sin\dfrac{17}{18}\pi\right)$

As $\sin\dfrac{17\pi}{18} = \sin\left(\pi - \dfrac{\pi}{18}\right) = \sin\dfrac{\pi}{18}$

$$\beta = \sin^2\frac{\pi}{18}\sin^2\frac{3\pi}{18}\sin^2\frac{5\pi}{18}\sin^2\frac{7\pi}{18}\sin\frac{9\pi}{18}$$

$$\beta = \left(\sin\frac{\pi}{18}\sin\frac{5\pi}{18}\sin\frac{7\pi}{18}\right)^2\sin\left(\frac{\pi}{2}\right)\sin^2\frac{3\pi}{18}$$

$$\beta = \left(\sin 10^\circ \sin 50^\circ \sin 70^\circ\right)^2\frac{1}{4}$$

$$= \left(\frac{\sin 30^\circ}{4}\right)^2\frac{1}{4}$$

$$= \frac{1}{256}$$

46. (BCD)

$$\cos\frac{x}{2}\cos\frac{x}{2^2}\cos\frac{x}{2^3}.........\cos\frac{x}{2^n}\frac{2\sin\frac{x}{2^n}}{2\sin\frac{x}{2^n}} = \frac{\sin(x)}{2^n\sin\left(\frac{x}{2^n}\right)}$$

$$\lim_{n\to\infty}\frac{\sin x}{2^n\sin\left(\frac{x}{2^n}\right)} = \frac{\sin x}{x}$$

$$f(x) = \begin{cases}\ln\left(\frac{\sin x}{x}\right); & x\in\left(0,\frac{\pi}{2}\right)\\ a; & x = 0\end{cases}$$

$$\lim_{x\to 0}\frac{f(x)}{x} = \lim_{x\to 0}\frac{1}{x}\ln\left(\frac{\sin x}{x}\right) = \lim_{x\to 0}\frac{1}{x}\ln\left(\frac{x-\frac{x^3}{3!}+....}{x}\right)$$

$$= \lim_{x\to 0}\frac{1}{x}\ln\left(1-\frac{x^2}{3!}+....\right) = \lim_{x\to 0}\frac{1}{x}\left(-\frac{x^3}{3!}+....\right) = 0$$

$$\lim_{x\to 0}\frac{f(x)}{x^2} = -\frac{1}{3!} = -\frac{1}{6}$$

$$\lim_{x\to 0}e^{f(x)} = e^0 = 1;\ \text{Also}\ \lim_{x\to 0}f(x) = 0 = a$$

47. (AC)

$$0 < \sin^{-1}x \Rightarrow x\in(0,1]$$

If $x\in(0,1]$ then $\sin^{-1}x\in\left(0,\frac{\pi}{2}\right]$

$$\Rightarrow \ln\left(\sin^{-1}x\right)\in\left(-\infty,\ln\frac{\pi}{2}\right]$$

48. (AB)

$$P = \frac{\lfloor 3 \cdot \lfloor 2}{\lfloor 4} = \frac{1}{2}$$

49. (1.07 – 1.10)

We have, $\cot\left[\sum_{n=1}^{23}\cot^{-1}\left(1+\sum_{k=1}^{n}2k\right)\right]$

$$\Rightarrow \cot\left[\sum_{n=1}^{23}\cot^{-1}(1+2+4+6+8+...+2n)\right]$$

$$\Rightarrow \cot\left[\sum_{n=1}^{23}\cot^{-1}\{1+n(n+1)\}\right]$$

$$\Rightarrow \cot\left[\sum_{n=1}^{23}\tan^{-1}\frac{1}{1+n(n+1)}\right]$$

$$\Rightarrow \cot\left[\sum_{n=1}^{23}\tan^{-1}\left\{\frac{(n+1)-n}{1+n(n+1)}\right\}\right]$$

$$\Rightarrow \cot\left[\sum_{n=1}^{23}\left(\tan^{-1}(n+1)-\tan^{-1}n\right)\right]$$

$$\Rightarrow \cot\left[\left(\tan^{-1}2-\tan^{-1}1\right)+\left(\tan^{-1}3-\tan^{-1}2\right)+\left(\tan^{-1}4-\tan^{-1}3\right)......+\left(\tan^{-1}24-\tan^{-1}23\right)\right]$$

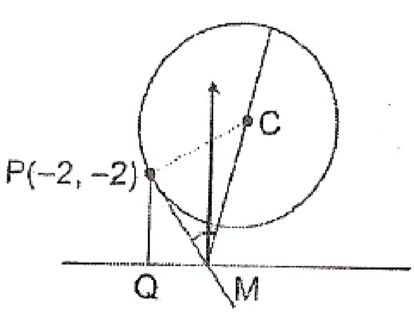
$$\Rightarrow \cot\left(\tan^{-1}24-\tan^{-1}1\right)$$

$$\Rightarrow \cot\left(\tan^{-1}\frac{24-1}{1+24\cdot(1)}\right)=\cot\left(\tan^{-1}\frac{23}{25}\right)$$

$$=\cot\left(\cot^{-1}\frac{25}{23}\right)=\frac{25}{23}$$

50. (0.50)

$$\tan 2\alpha = \frac{PC}{PM} = \frac{5}{PM} \qquad ...(1)$$



$$\sin(90-\alpha) = \frac{PQ}{PM}$$

$PM = \frac{5/2}{\cos\alpha} = \frac{5}{2\cos\alpha}$ ....(2)

$5\cot 2\alpha = \frac{5}{2\cos\alpha}$

$\Rightarrow 10\cot 2\alpha\cos\alpha = 5$

51. (0.51 – 0.54)

$\cos x(3\cos x+2)(2\cos x-1)=0$

$\Rightarrow x=(2m+1)\frac{\pi}{2}\cdot 2n\pi\pm\frac{\pi}{3}, 2k\pi\pm\cos^{-1}\left(\frac{-2}{3}\right)$

Smallest positive roots are

$\frac{\pi}{2}, \frac{\pi}{3}, \cos^{-1}\left(\frac{-2}{3}\right)$

Least difference between positive roots is $\frac{\pi}{6}$

52. (25.30)

$f(x)=23-\frac{16}{2^{(x-1)^2}} \leftarrow [7,23)$

$g(x)=\left[\frac{f(x)}{\mu}\right]$ is discontinuous when $\frac{f(x)}{\mu}$ is an integer

For $\mu\geq 23$ $\quad g(x)=0$ continuous $\forall x\in R$

For $1\leq\mu<23, \frac{f(x)}{\mu}$ becomes integer for some values of x & g(x) becomes discontinuous

$\therefore$ sum of values of $\mu=\frac{22\times 23}{2}=253$

$\Rightarrow A=253$

53. (1.00)

54. (1.10)

$T_r = {}^rC_2 - r = \frac{r^2-r}{2} - r = \frac{r^2}{2}-\frac{3r}{2}$

$\Sigma T_r = \sum_{r=5}^{10}\frac{r^2}{2}-\frac{3}{2}\sum_{r=5}^{10} r$

$=\left(\frac{1}{2}\sum_{r=1}^{10} r^2-\frac{3}{2}\sum_{r=1}^{10} r\right)-\left(\frac{1}{2}\left(\sum_{r=1}^{4} r^2\right)-\frac{3}{2}\left(\sum_{r=1}^{4} r\right)\right)$

$=110$