COMPLEX NUMBER

EXERCISE – 1

Q.1 [B]

$$\Rightarrow \sqrt{-2}\sqrt{-3} = \left(i\sqrt{2}\right)\left(i\sqrt{3}\right) = -\sqrt{6}$$

Q.2 [D]

$$\Rightarrow (1+i)^5(1-i)^5 = \left(1-i^2\right)^5 = 2^5$$

Q.3 [B]

$$(1+i)^4 + (1-i)^4 = \left[(1+i)^2 + (1-i)^2\right]^2 - 2(1+i)^2(1-i)^2$$

$$\Rightarrow \left[(2i)+(-2i)\right]^2 - 2\left(1-i^2\right)^2$$

$$\Rightarrow -2.2^2 = -8$$

Q.4 [C]

$$(1+i)^8 + (1-i)^8 = \left[(1+i)^4 + (1-i)^4\right]^2 - 2(1+i)^4(1-i)^4$$

$$\Rightarrow [-8]^2 - 2\left(1-i^2\right)^4$$

$$\Rightarrow 64 - 2(2)^4 = 32$$

Q.5 [A]

$$(1+i)^6 + (1-i)^6 = \left[(1+i)^3 + (1-i)^3\right]^2 - 2(1+i)^3(1-i)^3$$

$$\Rightarrow \left[1+3i^2+3i+i^3+1-i^3+3i^2-3i\right]^2 - 2\left(1-i^2\right)^3$$

$$\Rightarrow [2-6]^2 - 2(2)^3$$

Q.6 [A]

$$\Rightarrow (1+i)^{10} = \left[(1+i)^2\right]^5 = (2i)^5 = 32i$$

Q.7 [A]

$\Rightarrow 1+i^2+i^3-i^6+i^8 = 1-1-i-i^2+1 = 2-i$

Q.8 [B]

$\because i^4 = 1$

$\Rightarrow \therefore i^{4n+\lambda} = i^{\lambda}$ where $n \in I$; $i^{4n+2} = i^2 = -1$

$\therefore$ given expression will become

$$\Rightarrow \frac{1-1+1-1+1}{-1+1-1+1-1} - 1 = -2$$

Q.9 [D]

For given equation to be true

$(1-i)^n = 2^n$

$\Rightarrow n = 4m;\ m \in I$

$\Rightarrow \min n = 4$

Q.10 [A]

$$\left(\frac{-1+i}{1+i}\right)^n = \text{Real number}$$

$$\Rightarrow \left(\frac{-1+i}{1+i}\right)^n \left(\frac{1-i}{1-i}\right)^n = \frac{(1-i)^{2n}(-1)^n}{(1+1)^n} = \frac{(1+i^2-2i)^n(-1)^n}{2^n}$$

$$\Rightarrow \frac{2^n i^n}{2^n} = i^n$$

least n = 2

Q.11 [B]

$(a+ib)^5 = \alpha + i\beta$

$\Rightarrow i^5(-ai+b)^5 = \alpha + i\beta$

$$\Rightarrow (b-ai)^5 = \beta - \alpha i$$

Take complex conjugate then

$$\Rightarrow (b+ai)^5 = \beta + \alpha i$$

Q.12 [B]

$$\frac{1+2i}{1-i}$$

$$\Rightarrow \frac{1+2i}{1-i} \times \frac{1+i}{1+i}$$

$$\Rightarrow \frac{1+3i+2i^2}{2}$$

$$\Rightarrow \frac{-1+3i}{2} \qquad 2^{nd} \text{ quadrate}$$

Q.13 [A]

$$|z| = 1,\ w = \frac{z-1}{z+1} \qquad (z \neq -1)$$

Let $z = \cos\theta + i\sin\theta$

$$\Rightarrow \therefore w = \frac{(\cos\theta - 1) + i\sin\theta}{(\cos\theta + 1) + i\sin\theta} \neq 1$$

$$\Rightarrow \frac{-2\sin^2\frac{\theta}{2} + 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2} + 2i\sin\frac{\theta}{2} + \cos\frac{\theta}{2}}$$

$$\Rightarrow \frac{2i\sin\frac{\theta}{2}}{2\cos\frac{\theta}{2}}\left(\frac{\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}}{\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}}\right)$$

$$\Rightarrow w = i\tan\frac{\theta}{2}$$

$$\Rightarrow \therefore \text{Re}(w) = 0$$

Q.14 [C]

$$\frac{3+2i\sin\theta}{1-2i\sin\theta}=Ki;K\in R$$

$$\Rightarrow \frac{(3+2i\sin\theta)(1+2i\sin\theta)}{1+4\sin^2\theta}=Ki$$

$$\Rightarrow \therefore \frac{3-4\sin^2\theta}{1+4\sin^2\theta}=0 \ \left(\text{Re al part zero}\right)$$

$$\Rightarrow \sin^2\theta=\left(\frac{\sqrt{3}}{2}\right)^2=\sin^2 60^o$$

$$\Rightarrow \theta=n\pi\pm\frac{\pi}{3}$$

Q.15 [B]

Given $x=\frac{1}{x}$

$\Rightarrow x=\pm1$

Q.16 [C]

z = 1 + i

$\Rightarrow z^2=1+i^2+2i=2i$

Let z_1 is multiplication inverse

$\Rightarrow \therefore z^2 z_1=1$

$$\Rightarrow z_1=\frac{1}{z^2}=\frac{1}{zi}=\frac{-i}{z}$$

Q.17 [B]

$(x+iy)^{\frac{1}{3}}=a+ib$

$\Rightarrow x+iy=(a+ib)^3$

$$\Rightarrow x = a^3 - 3ab^2$$

$$\Rightarrow y = -b^3 + 3a^2 b$$

$$\Rightarrow \therefore \frac{x}{a} + \frac{y}{b} = \left(a^2 - 3b^2\right) + \left(-b^2 + 3a^2\right)$$

$$\Rightarrow 4\left(a^2 - b^2\right)$$

Q.18 [B]

$$\sqrt{3} + i = (a + ib)(c + id)$$

$$\Rightarrow \arg\left(\sqrt{3} + i\right) = \arg\left[(a + ib)(c + id)\right]$$

$$\Rightarrow \arg(a + ib) + \arg(c + id) = \tan^{-1}\frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan^{-1}\frac{b}{a} + \tan^{-1}\frac{d}{c} = \frac{\pi}{6}$$

Q.19 [C]

$$z_1 = 4 + i5$$

$$\Rightarrow z_2 = -3 + 2i$$

$$\Rightarrow \frac{z_1}{z_2} = \frac{4 + 5i}{-3 + 2i} = \frac{(4 + 5i)(-3 - 2i)}{9 + 4} = \frac{-12 + 10 - 15i - 8i}{13}$$

$$\Rightarrow \frac{-2}{13} - \frac{23}{13}i$$

Q.20 [C]

$$x + \frac{1}{x} = 2\cos\theta$$

$$\Rightarrow x^2 - 2x\cos\theta + 1 = 0$$

$$\Rightarrow x = \frac{2\cos\theta \pm \sqrt{4\cos^2\theta - 4}}{2}$$

$\Rightarrow x = \cos\theta \pm i\sin\theta$

Q.21 [A]

$3 - 2yi = 9^x - 7i$

$\Rightarrow 3 = 9^x$; $\quad -2y = -7$

$\Rightarrow x = \frac{1}{2}$

$\Rightarrow y = \frac{7}{2}$

Q.22 [B]

$$\frac{(1+i)x - 2i}{3+i} + \frac{(2-3i)y + i}{3-i} = i$$

$$\Rightarrow \left((1+i)x - 2i(3-i) + \left[(2-3i)y + i\right][3+i]\right) = i(3+1)$$

by comparing real & imaginary parts

$\Rightarrow$ x = 3, y = – 1

Q.23 [B]

$$x + iy = \frac{3}{2+\cos\theta + i\sin\theta} \times \left(\frac{2+\cos\theta - i\sin\theta}{2+\cos\theta - i\sin\theta}\right)$$

$$\Rightarrow \frac{6 + 2\cos\theta - 3i\sin\theta}{\left(4 + \cos^2\theta + 4\cos\theta + \sin^2\theta\right)}$$

$$\Rightarrow x = \frac{2(3+\cos\theta)}{5+4\cos\theta},\ y = \frac{-3\sin\theta}{5+4\cos\theta}$$

$$\Rightarrow x^2 + y^2 = \frac{4\left(9 + \cos^2\theta + 6\cos\theta\right) + 9\sin^2\theta}{(5+4\cos\theta)^2} = \frac{40 + 2y\cos\theta + 5\sin^2\theta}{(5+4\cos\theta)^2}$$

$$\Rightarrow \frac{8(3+\cos\theta)(5+4\cos\theta) - 3(5+4\cos\theta)^2}{(5+4\cos\theta)^2}$$

$\Rightarrow x^2 + y^2 = 4x - 3$

Q.24 [B]

$x = -5 + 2\sqrt{-4} = -5 + 4i$

$\Rightarrow x^2 - (-10)x + (41) = 0$

$\Rightarrow x^2 + 10x + 41 = 0$

$\Rightarrow x^2 = -10x - 41$

$\Rightarrow x^3 = -10x^2 - 41x = -10(-10x - 41) - 41x = 59x + 410$

$\Rightarrow \therefore x^4 + 9x^3 + 35x^2 - x + 4 = x^2(x^2 + 35) + 9x^3 - x + 4$

$\Rightarrow x^2(-10x - 6) + 9x^3 - x + 4$

$\Rightarrow -x^3 - 6x^2 - x + 4$

$\Rightarrow -59x + 410 + 60x + 41 \times 6 - x + 4$

$\Rightarrow -41 \times 4 + 4$

$\Rightarrow -160$

Q.25 [B]

(x + iy) (y – i3) = 4 + i

By comparing real & imaginary parts.

$\Rightarrow 2x + 3y = 4$ ....(1)

$\Rightarrow 2y - 3x = 1$ ....(2)

$\Rightarrow \therefore x = \frac{5}{13}, y = \frac{14}{13}$

Q.26 [D]

$z = \frac{1 - i\sin\alpha}{1 + 2i\sin\alpha} = k; \ \ k \in R$

$$\Rightarrow z=\frac{(1-i\sin\alpha)(1-2i\sin\alpha)}{1+4\sin^2\alpha}$$

$$\Rightarrow \therefore \text{Im}(z)=0$$

$$\Rightarrow -2\sin\alpha-\sin\alpha=0$$

$$\Rightarrow \alpha=n\pi;\quad n\in I$$

Q.27 [C]

$$z(2-i)=3+i$$

$$\Rightarrow z=\frac{3+i}{2-i}\times\frac{2+i}{2+i}=\frac{6-1+5i}{5}$$

$$\Rightarrow z=1+i=\sqrt{2}e^{\frac{i\pi}{4}}$$

$$\Rightarrow z^{20}=2^{10}e^{i5\pi}=2^{10}=1024$$

Q.28 [A]

$$\text{Re}\left(\frac{z-8i}{z+6}\right)=0$$

$$\Rightarrow z=\frac{z+i(y-8)}{(x+6)+iy}=\frac{[x+i(y-8)][(x+6)-iy]}{(x+6)^2+y^2}$$

$$\Rightarrow \text{Re}(z)=x(x+6)+y(y-8)=0$$

$$\Rightarrow x^2+y^2+6x-8y=0$$

Q.29 [B]

$$z=\frac{2+5i}{4-3i}\times\frac{4+3i}{4+3i}=\frac{8-15+26i}{25}$$

$$\Rightarrow z=\frac{-7}{25}+\frac{i26}{25}$$

$$\Rightarrow \bar{z}=\frac{-7}{25}-\frac{i26}{25}$$

Q.30 [B]

$z_1 + z_2 = \text{Real}$

$\Rightarrow z_1 z_2 = \text{Real}$

$\Rightarrow \therefore z_1 \,\&\, z_2$ are complex conjugate

$\Rightarrow z_1 = \overline{z_2}$

Q.31 [B]

z = x + iy (in 3rd quadrate)

$\Rightarrow x < 0,\ y < 0$

$\Rightarrow \bar{z} = x - iy = x + i(y)$ 2nd quadrate

Q.32 [A]

$(z+3)(\bar{z}+3)$

$\Rightarrow (z+3)\left(\overline{z+3}\right)$

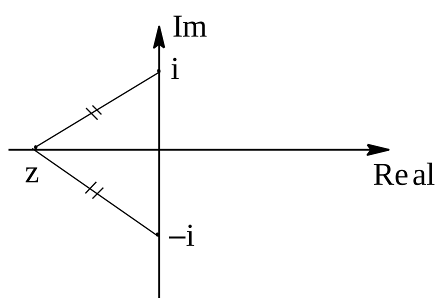
$\Rightarrow |z+3|^2$

Q.33 [A]

$\Rightarrow |z_1 z_2| = |z_1||z_2| = 1$

Q.34 [A]

$|z+1| = |z-i|$



Locus of z wier be Real axis

Q.35 [B]

$$\frac{z-1}{z+1}=ki;k\in R$$

$$\Rightarrow \text{Re}\left[\frac{(x-1)+iy}{(x+1)+iy}\times\frac{(x+1)-iy}{(x+1)-iy}\right]=0$$

$$\Rightarrow (x-1)(x+1)+y^2=0$$

$$\Rightarrow x^2+y^2=1$$

$$\Rightarrow |z|^2=1$$

$$\Rightarrow |z|=1$$

Q.36

$$\Rightarrow |2z-1|+|3z-2|$$

Q.37 [B]

$$|z_1+z_2|^2+|z_1-z_2|^2$$

$$\Rightarrow (z_1+z_2)(\overline{z_1}+\overline{z_2})+(z_1-z_2)(\overline{z_1}-\overline{z_2})$$

$$\Rightarrow 2\left(|z_1|^2+|z_2|^2\right)$$

Q.38

$$\frac{2z_1}{3z_2}=ki;k\in R$$

$$\Rightarrow \frac{2z_1\overline{z_2}}{3|z_2|^2}=ki$$

$$\Rightarrow \therefore \text{Re}\left(z_1\overline{z_2}\right)=0$$

$$\Rightarrow z_1\overline{z_2}+\overline{z_1}z_2=0$$

$$\Rightarrow \therefore \left|\frac{z_1 - z_2}{z_1 + z_2}\right|^2 = \left(\frac{z_1 - z_2}{z_1 + z_2}\right)\left(\frac{\overline{z_1} - \overline{z_2}}{\overline{\overline{z_1} + \overline{z_2}}}\right)$$

$$\Rightarrow \frac{|z_1|^2 + |z_2|^2 - \left(z_1\overline{z_2} + z_2\overline{z_1}\right)}{|z_1|^2 + |z_2|^2 + \left(z_1\overline{z_2} + z_2\overline{z_1}\right)} = \frac{|z_1|^2 + |z_2|^2}{|z_1|^2 + |z_2|^2}$$

Q.39

$$|z_1| = |z_2| = |z_3| = \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$$

$$\Rightarrow |z_1 + z_2 + z_3| = |\overline{z_1} + \overline{z_2} + \overline{z_3}| = \left|\frac{z_1\overline{z_1}}{z_1} + \frac{z_2\overline{z_2}}{z_2} + \frac{z_3\overline{z_3}}{z_3}\right|$$

$$\Rightarrow \left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right| = 1$$

Q.40

$$\left|z_1 + \sqrt{z_1^2 - z_2^2}\right| + \left|z_1 - \sqrt{z_1^2 - z_2^2}\right|$$

Let $z_3 = \sqrt{z_1^2 - z_2^2}$ $\quad z_3^2 = z_1^2 - z_2^2$

$$\Rightarrow \left[|z_1 + z_3| + |z_1 - z_3|\right]^2 = |z_1 + z_3|^2 + |z_1 - z_3|^2 + 2|z_1 + z_3||z_1 - z_3|$$

$$\Rightarrow 2\left(|z_1|^2 + |z_3|^2\right) + 2|z_1^2 - z_3^2|$$

$$\Rightarrow 2\left(|z_1|^2 + |z_3|^2\right) + 2|z_2|^2$$

$$\Rightarrow 2\left(|z_1|^2 + |z_2|^2\right) + 2|z_1^2 - z_2^2|$$

$$\Rightarrow \left(|z_1 - z_2|^2 + |z_1 + z_2|^2\right) + 2|z_1 - z_2||z_1 + z_2|$$

$$\Rightarrow \left[|z_1 - z_2| + |z_1 + z_2|\right]^2$$

Q.41 [C]

$$\left|\frac{z-4}{z-8}\right|=1$$

Locus of Z will be x = 6

$\Rightarrow \therefore z=6+iy$

$$\Rightarrow \left|\frac{z-12}{z-8i}\right|=\frac{5}{3}$$

$\Rightarrow 3\left|-6+iy\right|=5\left|6+i\left(y-8\right)\right|$

$\Rightarrow 9\left(36+y^2\right)=25\left(36+\left(y-8\right)^2\right)$

$\Rightarrow y^2-25y+136=0$

$\Rightarrow$ y = 8, 17

$\Rightarrow \therefore z=6+8i, 6+i17$

Q.42 [A]

$\left|z-4\right|<\left|z-2\right|$

$\Rightarrow \left|z-4\right|^2<\left|z-2\right|^2$

$\Rightarrow \left|z\right|^2-4\left(\bar{z}+z\right)+16<\left|z\right|^2-2\left(z+\bar{z}\right)+4$

$\Rightarrow 2\left(z+\bar{z}\right)>12$

$\Rightarrow 2\left(2x\right)>12$

$\Rightarrow$ x > 3

$\Rightarrow$ Re (z) > 3

Q.43 [B]

$z=1+i\tan\alpha$

$$\Rightarrow \pi<\alpha<\frac{3\pi}{2}$$

$\Rightarrow |z| = \sqrt{1+\tan^2\alpha} = |\sec\alpha|$

$\Rightarrow |z| = -\sec\alpha$ (3rd quadrant)

Q.44 [A, B]

$$\Rightarrow \left|\frac{|\bar{z}|^2}{z\bar{z}}\right| = \left|\frac{|z|^2}{|z|^2}\right| = 1 = \left|\frac{\bar{z}}{z}\right|$$

Q.45 [C]

$$|z_1 + z_2| = \left|\frac{1}{z_1} + \frac{1}{z_2}\right|$$

$$\Rightarrow |z_1 + z_2| = \left|\frac{z_1 + z_2}{z_1 z_2}\right|$$

$$\Rightarrow |z_1 z_2| = 1$$

Q.46 [D]

$$z^2 + |z|^2 = 0$$

$$\Rightarrow z^2 = -|z|^2$$

$$\Rightarrow z = i|z|$$

$\Rightarrow$ Real (z) = 0

$\Rightarrow \therefore z = iy$

So infinite solution.

Q.47

$$\Rightarrow |z| = \max\{|z-2|, |z+2|\}$$

Q.48 [C]

$z = -1 + i\sqrt{3}$ z lies in 2nd quadrant

$$\Rightarrow \therefore \arg(z) = \tan^{-1}\left(-\sqrt{3}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Q.49

$z = -1 - i\sqrt{3}$; z lies in 3rd quadrant

$$\Rightarrow \arg(z) = \pi + \tan^{-1}\left(\sqrt{3}\right) = \frac{4\pi}{3}$$

Q.50 [A]

$$z = \frac{1 + i\sqrt{3}}{\sqrt{3} + i} \times \frac{\sqrt{3} - i}{\sqrt{3} - i} = \frac{\sqrt{3} + \sqrt{3} + 3i - i}{4}$$

$$\Rightarrow z = \frac{\sqrt{3} + i}{2}$$

$$\Rightarrow \arg(z) = \tan^{-1}\frac{1}{\sqrt{3}} = \frac{\pi}{6}$$

Q.51 Repeated Q.50

Q.52

$$z = \frac{13 - 5i}{4 - 9i}$$

$\Rightarrow$ arg (z) = arg (13 – 5i) – arg (4 – 9i)

$$\Rightarrow \tan^{-1}\left(\frac{-5}{13}\right) - \tan^{-1}\left(\frac{-9}{4}\right)$$

$$\Rightarrow \left[-\tan^{-1}\frac{5}{13}\right] - \left[-\tan^{-1}\frac{9}{4}\right]$$

$$\Rightarrow \tan^{-1}\left[\frac{\frac{9}{4} - \frac{5}{13}}{1 + \frac{9}{4} \times \frac{5}{13}}\right] = \tan^{-1}\left(\frac{97}{97}\right) = \frac{\pi}{4}$$

Q.53 [C]

$$z=\frac{1-i\sqrt{3}}{1+i\sqrt{3}}$$

$$\Rightarrow \arg(z)=\arg\left(1-i\sqrt{3}\right)-\arg\left(1+i\sqrt{3}\right)$$

$$\Rightarrow \left(-\frac{\pi}{3}\right)-\left(\frac{\pi}{3}\right)=\frac{-2\pi}{3}=\frac{4\pi}{3}$$

Q.54 [D]

$$z=1-\cos\alpha+i\sin\alpha$$

$$\Rightarrow \operatorname{amp}(z)=\tan^{-1}\left(\frac{\sin\alpha}{1-\cos\alpha}\right)$$

$$\Rightarrow \tan^{-1}\left(\frac{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}{2\sin^{2}\frac{\alpha}{2}}\right)$$

$$\Rightarrow \tan^{-1}\left(\cot\frac{\alpha}{2}\right)=\frac{\pi}{2}-\frac{\alpha}{2}$$

Q.55 [B]

$$z=\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}=1\cdot e^{\frac{i\pi}{6}}$$

$$\Rightarrow |z|=1,\arg(z)=\frac{\pi}{6}$$

Q.56 [B]

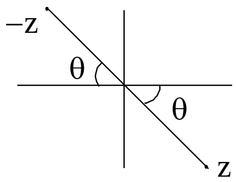
$$\arg(z)=\theta$$

$$\Rightarrow \arg(\bar{z})=-\theta$$

Q.57

arg (z) < 0

Let arg (z) = $-\theta;\ \theta>0$



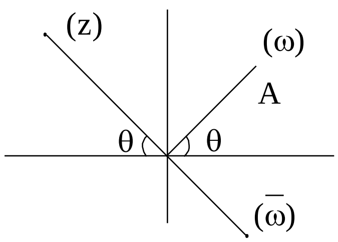
then arg (– z) – arg (z)

$(\pi-\theta)-(-\theta)=\pi$

Q.58 [D]

$|z|=|\omega|$

$\Rightarrow \arg(z)+\arg(\omega)=\pi$



$\Rightarrow \therefore z=-\overline{\omega}$

Q.59

Re(z) < 0

$\Rightarrow$ Im (z) = 0

$\Rightarrow \arg(z)\pi$

Q.60 [D]

if $\arg(z)=\theta$

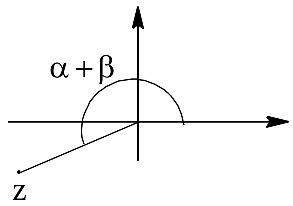
$\Rightarrow$ then $\arg(\overline{z})=-\theta$

Q.61 [C]

$\arg(z_1)=\alpha$

$\Rightarrow \arg(z_2)=\beta$

given $\alpha+\beta>\pi$



$\Rightarrow z_1z_2 = r_1r_2e^{i(\alpha+\beta)}$

Principal argument $\arg(z_1\times z_2) = -(2\pi-\alpha+\beta)$

$\Rightarrow \alpha+\beta-2\pi$

Q.62 [B]

z = – 1

$$\Rightarrow \arg\left(z^{\frac{2}{3}}\right) = \frac{2}{3}\arg(z) = \frac{2}{3}\arg(-1) = \frac{2}{3}\pi$$

Q.63 [B]

z = x + iy

$\Rightarrow$arg (z – 1) = arg (z + 3i)

$$\Rightarrow \tan^{-1}\frac{y}{x-1} = \tan^{-1}\frac{y+3}{x}$$

$$\Rightarrow \therefore \frac{y}{x-1} = \frac{y+3}{x}$$

$\Rightarrow$xy = xy + 3x – y – 3

$$\Rightarrow \frac{x-1}{y} = \frac{1}{3}$$

Q.64 [C]

$(1+i)^n + (1-i)^n$

$$\Rightarrow\left[\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)\right]^{n}+\left[\sqrt{2}\left(\cos\left(\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)\right]^{n}$$

$$\Rightarrow 2^{\frac{n}{2}}\left[2\cos^{n}\frac{\pi}{4}\right]$$

$$\Rightarrow\left(\sqrt{2}\right)^{n+2}\cos\left(\frac{n\pi}{4}\right)$$

Q.65 [A]

$$y=\cos\theta+i\sin\theta$$

$$\Rightarrow\frac{1}{y}=\bar{y}=\cos\theta-i\sin\theta$$

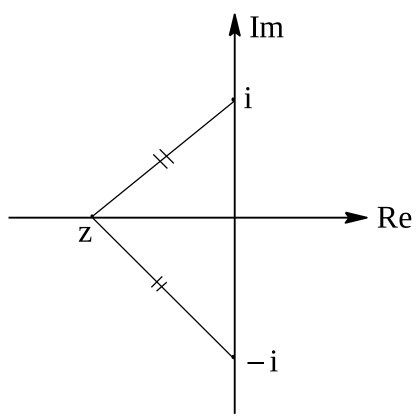
$$\Rightarrow y+\frac{1}{y}=2\cos\theta$$

Complex Number

Exercise – 2

Q.1 [B]

$$w=\frac{1-iz}{z-i}=\frac{-i(z+i)}{(z-i)}$$



$$|w|=\left|\frac{z+i}{z-i}\right|=1$$

$$|z+i|=|z-i|$$

z lies on real axis.

Q.2 [C]

$$|z|=|z_1-z_2|=|-3-i|=5$$

Q.3 [B]

$$z\bar{z}+a\bar{z}+\bar{a}z+b=0;\ b\in R$$

radius of the circle $=|a|^2-b>0$

$$\therefore |a|^2>b$$

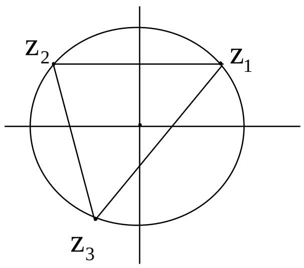
Q.4

$|z_1|=|z_2|=|z_3|=r$ (let's take)

Let $z_1 = re^{i\theta}$

$z_2 = re^{i\left(\theta+\frac{2\pi}{3}\right)}$

$z_3 = re^{i\left(\theta-\frac{2\pi}{3}\right)}$



$\therefore z_1 + z_2 + z_3 = r(0) = 0$

Q.5 [D]

$G\left(\frac{z_1+z_2+z_3}{3}\right) \qquad A(z_1)$

$$\therefore \text{ mid point of AG } z = \frac{\frac{z_1+z_2+z_3}{3}+z_1}{2} = 0$$

$\therefore 4z_1 + z_2 + z_3 = 0$

Q.6 [B]

$|z_1| = 12, \ |z_2 - 3 - 4i| = 5$

$|z_1 - z_2| = |z_1 + (-z_2 + 3 + 4i) + (-3 - 4i)|$

$||z_1| - |z_2 - 3 - 4i|| \le |z_1 + (-z_2 + 3 + 4i)| \le |z_1 - z_2| + |3 + 4i|$

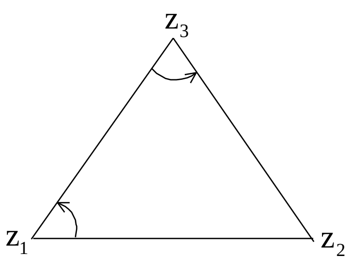
$7 \le |z_1 - z_2| + 5$

$\therefore |z_1 - z_2| \ge 2$

$|z_1 - z_2|_{min} = 2$

Q.7 [B]

For



$$\frac{z_3 - z_1}{z_2 - z_1} = e^{\frac{i\pi}{3}} = \frac{z_2 - z_3}{z_1 - z_3}$$

$$-(z_1 - z_3)^2 = (z_2 - z_3)(z_2 - z_1)$$

$$-(z_1^2 + z_3^2 - 2z_1z_3) = z_2^2 - z_1z_2 - z_2z_3 + z_1z_3$$

$$z_1^2 + z_2^2 + z_3^2 = z_1z_2 + z_2z_3 + z_3z_1$$

Q.8 [C]

Let $\arg(z) = \theta$

Then $\arg(-iz) = \arg(-i) + \arg(z) = \frac{-\pi}{2} + \theta$

$$\therefore \arg(z) - \arg(-iz) = \frac{\pi}{2}$$

Q.9 [C]

$$\text{Re}\left(\frac{z+4}{2z-i}\right) = \frac{1}{2}$$

$$\frac{z+4}{2z-i} + \frac{\bar{z}+4}{2\bar{z}+i} = 1$$

$$(z+4)(2\bar{z}+i) + (\bar{z}+4)(2z-i) = (2z-i)(2\bar{z}+i)$$

$$2|z|^2 + iz + 8\bar{z} + 4i + 2|z|^2 - i\bar{z} + 8z - 4i = 4|z|^2 + 2zi - 2i\bar{z} + 1$$

$$zi - i\bar{z} - 8\bar{z} - 8z + 4i + 1 = 0$$

$$z(i-8) - \bar{z}(8+i) + 4i + 1 = 0$$

$$z(8-i) + \bar{z}(8+i) - 4i - 1 = 0$$

This is equation of straight line

Q.10 [C]

$$\frac{z_1}{z_2} + \frac{z_2}{z_1} = 1$$

$$z_1^2 + z_2^2 = z_1 z_2$$

For equilateral triangle with vertices z_1, z_2, z_3

$$z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$$

if $z_3 = 0$

$$z_1^2 + z_2^2 = z_1 z_2$$

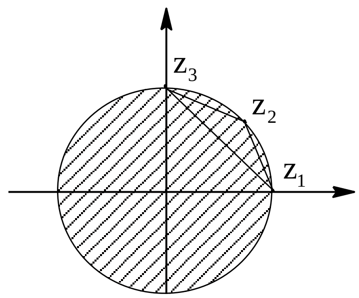
Q.11

$$z_1 = 1, z_2 = \frac{1+i}{\sqrt{2}}, z_3 = i$$

$$|z_1 - z_3| = \sqrt{2}$$

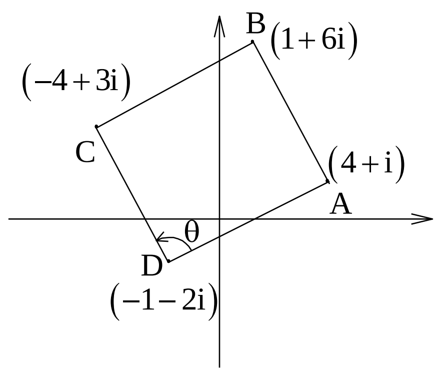
$$|z_2 - z_3| = \left|\frac{1+i}{\sqrt{2}} - i\right| = \left|\frac{1}{\sqrt{2}} + i\left(\frac{1}{\sqrt{2}} - 1\right)\right| = \sqrt{\frac{1}{2} + \frac{1}{2} + 1 - \sqrt{2}} = \sqrt{2 - \sqrt{2}}$$

$$|z_1 - z_2| = \sqrt{\left(1 - \frac{1}{\sqrt{2}}\right)^2 + \frac{1}{2}} = \sqrt{2 - \sqrt{2}}$$



Triangle is isoleces triangle

Q.12 [B]



$$|AB| = |BC| = |CD| = |DA|$$

$$\frac{Z_C - Z_D}{Z_A - Z_D} = \frac{-3+5i}{5+3i} = i = e^{\frac{i\pi}{2}}$$

$$\therefore \theta = \frac{\pi}{2}$$

$\therefore$ ABCD is square

Q.13 [B]

90^o

Q.14 [C]

By triangle inequality

$$\left||z_1| - |z_2|\right| \le |z_1 + z_2| \le |z_1| + |z_2|$$

Q.15

Q.16

$(z_1 - z_2) = \lambda(z_2 - z_3)$ for collinear

$$(3-2i) = \lambda\left(-2+i\left(3-\frac{a}{3}\right)\right)$$

$$3 = -2\lambda$$

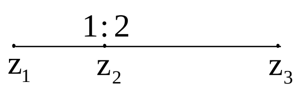
$$-2 = \lambda\left(3-\frac{a}{3}\right)$$

$$-2 = \frac{-3}{2}\left(3-\frac{a}{3}\right)$$

a = 5

Q.17 [B]

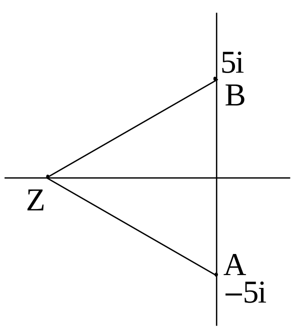
$$2z_1 - 3z_2 + z_3 = 0$$



$$z_2 = \frac{2z_1 + z_3}{2+1}$$

Collinear points.

Q.18 [A]



$$|Z-5i| = |Z+5i|$$

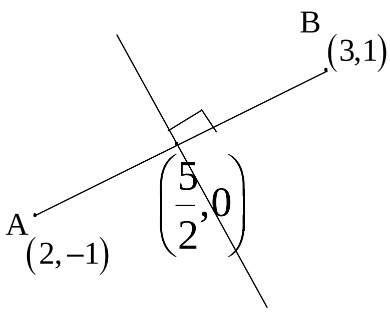
So locus of z will perpendicular bisector of AB or z lies on real axis.

$\therefore x = 0$

Q.19 [A]

$$|Z-(2-i)|=|Z-(3+i)|$$

Locus of z will be perpendicular bisector of line segment AB. A(2, – 1), B (3, 1)



$$m_{AB}=\frac{1+1}{1}=2$$

$\therefore$ locus of z is

$$y-0=-\frac{1}{2}\left(x-\frac{5}{2}\right)$$

$$x+2y=\frac{5}{2}$$

Q.20 [D]

$$(Z-2-3i)=\frac{\pi}{4}$$

$$\text{amp}(x-2)+i(y-3)=\frac{\pi}{4}$$

$$\tan^{-1}\left(\frac{y-3}{x-2}\right)=\frac{\pi}{4}$$

y – 3 = x – 2

x – y + 1 = 0

Q.21

$$\arg\left(\frac{z-2}{z+2}\right)=\frac{\pi}{6}$$

$$\arg(z-2)-\arg(z+2)=\frac{\pi}{6}$$

$$\tan^{-1}\left(\frac{y}{x-2}\right)-\tan^{-1}\left(\frac{y}{x+2}\right)=\frac{\pi}{6}$$

$$\tan^{-1}\left[\frac{\left(\frac{y}{x-2}-\frac{y}{x+2}\right)}{1+\left(\frac{y^2}{x^2-4}\right)}\right]=\frac{\pi}{6}$$

$$\frac{yx+2y-xy+2y}{x^2-4+y^2}=\sqrt{3}$$

$$x^2+y^2=4+\frac{4y}{\sqrt{3}}$$

$$x^2+y^2-\frac{4y}{\sqrt{3}}-4=0$$

$$z=\lambda+3+i\sqrt{5-\lambda^2}$$

It is a circle.

Q.22

$$x=\lambda+3 \ \& \ y=\sqrt{5-\lambda^2}$$

$$x=\lambda+3 \& y=\sqrt{5-\lambda^2}$$

$$y^2=5-(x-3)^2$$

$$(x-3)^2+y^2=5$$

Q.23 Repeated Question (21) of Ex. (2)

Q.24 [D]

$$|z-2|=2|z-3|$$

$$(x-2)^2+y^2=4\left((x-3)^2+\left(y^2\right)\right)$$

$$3x^2+3y^2-20x+32=0$$

$$x^2+y^2-\frac{20}{3}x+\frac{32}{3}=0$$

$$r=\sqrt{\left(\frac{10}{3}\right)^2-\frac{32}{3}}=\sqrt{\frac{4}{9}}=\frac{2}{3}$$

Q.25 [A]

$$i^{\frac{1}{3}}=\left(e^{i\frac{\pi}{2}}\right)^{\frac{1}{3}}=e^{i\frac{\pi}{6}}$$

$$\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}$$

$$\frac{\sqrt{3}+i}{2}$$

Q.26 [D]

$$\left(-1+i\sqrt{3}\right)^{20}=\left[2\left(e^{i\frac{2\pi}{3}}\right)\right]^{20}$$

$$2^{20}e^{i\frac{40\pi}{3}}$$

$$1^{20}e^{i\frac{4\pi}{3}}=2^{20}\left(\frac{-1-i\sqrt{3}}{2}\right)$$

Q.27 [A]

$$z=\frac{\sqrt{3}+i}{2}=-i\left(\frac{-1+i\sqrt{3}}{2}\right)$$

$$z=-i\omega$$

$$z^{69}=(-i)^{69}\,\omega^{69}=-i$$

Q.28 [C]

$$(\sin\theta + i\cos\theta)^n = \left[\cos\left(\frac{\pi}{2} - \theta\right) + i\sin\left(\frac{\pi}{2} - \theta\right)\right]^n$$

$$\cos\left(\frac{n\pi}{2} - n\theta\right) + i\sin\left(\frac{n\pi}{2} - n\theta\right)$$

Q.29

$$z = \left(\frac{\cos\pi}{3} + i\sin\frac{\pi}{3}\right)^{\frac{3}{4}} = (\cos\pi + i\sin\pi)^{\frac{1}{4}}$$

$$(-1)^{\frac{1}{4}}$$

$$z^4 = -1$$

$$z = \left(\cos\left(\frac{2k\pi + \pi}{4}\right) + i\sin\left(\frac{2k\pi + \pi}{4}\right)\right)$$

$$z_1 = e^{i\frac{\pi}{4}}$$

$$z_2 = e^{i\frac{3\pi}{4}}$$

$$z_3 = e^{i\frac{5\pi}{4}}$$

$$z_3 = e^{i\frac{7\pi}{4}}$$

Product of roots $= e^{i\left(\frac{\pi}{4} + \frac{3\pi}{4} + \frac{5\pi}{4} + \frac{7\pi}{4}\right)}$

$$e^{i(4\pi)} = 1$$

Q.30 [D]

$$\left(\frac{1 + \cos\theta + i\sin\theta}{i + \sin\theta + i\cos\theta}\right)^n = \cos n\theta + i\sin n\theta$$

$$\left[\frac{2\cos\frac{\theta}{2}\left(e^{i\frac{\theta}{2}}\right)}{2\cos\frac{\theta}{2}\left(\sin\frac{\theta}{2}+i\cos\frac{\theta}{2}\right)}\right]^n$$

$$\left[\frac{e^{i\frac{\theta}{2}}}{ie^{i\frac{\theta}{2}}}\right]^n = \left[-ie^{i\theta}\right]^n$$

$$(-i)^n(\cos n\theta + i\sin\theta)$$

So, n = 4

Q.31 [C]

$$\left[\frac{1-\cos\frac{\pi}{10}+i\sin\frac{\pi}{10}}{1-\cos\frac{\pi}{10}-i\sin\frac{\pi}{10}}\right]^{10} = \left[\frac{2\sin\frac{\pi}{10}\left(\sin\frac{\pi}{20}+icoc\frac{\pi}{20}\right)}{\left(2\sin\frac{\pi}{20}\right)\left(\sin\frac{\pi}{20}-i\cos\frac{\pi}{20}\right)}\right]^{20}$$

$$\left[\frac{i\,e^{-i\frac{\pi}{20}}}{-ie^{i\frac{\pi}{20}}}\right]^{20} = \left[-e^{-i\frac{\pi}{10}}\right]^{20} = e^{-i2\pi}$$

1

Q.32 [D]

$$\sum_{k=1}^{6}\left(\sin\frac{2\pi k}{7}-i\cos\left(\frac{2\pi k}{7}\right)\right)$$

$$\sum_{k=1}^{6}-ie^{i\frac{2\pi k}{7}} \qquad \text{Let } \alpha = e^{i\frac{2\pi}{7}}$$

$$-i\left[\alpha+\alpha^2+\alpha^3+\alpha^4+\alpha^5+\alpha^6\right]$$

– i [–1] = i

Q.33 [B]

$$\cos\left(\theta+\frac{\theta}{2}+\frac{\theta}{2^2}+\frac{\theta}{2^3}+......\right)+\sin\left(\theta+\frac{\theta}{2}+\frac{\theta}{2^2}+\frac{\theta}{2^3}+.......\right)$$

$$\cos(2\theta)+i\sin 2\theta$$

Q.34 [C]

$$z^3=-1$$

$$z=-1,-\omega,-\omega^2$$

where ω is cube root of unity.

$$z_1z_2z_3=(-1)(-\omega)(-\omega^2)=-1$$

Q.35

$$\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)=\left[\frac{2\left(e^{i\frac{\pi}{3}}\right)}{2e^{-i\frac{\pi}{3}}}\right]^n=\left[e^{i\frac{2\pi}{3}}\right]^n$$

$$e^{i\frac{2\pi n}{3}}$$

it will be integer if n = 3.

Q.36 [C]

$$(1+\omega)^7=A+B\omega$$

$$(-\omega^2)^7=A+B\omega$$

$$(-\omega^2)=A+B\omega$$

$$1+\omega=A+B\omega$$

A = 1 = B

Q.37 [C]

$$(3t\omega+3\omega^2)^4=(-3\omega+\omega)^4=(-2\omega)^4=16\omega$$

Q.38 [A]

$$\left(3+\omega^2+\omega^4\right)^6=\left(3+\omega^2+\omega\right)^6=\left(2+0\right)^6=64$$

Q.39 [C]

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1+\omega+\omega^2=0$$

Q.40 [B]

Given question is wrong. Actual question is

$$\left(z+1\right)^3=8\left(z-1\right)^3$$

$$\left(\frac{z+1}{z-1}\right)=\left(8\right)^{\frac{1}{3}}$$

$$\frac{z+1}{z-1}=2,2\omega,2\omega^2$$

$$z=3,\frac{2\omega+1}{2\omega-1},\frac{2\omega^2+1}{2\omega^2-1}$$

$$\therefore z_1+z_2+z_3=\frac{27}{7}$$

$$\text{Re}\left(z_1+z_2+z_3\right)=\frac{27}{7}$$

COMPLEX NUMBERS

Ex. 3

Q.1 (c)

$x = 9^{\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots\infty \text{ terms}} \Rightarrow x = 9^{\frac{1/3}{1-1/3}}$ or $x = 3.$

$y = 4^{\frac{1}{3}-\frac{1}{9}+\frac{1}{27}-\ldots\infty \text{ terms}} \Rightarrow y = 4^{\frac{1/3}{1+1/3}}$ or $y = \sqrt{2}.$

$z = \sum_{r=1}^{\infty} \frac{1}{(1+i)^r} \Rightarrow z = \frac{1/(1+i)}{1-1/(1+i)}$ or $z = -i.$

Now $x + yz = 3 - \sqrt{2}i \Rightarrow \arg(x+yz) = -\tan^{-1}\frac{\sqrt{2}}{3}.$

Q.2 (c)

$\overline{Z} + i\overline{W} = 0 \Rightarrow Z - iW = 0$ or $\frac{Z}{W} = i.$

Now $\arg\left(\frac{Z}{W}\right) = \frac{\pi}{2} \Rightarrow \arg(ZW) + \arg\left(\frac{Z}{W}\right) = \frac{3\pi}{2}$

$\Rightarrow \arg(Z^2) = \frac{3\pi}{2}$ or $\arg(Z) = \frac{3\pi}{4}.$

Q.3 (d)

Let $P(x) = x^6 + 4x^5 + 3x^4 + 2x^3 + x + 1$

or $P(x) = x^6 - x^3 + (4x^3 - x^2 + 1)(x^2 + x + 1).$

Now for $x = \omega$ & ω^2, $x^6 - x^3 = x^2 + x + 1 = 0$, hence $P(\omega) = 0 = P(\omega^2).$

$P(x)$ is divisible be $(x-\omega)(x-\omega^2).$

Q.4 (c)

$\cos r\theta + i\sin r\theta = e^{ir\theta} \Rightarrow (\cos\theta + i\sin\theta)(\cos 2\theta + i\sin 2\theta)\ldots(\cos n\theta + i\sin n\theta) = \sum_{r=1}^{n} e^{ir\theta}$

Hence $e^{i\frac{n(n+1)}{2}\theta} = 1$ or $\cos\frac{n(n+1)}{2}\theta + i\sin\frac{n(n+1)}{2}\theta = 1.$

$\Rightarrow \frac{n(n+1)}{2}\theta = 2m\pi$ or $\theta = \frac{4m\pi}{n(n+1)}.$

Q.5 (c)

Let $Z = \cos\theta + i\sin\theta$.

$$\left|\frac{Z}{\overline{Z}} + \frac{\overline{Z}}{Z}\right| = 1 \Rightarrow \left|Z^2 + \overline{Z}^2\right| = 1 \text{ or } 2\left|\cos 2\theta\right| = 1.$$

Now $\cos 2\theta = \pm\frac{1}{2} \Rightarrow \theta = \frac{n\pi}{2} \pm \frac{\pi}{6}$.

As $\theta \in (0, 2\pi)$, therefore $\theta = \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{6}$.

Q.6 (A)

$\mu^2 - 2\mu + 2 = 0 \Rightarrow (\mu - 1)^2 = -1$ or $\mu - 1 = \pm i$, hence $\alpha - 1 = i$ & $\beta - 1 = -i$.

Now $(x + \mu)^n = (\cot\theta + \mu - 1)^n$

$$\frac{(x+\alpha)^n - (x+\beta)^n}{\alpha - \beta} = \frac{(\cot\theta + i)^n - (\cot\theta - i)^n}{2i}$$

$$\Rightarrow \frac{(x+\alpha)^n - (x+\beta)^n}{\alpha - \beta} = \frac{(\cos\theta + i\sin\theta)^n - (\cos\theta - i\sin\theta)^n}{2i\sin^n\theta}$$

or $$\frac{(x+\alpha)^n - (x+\beta)^n}{\alpha - \beta} = \frac{\sin n\theta}{\sin^n\theta}.$$

Q.7 (d)

$|Z - 4| = \text{Re}(Z) \Rightarrow (x-4)^2 + y^2 = x^2$ or $y^2 = 8(x-2)$.

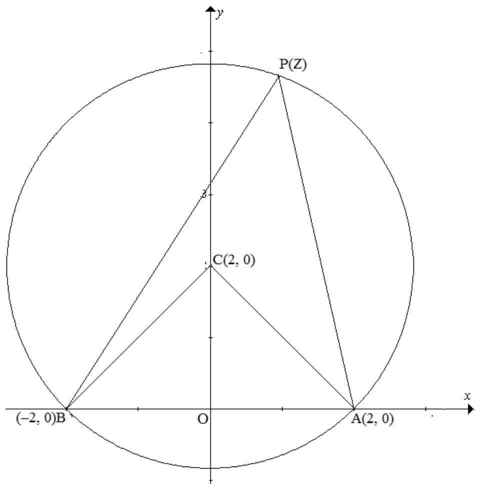
Now greatest positive $\arg(Z)$ will be greatest slope angle of tangent from origin to this parabola.

Equation of any tangent of slope m will be $y = m(x-2) + \frac{2}{m}$.

As it has to be drawn from $(0,0)$, hence $m = 1$.

$\therefore$ Greatest positive $\arg(Z) = \frac{\pi}{4}$.

Q.8 (b)



As shown in figure

both A(2, 0) & B(–2, 0) lie on the circle

$|Z-2i|=2\sqrt{2}$.

Center of the circle is C(0, 2) & radius is $2\sqrt{2}$.

Now $\angle APB=\angle ACO$.

Hence $\arg\left(\frac{Z-2}{Z+2}\right)=\frac{\pi}{4}$.

Q.9 (b)

$S=1+2\alpha+3\alpha^2+4\alpha^3+\ldots+n\alpha^{n-1}$ ...(i)

Multiply throughout by α to get

$\alpha S=\alpha+2\alpha^2+2\alpha^3+\ldots(n-1)\alpha^{n-1}+n\alpha^n$ ...(ii)

Subtract (ii) from (i) to get

$(1-\alpha)S=1+\alpha+\alpha^2+\alpha^3+\ldots+\alpha^{n-1}-n\alpha^n$

Now $1+\alpha+\alpha^2+\alpha^3+\ldots+\alpha^{n-1}=0\Rightarrow S=\frac{-n}{1-\alpha}$.

Q.10 (c)

Let $Z=a+ib$ & $\frac{2}{Z}=x+iy$, then $\frac{2(a-ib)}{a^2+b^2}=x+iy$.

As $a^2+b^2=1$, thus $x=2a$ & $y=-2b$ or $x^2+y^2=4$.

Required locus is a circle of radius 2.

Q.11 (a)

$\angle AOB=\frac{\pi}{2}$ & $OA=OB\Rightarrow Z_2=iZ_1$. Hence $\frac{Z_2}{Z_1}$ is purely imaginary.

Q.12 (c)

$\beta+\gamma=\alpha+\alpha^2+\ldots+\alpha^6=-1$ &

$$\beta\times\gamma=\left(\cos\frac{2\pi}{7}+\cos\frac{4\pi}{7}+\cos\frac{8\pi}{7}\right)\left(\cos\frac{6\pi}{7}+\cos\frac{10\pi}{7}+\cos\frac{12\pi}{7}\right)=2$$

hence required equation is $Z^2+Z+2=0$.

Q.13 (c)

Let $Z_1=e^{i\alpha}, Z_2=e^{i\beta}$ & $Z_3=e^{i\gamma}$.

Now $\cos\alpha+2\cos\beta+3\cos\gamma=\sin\alpha+2\sin\beta+3\sin\gamma=0\Rightarrow Z_1+2Z_2+3Z_3=0$

$\Rightarrow Z_1{}^3+8Z_2{}^3+27Z_3{}^3=18Z_1Z_2Z_3$

$\therefore \sin 3\alpha+8\sin 3\beta+27\sin 3\gamma=18\sin(\alpha+\beta+\gamma)$.

Q.14 (A)

Let $Z=k(\cos A+i\sin A)$ & $W=k(\cos B+i\sin B)$

Now $\alpha=\dfrac{Z-\overline{W}}{k^2+Z\overline{W}}\Rightarrow\alpha=\dfrac{(\cos A-\cos B)+i(\sin A+\sin B)}{k+k(\cos A+i\sin A)(\cos B-i\sin B)}$

or $\alpha=\dfrac{(\cos A-\cos B)+i(\sin A+\sin B)}{k\{1+\cos(A-B)+i\sin(A-B)\}}$

$$\Rightarrow\alpha=\frac{\sin\frac{A+B}{2}\left(\sin\frac{B-A}{2}+i\cos\frac{A-B}{2}\right)}{k\cos\frac{A-B}{2}\left(\cos\frac{A-B}{2}+i\sin\frac{A-B}{2}\right)}$$

$$\Rightarrow\alpha=\frac{\sin\frac{A+B}{2}\left(-\sin\frac{A-B}{2}+i\cos\frac{A-B}{2}\right)}{k\cos\frac{A-B}{2}\left(\cos\frac{A-B}{2}+i\sin\frac{A-B}{2}\right)}\times\frac{\cos\frac{A-B}{2}-i\sin\frac{A-B}{2}}{\cos\frac{A-B}{2}-i\sin\frac{A-B}{2}}$$

$$\Rightarrow\alpha=\frac{i\sin\frac{A+B}{2}}{k\cos\frac{A-B}{2}}.$$

Hence Re(Z) = 0.

Q.15 (c)

$|Z^2+k|+k=|Z^2|\Rightarrow|Z^2+k|+k=|Z^2+k-k|$

Hence $\arg(Z^2)=-\arg(k)$ or $\arg(Z^2)=\pi$.

$\therefore\ \arg(Z)=\dfrac{\pi}{2}$.

Q.16 (b)

Let $f(Z) = (Z^2+1)Q(Z) + aZ + b,$

where $Q(Z)$ is the qoutient when $f(Z)$ is divided by Z^2+1

Now $f(i) = i$ & $f(-i) = 1+i$, hence

$ai + b = i$ & $-ai + b = 1+i.$

Solving these equations simultaneously gives

$b = \frac{1+2i}{2}$ & $a = \frac{i}{2}.$

$\therefore$ remainder when $f(Z)$ is divided by Z^2+1 is $\frac{1+2i}{2} + \frac{iZ}{2}.$

Q.17 (a)

$a|Z_1| = b|Z_2| \Rightarrow \left|\frac{Z_1}{Z_2}\right| = \frac{b}{a}$ or $\frac{aZ_1}{bZ_2} = e^{i\theta}$ & $\frac{aZ_2}{bZ_1} = e^{-i\theta}.$

Hence $\frac{aZ_1}{bZ_2} + \frac{aZ_2}{bZ_1} = 2\cos\theta.$

$\left(\frac{aZ_1}{bZ_2}, \frac{aZ_2}{bZ_1}\right)$ lies on real axis between $(-2,0)$ & $(2,0)$.

Q.18 (c)

For n^{th} roots of unity

$1 + \omega + \omega^2 + \omega^3 + \ldots + \omega^{n-1} = 0$

Also let $S = 1 + 2\omega + 3\omega^2 + \ldots + n\omega^{n-1},$ then

$$\omega S = \quad \omega + 2\omega^2 + \ldots + (n-1)\omega^{n-1} + n\omega^n$$

From the above two relations we get $S = \frac{n}{\omega - 1}$

Now $\sum_{r=1}^{n}(ar+b)\omega^{r-1} = a\sum_{r=1}^{n} r\,\omega^{r-1} + b\sum_{r=1}^{n}\omega^{r-1}$

Or $\sum_{r=1}^{n}(ar+b)\omega^{r-1} = \frac{an}{\omega - 1}.$

Q.19 (b)

$(Z+ab)^3 = a^3 \Rightarrow Z = a - ab,\ a\omega - ab$ & $a\omega^2 - ab.$

Now side length $|(a-ab)-(a\omega-ab)| = |a(1-\omega)|$ i.e. $\sqrt{3}|a|.$

Q.20 (d)

$|\omega Z - 1 - \omega^2| = a \Rightarrow |Z+1| = a.$

Given $|Z+1| = a$ & $|Z-1| \le 2.$

Now $||Z+1|-2| \le |Z-1| \Rightarrow |a-2| \le 2$ or $0 \le a \le 4.$

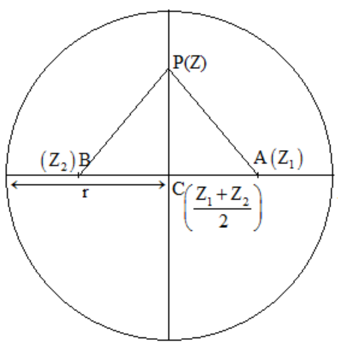
Q.21 (a)

$|Z^2 + 2Z\cos\alpha| \le |Z|^2 + 2|Z||\cos\alpha|$

Now $|Z| < \sqrt{2}-1$ & $\cos\alpha \le 1$, hence $|Z^2 + 2Z\cos\alpha| < (\sqrt{2}-1)^2 + 2(\sqrt{2}-1).$

Or $|Z^2 + 2Z\cos\alpha| < 1.$

Q.22 (b)



Consider a circle having center at $C\left(\frac{Z_1+Z_2}{2}\right)$ and radius r.

Now $A(Z_1)$ & $B(Z_2)$ will be two points on a diameter such that AC = BC.

Also P(Z) will be a point on the perpendicular diameter as given PA = PB.

Clearly area will be maximum when CP = r.

Hence max. area = $\frac{1}{2}|Z_1 - Z_2| r$.

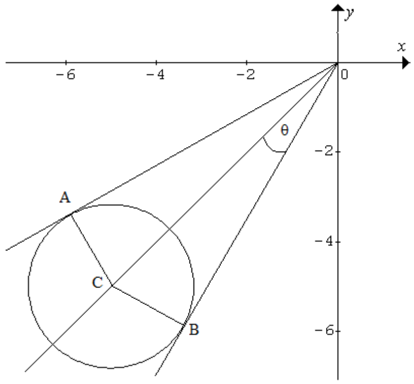
Q.23 (d)

$|Z_2 + iZ_1| = |Z_2| + |iZ_1| \Rightarrow \arg(Z_2) = \arg(iZ_1)$ or $\arg(Z_2) - \arg(Z_1) = \frac{\pi}{2}.$

Let $Z_1 = 3$ & $Z_2 = 4i$, then $\frac{Z_2 - iZ_1}{1-i} = \frac{i(1+i)}{2}$ or $\frac{-1+i}{2}$

$$\text{Area} = \frac{1}{2} \times \left\| \begin{matrix} 1 & 3 & 0 \\ 1 & 0 & 4 \\ 1 & -\frac{1}{2} & \frac{1}{2} \end{matrix} \right\| = \frac{25}{4}.$$

Q.24 (a)



$\left|Z+5+5i\right| \le \frac{5\sqrt{3}-5}{2}$ represents a circle with center at $(-5, -5)$ and radius $\frac{5\sqrt{3}-5}{2}$.

Now OC = $5\sqrt{2}$ & BC = $\frac{5\sqrt{3}-5}{2}$, thus

$\sin\theta = \frac{\sqrt{3}-1}{2\sqrt{2}}$ or $\theta = \frac{\pi}{12}$.

Now angle made by OC with positive real axis is $\frac{5\pi}{4}$, therefore angle made by OB & OC with positive real axis are $\frac{4\pi}{3}$ & $\frac{7\pi}{6}$.

Hence least arg(Z) = $-\frac{5\pi}{6}$.

Q.25 (a)

$\text{Case I}: \left|Z-1\right| < \left|Z+1\right| \,\&\, \left|Z\right| = \left|Z-1\right|$

$\text{Case II}: \left|Z-1\right| > \left|Z+1\right| \,\&\, \left|Z\right| = \left|Z+1\right|$

$\Rightarrow x > 0, \text{ then } x = \frac{1}{2} \,\&\, x < 0, \text{ then } x = -\frac{1}{2}.$

Now $Z + \overline{Z} = 2\,\mathrm{Re}(Z)$, thus $Z + \overline{Z} = 1$ or -1.

Q.26 (b)

$$\arg\left(\frac{Z_1 - \frac{Z}{|Z|}}{\frac{Z}{|Z|}}\right) = \frac{\pi}{2} \Rightarrow \frac{Z_1 - \frac{Z}{|Z|}}{\frac{Z}{|Z|}} = \left|Z_1 - \frac{Z}{|Z|}\right| e^{-\frac{\pi}{2}i}$$

$$\Rightarrow Z_1 - \frac{Z}{|Z|} = 3i\frac{Z}{|Z|} \text{ or } Z_1 = (3i+1)\frac{Z}{|Z|}.$$

Hence $\left|Z_1\right| = \sqrt{10}$.

Q.27 (b)

The required complex vector will be $\frac{\lambda}{2}\left(\frac{Z_1}{|Z_1|} + \frac{Z_2}{|Z_2|}\right)$ i.e. $\frac{\lambda}{2}\left(\frac{3+\sqrt{3}i}{2\sqrt{3}} + \frac{2\sqrt{3}+6i}{4\sqrt{3}}\right)$.

Hence any complex number of form $\mu(1+i)$ will lie along the angle bisector.

Q.28 (a)

$$\left|Z-2+2i\right| \le \left|\left|Z\right|-\left|2-2i\right|\right| \Rightarrow -1 \le \left|Z\right|-2\sqrt{2} \le 1.$$

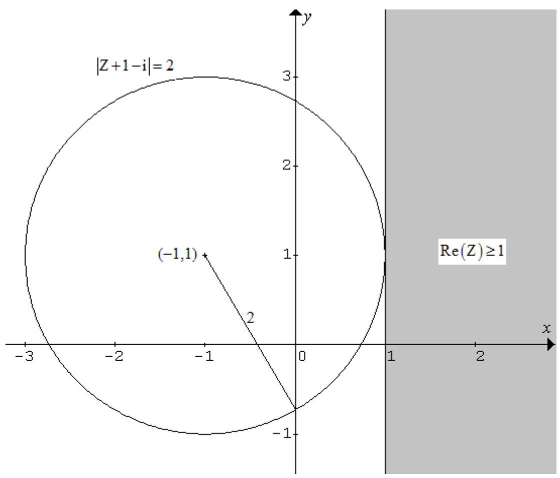
Hence least value of $|Z|$ is $2\sqrt{2}-1$.

Also arg(Z) = arg(2 – 2i).

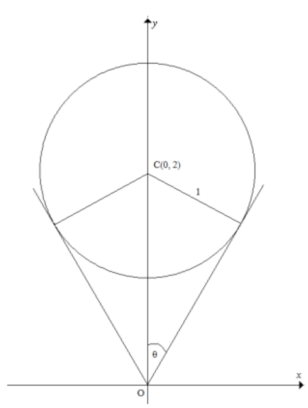
$$\therefore Z = \frac{2\sqrt{2}-1}{\sqrt{2}}(1-i).$$

Q.29 (b)

Refer the adjoining figure.



Q.30 (a)

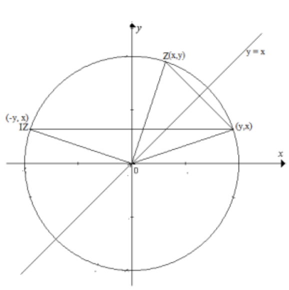


As shown in figure range of arg(Z) will be

from $\frac{\pi}{2}-\theta$ to $\frac{\pi}{2}+\theta$, where $\sin\theta = \frac{1}{2}$ i.e. $\theta = \frac{\pi}{6}$.

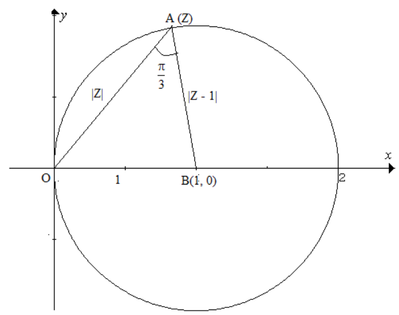
Hence $\text{Arg}(\alpha) \in \left[\frac{\pi}{3}, \frac{2\pi}{3}\right]$.

Q.31 (b)



Rotation of Z(x + iy) about the origin gives iZ(–y + ix).
Then reflection in Imaginary-Axis gives (y + ix),
which is equivalent to reflection of Z in the line x = y.
Hence T_1 is equivalent to composite of T_2 & T_3.

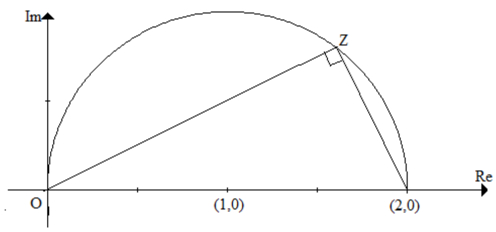
Q.32 (d)



Refer the adjoining figure.
By cosine formula,

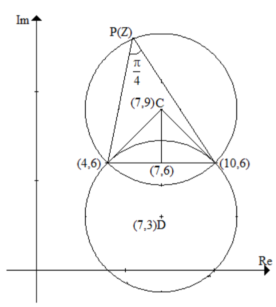
$$\cos\frac{\pi}{3}=\frac{|Z|^2+1-1}{2|Z|}\Rightarrow |Z|^2=|Z| \text{ or } |Z|=1.$$

Q.33 (c)



Refer the adjoining figure.

Q.34 (c)



Angles in same segment of a circle are equal, hence Z will move on major arc of two circles passing through (4, 6) & (10, 6) and of radius $3\sqrt{2}$ as shown in adjoining figure.

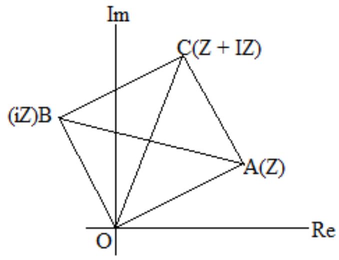
Q.35 (d)

$\cos\frac{2k\pi}{11}+i\sin\frac{2k\pi}{12}$ is 12th root of unity for $k=0,1,2,\ldots,11$.

Now $\sum_{k=0}^{11}\left(\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n}\right)=0$ & $\sum_{k=0}^{11}\left(\sin\frac{2k\pi}{n}-i\cos\frac{2k\pi}{n}\right)=-i\sum_{k=0}^{11}\left(\cos\frac{2k\pi}{n}+i\sin\frac{2k\pi}{n}\right)$,

hence $\sum_{k=1}^{11}\left(\sin\frac{2k\pi}{n}-i\cos\frac{2k\pi}{n}\right)=i.$

Q.36 (d)



$A(Z)\,\&\,B(iZ)$ are such that $OA \perp OB$.

Also $C(Z+iZ)$ will be such that OC is diagonal of Square OACB as shown in adjoining figure.

Hence required area is $\frac{1}{2}|Z|^2$.

Q.37 (c)

Let $P(Z)=(Z-1-i)(Z-1+i)Q(Z)+aZ+b$

Now $P(1+i)=3+4i \Rightarrow (1+i)a+b=3+4i$ ...(i)

& $P(1ii)=3+4i \Rightarrow (1-i)a+b=4-3i$ ...(ii)

From (i) & (ii)

$a=\left(\frac{7+i}{2}\right), b=0.$

Q.38 (a)

Note that triangle AOB is right angled isosceles triangle, hence C will be midpoint of AB.

Q.39 (b)

$(a+ib)^n=(a-ib)^n \Rightarrow e^{i(n\theta)}=e^{-i(n\theta)}$, where $\theta=\tan^{-1}\frac{b}{a}$

$\Rightarrow e^{i(2n\theta)}=1 \Rightarrow \tan^{-1}\frac{b}{a}=\frac{\pi}{n} \Rightarrow \frac{b}{a}=\tan\frac{\pi}{n}$

Clearly least positive integral value of n is 3 such that $\frac{b}{a}$ is defined and not zero.

Q.40 (a)

$|2Z_1+Z_2|\le 2|Z_1|+|Z_2| \Rightarrow |2Z_1+Z_2|\le 4.$