IIT – JEE: 2026 **MAJOR TEST - 1** **DATE: 28/07/24**

ADVANCED **ANSWER KEY**

PAPER – 1 (Code – 01)

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	C	**20.**	C	**39.**	D
2.	B	**21.**	A	**40.**	A
3.	B	**22.**	A	**41.**	B
4.	A	**23.**	C	**42.**	D
5.	D	**24.**	C	**43.**	B
6.	C	**25.**	A	**44.**	B
7.	B	**26.**	B	**45.**	D
8.	B	**27.**	A	**46.**	D
9.	B	**28.**	C	**47.**	A
10.	B	**29.**	B	**48.**	B
11.	B, C	**30.**	B, C	**49.**	B, C, D
12.	B	**31.**	A, B	**50.**	D
13.	A, B, C	**32.**	C, D	**51.**	B, C, D
14.	A, B	**33.**	B, C	**52.**	A, B
15.	D	**34.**	A, D	**53.**	B, C, D
16.	A	**35.**	C	**54.**	C
17.	D	**36.**	D	**55.**	D
18.	C	**37.**	C	**56.**	A
19.	D	**38.**	B	**57.**	B

PART (A) : PHYSICS

1. (C)

Since, $|\vec{\mathbf{a}}_1+\vec{\mathbf{a}}_2|=\sqrt{3}$, and if angle between then is θ, then $(\sqrt{3})^2=1^2+1^2+2.11\cos\theta$ *i.e.*, $\theta=60°$

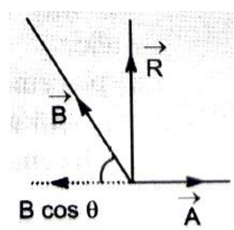
$$(\vec{\mathbf{a}}_1-\vec{\mathbf{a}}_2)\cdot(2\vec{\mathbf{a}}_1+\vec{\mathbf{a}}_2)=2a_1^2+(\vec{\mathbf{a}}_1\cdot\vec{\mathbf{a}}_2)-(2\vec{\mathbf{a}}_1\cdot\vec{\mathbf{a}}_2)-a_2^2$$

$$=2a_1^2-(\vec{\mathbf{a}}_1\cdot\vec{\mathbf{a}}_2)-a_2^2$$

$$=2-1.1.\frac{1}{2}-1=\frac{1}{2}$$

2. (B)

As shown in the figure, $A=B\cos\theta$



$$\cos\theta=\frac{A}{B}$$

Required angle $=\pi-\theta$

$$=\cos^{-1}\left(-\frac{A}{B}\right)$$

3. (B)

$$|\vec{\mathbf{A}}|=2, |\vec{\mathbf{B}}|=2\sqrt{2}$$

$$\vec{\mathbf{A}}.\vec{\mathbf{B}}=\vec{\mathbf{A}}\times\vec{\mathbf{B}}$$

$\Rightarrow$ $\theta=45°$

$$\left|\frac{\vec{\mathbf{A}}+\vec{\mathbf{B}}}{\vec{\mathbf{A}}-\vec{\mathbf{B}}}\right|=\sqrt{\frac{|\vec{\mathbf{A}}|^2+|\vec{\mathbf{B}}|^2+2\vec{\mathbf{A}}.\vec{\mathbf{B}}}{|\vec{\mathbf{A}}|^2+|\vec{\mathbf{B}}|^2-2\vec{\mathbf{A}}.\vec{\mathbf{B}}}}$$

$$=\sqrt{5}$$

4. (A)

$$y=\frac{x^2+1}{x+1}$$

$$\frac{dy}{dx}=\frac{(x+1)(2x)-(x^2+1)(1)}{(x+1)^2}$$

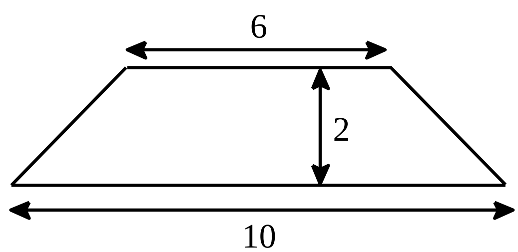
$$=\frac{2x^2+2x-x^2-1}{(x+1)^2}=\frac{x^2+2x-1}{(x+1)^2}$$

5. (D)

$xy = 4 \Rightarrow x.\frac{dy}{dx} + y = 0$

$\frac{dy}{dx} = -\frac{y}{x}$

6. (C)



a-t curve area = change in velocity

$\frac{1}{2} \times 16 \times 2 = v - 0$

v = 16 m/s

7. (B)

$v = u + at$

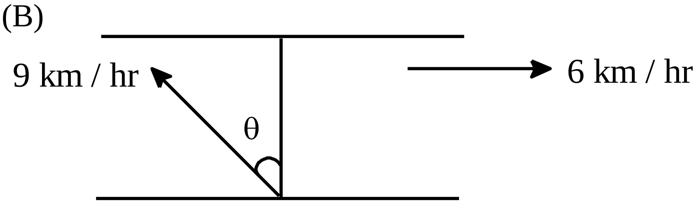
$200 = 100 + 10\,t \Rightarrow t = 10s$

8. (B)

Time of flight $7 = \frac{2u}{g} = 4$

$u = \frac{40}{2} = 20\,m/s$

9. (B)



$9 \sin\theta = 6$

$\sin\theta = \frac{2}{3}$

10. (B)

$x = t^2$

$v = 2t$

$a = 2$

11. (B, C)

$v = \frac{dx}{dt} = -9 + t^2 = 0 \Rightarrow t = 3\text{s}$

The particle's velocity is getting zero at t = 3s, where it changes its direction of motion.

For $0 < t < 3\text{s}$, v is negative, a is positive, so particle is slowing down.

For $t > 3$, both v and a are positive, so the particle is speeding up.

12. (B)

Let $\vec{a} = a_x\hat{i} + a_y j + a_z k = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt} j + \frac{dv_z}{dt} k$

As $\vec{a}$ is constant, so its magnitude as well as direction is not changing, but v_x, v_y and v_z all can be varying (any combination of these if a_x, a_y or $a_z = 0$, then corresponding velocity component may be constant).

$|\vec{v}| = v\sqrt{v_x^2 + v_y^2 + v_z^2}$

$$\frac{d|\vec{v}|}{dt} = \frac{1}{2\sqrt{v_x^2 + +v_y^2 + v_z^2}} \times \left[2v_x\frac{dv_x}{st} + 2v_y\frac{dv_y}{st} + 2v_z\frac{dv_z}{st}\right] = \frac{\vec{v}.\vec{a}}{v}$$

Which is *a* variable quantity.

$\left|\frac{d\vec{v}}{dt}\right| = |\vec{a}|$, which would be constant

$$\frac{dv^2}{dt} = \frac{d\left(v_x^2 + v_y^2 + v_z^2\right)}{dt} = 2\vec{v}.\vec{a} \qquad \text{(}a\text{ variable quantity)}$$

$$\frac{d\left(\vec{v}/v\right)}{dt} = \frac{d}{dt}\left[\frac{v_x\hat{i} + v_y j + v_z k}{\sqrt{v_x^2 + v_y^2 + v_z^2}}\right] \qquad \text{(}a\text{ variable quantity)}$$

13. (A, B, C)

Given, $\frac{dv}{dt} = 0.5t$.

Integrate to get, $v - 16 = 0.25t^2$

Direction changes after v becomes negative, which occurs at $0 - 16 = -0.25t^2$ or,

$t = \sqrt{16 \times 4} = 8$ seconds.

Velocity at 10 seconds is $v = 16 - 0.25 \times 100 = -9$.

Thus speed is magnitude, which is 9 m/s,

Integrate the velocity equation once more:

$\frac{ds}{dt} = 16 - 0.25t^2$ or, $s - s_0 = 16 - \frac{1}{12}t^3$ Assume

$s_0 = 0$ for distance calculations (becomes its relative to some initial point)

In 4 seconds, distance $=16\times 4-\frac{1}{12}\times 64=64-\frac{16}{3}$

which is nearly 58.67 m.

In 10 seconds, distance is sum of distance till 8 seconds and then 9th and 10th second (because velocity becomes negative, so displacement starts decreasing, thus we need to break it down)

$$=16\times 8-\frac{1}{12}\times 512+\left(-16\times 10+\frac{1}{12}\times 1000+s_8\right)$$

$$=85.33+(-76.67+85.33)=94\,\text{m}$$

14. (A, B)

$$R_2=2R$$

$$\frac{U_2^2\sin 2\theta}{g}=\frac{2U^2\sin 2\theta}{g}$$

$$U_2=\sqrt{2}\,U$$

$$T_2=\frac{2U_2\sin\theta}{g}=\sqrt{2}\,T$$

$$H_2=\frac{U_2^2\sin^2\theta}{g}=2(H)$$

15. (D)

$$\vec{S}=(t-1)\hat{i}+\left(3t-t^2\right)\hat{j}$$

$$\vec{v}=\hat{i}+(3-2t)\hat{j}$$

$$\vec{a}=-2\hat{j}$$

$$t=0 \quad s=-\hat{i}$$

$$t=1 \quad \vec{v}=\hat{i}+\hat{j}=|\vec{v}|=\sqrt{2}$$

16. (A)

Distance covered = area of speed – time graph $=\frac{1}{2}\times(4+2)\times 4+\frac{1}{2}(4+2)\times 2=18\,\text{m}$

17. (D)

Displacement from the origin will be equal to the final displacement of the particle which is equal to the area of the velocity-time graph, i.e.,

$$\frac{1}{2}\times(4+2)\times 4-\frac{1}{2}(4+2)\times 2=6\,\text{m}$$

18. (C)

19. (D)

$\vec{F}_1 + \vec{F}_2 = m \cdot \vec{a}$

$\left(70\hat{j} + \vec{F}_2\right) = 5\left(10\cos 53^o\right)\hat{i} + \left(10\sin 53^o\right)\hat{j}$

$\Rightarrow \left(70\hat{j} + \vec{F}_2\right) = 5\left(10\times\frac{3}{5}\hat{i} + 10\times\frac{4}{5}\hat{j}\right)$

$\left(70\hat{j} + \vec{F}_2\right) = 30\hat{i} + 40\hat{j}$

$\therefore \quad \vec{F}_2 = 30\hat{i} - 30\hat{j}$

At $t = 10$ seconds

$v = \vec{u} + \vec{at} = 0 + 10\left(6\hat{i} + 8\hat{j}\right) = \left(60\hat{i} + 80\hat{j}\right)$ m/s

PART (B) : CHEMISTRY

20. (C)

$L = mvr = n\frac{h}{2\pi}$

For lowest p energy level $n = 1$

$L = \frac{h}{2\pi}$

21. (A)

$m = \frac{89}{0.22} \times 100 = 4.05 \times 10^4$

22. (A)

C	70.8	$\frac{70.8}{12} = 5.9$	$\frac{5.9}{0.29} \simeq 20$
H	6.2	$\frac{6.2}{1} = 6.2$	$\frac{6.2}{0.29} \simeq 21$
N	4.1	$\frac{4.1}{14} = 0.29$	$\frac{0.29}{0.29} = 1$
O	18.9	$\frac{18.9}{16} = 1.18$	$\frac{1.18}{0.29} \simeq 4$

$C_{20}H_{21}NO_4$

23. (C)

$Cl \rightarrow \{Ne\}3s^2 3p^5$

$n = 3,\ \ell = 1$

24. (C)

mmol of HNO_3 = 25 × 3 =75 ;

mmol of HNO_3 = 75 × 4 = 300

∴ Total mmol = 375

Thus, 375 = M × 100

∴ M = 3.75

25. (A)

$(4a+96)$ g X_4O_6 has $4a$ g X

∴ 10g X_4O_6 has $\left[\frac{4a \times 10}{40a+96}\right]$ g X

∴ $\frac{4a \times 10}{4a+96} = 5.72$

∴ Atomic mass of X, $a = 32\ \text{g mol}^{-1}$

26. (B)
mmol of NaOH = 10 × 0.5 × 2 = 10

$\therefore \quad \frac{w}{40} \times 1000 = 10$

$\therefore \quad w = 0.4 \text{ g}$

27. (A)
It is experimental evidence for particle nature of electron.

28. (C)
According to de Broglie wavelength $\lambda = \frac{h}{mu} = \frac{h}{p}$

29. (B)
Bohr model is based on one electron system.

30. (B, C)
As per theory

31. (A, B)
No. of non spherical node $= \ell$
(nodal plane)
Shape depends only on azimuthal quantum, no. for (m = –1, 0, +1) = 6 electrons

32. (C, D)

$KClO_3 \longrightarrow KCl + \frac{3}{2} O_2$

$\frac{n_{KCl}}{1} = \frac{n_{O_2}}{3/2}$

$W_{KCl} = \frac{5}{32} \times \frac{2}{3} \times 74.5 = 7.76 \text{ gm}$

$n_{KCl} = 0.104$

33. (B, C)
1 gm-atom = 1 mole atom of N
= 14 gm
= 11.35 lit. N_2 at STP

34. (A, D)

$Rb(37) = \{K_r\} 5s^1$

$n = 5, \ell = 0, m = 0, s = +1/2 \text{ or } -1/2$

35. (C)

$$2H + S + 2O_2 \rightarrow H_2SO_4$$

Moles $n =$ 2.5 1 2.75

$\frac{n}{5.C}$ $\frac{2.5}{2}$ L.R $\frac{2.75}{2}$

36. (D)

After $r \times n$ (moles): 0.5 → H, 0 → S, 0.75 → O_2

Total weight $= 0.5 \times 1 + 0.75 \times 32 = 24.5\,g$

37. (C)

38. (B)

$$13.6\,Z^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 13.6 \times 4\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$Z = 1, n_1 = 2, n_2 = 6$

(hit & trial)

PART (C) : MATHEMATICS

39. (D)

$\sin^3\theta + \cos^3\theta = 0$

$\sin^3\theta = -\cos^3\theta$

$\tan^3\theta = -1$

$\tan\theta = -1$

$\Rightarrow \theta = \frac{-\pi}{4}$

40. (A)

$\frac{3\cos\theta + 2\sin\theta}{2\cos\theta - \sin\theta} = \frac{3 + 2\tan\theta}{2 - \tan\theta}$

$= \frac{3 + 2\left(\frac{1}{2}\right)}{2 - \frac{1}{2}} = \frac{4}{3/2} = \frac{8}{3}$

41. (B)

$\sin 163^\circ \cos 347^\circ + \sin 73^\circ \sin 167^\circ$

$= \sin 17^\circ \cos 13^\circ + \cos 17^\circ \sin 13^\circ$

$= \sin(17^\circ + 13^\circ)$

$= \sin 30^\circ = \frac{1}{2}$

42. (D)

$(a^2 + c^2)x^2 + 2(ab + cd)x + (b^2 + d^2) = 0$

$\because$ Roots are Equal $D = 0$

$\cancel{4}(ab + cd)^2 - \cancel{4}(a^2 + c^2)(b^2 + d^2) = 0$

$a^2b^2 + c^2d^2 + 2abcd - a^2b^2 - a^2d^2 - b^2c^2 - c^2d^2 = 0$

$a^2d^2 - 2abcd + b^2c^2 = 0$

$(ad - bc)^2 = 0 \qquad ad = bc$

43. (B)

$\frac{\cos 6x + 6\cos 4x + 15\cos 2x + 10}{\cos 5x + 5\cos 3x + 10\cos x}$

$= \frac{(\cos 6x + \cos 4x) + 5(\cos 4x + \cos 2x) + 10(\cos 2x + 1)}{\cos 5x + 5\cos 3x + 10\cos x}$

$(\text{use} \cos C + \cos D) = 2\cos x$

44. (B)

Let $f(x)=x^5-6x^2-4x+5=0$

Then the number of change of sign in $f(x)$ is 2 therefore $f(x)$ can have at most two positive real roots.

Now, $f(-x)=-x^5-6x^4+4x+5=0$

Then the number of change of sign is 1.

Hence $f(x)$ can have at most one negative real root. So that total possible number of real roots is 3.

45. (D)

$x^2-10x-21=0$ has roots α and β

$\therefore \quad \alpha+\beta=10$ and $\alpha\beta=-21$

$A_n=\alpha^n+\beta^n$ and $A_{n+2}+\mu A_{n+1}+v A_n=0$

$\Rightarrow \quad \alpha^{n+2}+\beta^{n+2}+\mu\left(\alpha^{n+1}+\beta^{n+1}\right)+v\left(\alpha^n+\beta^n\right)=0$

$\Rightarrow \quad \alpha^n\left(\alpha^2+\mu\alpha+v\right)+\beta^n\left(\beta^2+\mu\beta+v\right)=0 \ \forall n\in N$

$\therefore \quad \alpha^2+\mu\alpha+v=0$ and $\beta^2+\mu\beta+v=0$ is same as $\alpha^2-10\alpha-21=0$ and $\beta^2-10\beta-21=0$

$\therefore \quad \mu=-10$ & $v=-21$

$\therefore \quad \mu+v=-31$

46. (D)

Given equation is $8x^2-3x+27=0$,

Also roots are α and β

$\alpha+\beta=\frac{3}{8};\ \alpha\beta=\frac{27}{8}$

$$\Rightarrow \left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)^{1/3}=\left(\frac{\alpha^3+\beta^3}{\alpha\beta}\right)^{1/3}$$

$$=\left(\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}\right)^{1/3}$$

$$=\left(\frac{\left(\frac{3}{8}\right)^3-3\left(\frac{27}{8}\right)\left(\frac{3}{8}\right)}{\frac{27}{8}}\right)^{1/3}$$

$$=\left(\frac{27}{8^3\left(\frac{27}{8}\right)}-\frac{3\left(\frac{27}{8}\right)\left(\frac{3}{8}\right)}{\left(\frac{27}{8}\right)}\right)^{1/3}$$

$$=\left(\frac{1}{8^2}-\frac{9}{8}\right)^{1/3}$$

$$\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)^{1/3}=\left(\frac{-71}{64}\right)^{1/3}$$

47. (A)

$x^2+ax+b+1=0$ has positive integral roots α and β.

Hence, $(\alpha+\beta)=-a$ and $\alpha\beta=b+1$

$$\Rightarrow \quad a^2+b^2=(\alpha+\beta)^2+(\alpha\beta-1)^2$$

$$=\alpha^2+2\alpha\beta+\beta^2+\alpha^2\beta-2\alpha\beta+1$$

$$=\alpha^2\left(1+\beta^2\right)+1\left(1+\beta^2\right)$$

$$=\left(1+\alpha^2\right)\left(1+\beta^2\right)$$

$\Rightarrow \quad a^2+b^2$ can be equal to 50 (Since other options have prime numbers)

48. (B)

The discriminants of the given equations are $D_1=a^2+12b,\ D_2=c_2-4b$ and $D^3=d^2-8b$

$\therefore \quad D_1+D_2+D_3=a^2+c^2+d^2\geq 0$

Hence, at least one of D_1, D_2, D_3 is non-negative.

Therefore, the equation has at least two real roots.

49. (B, C, D)

Range of $\sin^4 x+\cos^2 x$ in $\left[\frac{3}{4},1\right]$

$\therefore \quad P$ cannot be equal to 2, 3, 4.

$$P=1-\sin^2 x+\sin^4 x$$

$$=\left(\sin^2 x-\frac{1}{2}\right)^2+\frac{3}{4}$$

$$P\in\left[\frac{3}{4},1\right]$$

50. (D)

A, B, C are only true if $a,b,c\in P/Q/R$ respectively

(A) Non-real roots for real polynomial appears in conjugate pairs.

(B) Irrational roots for rational polynomial appears in conjugate pairs.

(C) This is also for real polynomial.

51. (B, C, D)

$$\sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$$

$$\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

52. (A, B)
Apply common Root Condition

53. (B, C, D)
Same as above, we get $a+b=-1$

54. (C)

$$(1+m)x^2-2(1+3m)x+(1+8m)=0 \quad m\in R-\{-1\}$$

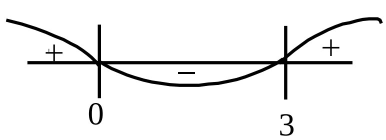
Imaginary roots so $D<0$

$$4(1+3m)^2-4c(1+m)(1+8m)<0$$

$$19m^2+6m-1-9m-8m^2<0$$

$$m^2-3m<0$$

$$m(m-3)<0$$

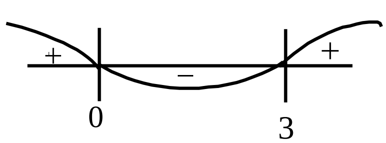


$m\in(0,3)$

Integers between 0 to 3 $\rightarrow$ 1, 2

55. (D)
Both roots +ve product of roots > 0 if both +ve

$$\frac{(1+8m)}{(1+m)}>0$$



$(-\infty,-1)\cup\left(-\frac{1}{8},\infty\right)$ ….. (2)

For $D\geq 0$

$m\in(-\infty,0]\cup[3,\infty)$ from (1) ….. (3)

Intersection of (2) of (3) is result

$$(-\infty,-1)\cup\left(\frac{-1}{8},0\right]\cup[3,\infty)$$

56. (A)

$$\sin\alpha+\sin\beta=3$$

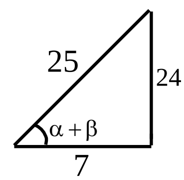
$$2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}=3 \qquad \ldots.(1)$$

$$\cos\alpha+\cos\beta=4$$

$$2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}=4 \qquad \ldots.(2)$$

$$\tan\frac{\alpha+\beta}{2}=\frac{3}{4}$$

$$\tan(\alpha+\beta)=\frac{2\tan\left(\frac{\alpha+\beta}{2}\right)}{1-\tan^2\left(\frac{\alpha+\beta}{2}\right)}=\frac{2\times\frac{3}{4}}{1-\frac{9}{16}}=\frac{6}{4}\times\frac{16}{7}=\frac{24}{7}$$



57. (B)

$$\cos(\alpha+\beta)=\frac{7}{25}$$

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ANSWER KEY

PAPER – 2 (Code – 02)

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14.	B	**33.**	A, C	**52.**	B
15.	4	**34.**	5	**53.**	4
16.	5	**35.**	3	**54.**	1
17.	5	**36.**	9	**55.**	0
18.	6	**37.**	1	**56.**	3
19.	4	**38.**	2	**57.**	1

PART (A) : PHYSICS

1. (C)

Component of $(\vec{a}-\vec{b})$ along $(\vec{a}+\vec{b})$

$$=\left(\vec{a}-\vec{b}\right)\cdot n=\left(\vec{a}-\vec{b}\right)\cdot\frac{\left(\vec{a}+\vec{b}\right)}{\sqrt{a^2+b^2}}=\frac{a^2-b^2}{\sqrt{a^2+b^2}}$$

2. (C)

$$|A+B|^2=|C|^2$$

$$\Rightarrow\ |A|^2+|B|^2+2|A||B|\cos\theta=|C|^2$$

$$\Rightarrow\ 12^2+5^2+2\times12\times5\times\cos\theta=13^2$$

$$\Rightarrow\ 144+25+120\cos\theta=169$$

$$\Rightarrow\ 169+120\cos\theta=169$$

$$\cos\theta=0$$

$$\theta=\frac{\pi}{2}$$

3. (B)

$$(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})$$

$$=\vec{A}\times\vec{A}-\vec{A}\times\vec{B}+\vec{B}\times\vec{A}-\vec{B}\times\vec{B}$$

$$=-\left(-\vec{B}\times\vec{A}\right)+\vec{B}\times\vec{A}$$

$$=2\left(\vec{B}\times\vec{A}\right)$$

4. (C)

$$y=0.5x-0.04x^2$$

Range $y=0$

$$x=0,x=\frac{0.5}{0.04}=\frac{50}{4}=12.5\,\text{m}$$

$$\frac{dy}{dx}=0.5 \qquad\qquad (e\,x=0)$$

$$=\tan\theta=\frac{1}{2}$$

$$\text{Range}=\frac{U^2\sin2\theta}{g}=\frac{25}{2}$$

$$\frac{U^2}{10}\times\frac{1}{\sqrt{5}}\times\frac{2}{\sqrt{5}}=\frac{25}{2}$$

$$U^2=\frac{625}{4}\Rightarrow U=\frac{25}{2}=12.5\,\text{sec}$$

5. (D)

Let $1+\tan x = t$

Differentiate

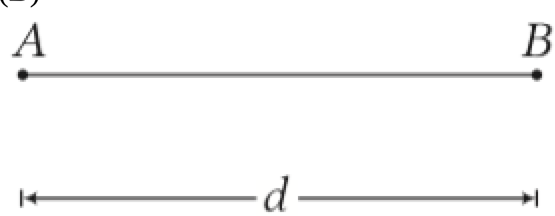
$\sec^{2x} dx = dt$

$\therefore \int \frac{dt}{t}$

$= \ln(t) + c$

$= \ln(1+\tan x) + c$

6. (B)



$u : v_{rg}$ (velocity of river w.r.t. ground)

$v = v_{mr}$ (velocity of man w.r.t. river)

Along the flow, $v_{mg} = v + u$, $3 = \frac{d}{v+u}$

where, v_{mg} is velocity of man w.r.t. ground.

Opposite to the flow,

$v_{mg} = v - u,\ 6 = \frac{d}{v-u}$

$\Rightarrow \quad t_0 = \frac{d}{v} = 4\,\text{h}$

7. (B)

Time interval between 8th and 3rd second,

$\Delta t = 8 - 3 = 5\text{s}, \quad i.e.\ \Delta t = 5\text{s}$

Change in velocity,

$\Delta v = 20 - 0 = 20\ \text{ms}^{-1}$

$\therefore$ Average acceleration $= \frac{\Delta v}{\Delta t} = \frac{20}{5} = 4\ \text{ms}^{-2}$

8. (A)

$t_1 = \frac{x}{2v_1}$

$t_2 = \frac{x}{2v_2}$

$$v_{avg} = \frac{x}{t_1 + t_2} = \frac{2v_1 v_2}{v_1 + v_2}$$

9. (A)

$v = an$

$s = \frac{1}{2}a\left\{\left(n^2\right) - (n-2)^2\right\}$

$\frac{1}{2}\frac{v}{n}\{2 \times 2(n-1)\}$

$\frac{2v}{n}(n-1)$

10. (B)

$$R = \frac{2u_x u_y}{g} = \frac{2 \times 6 \times 8}{10} = 9.6\text{ m}$$

11. (ABD)

A scalar cannot be added to a vector, hence options (A) and (D) are wrong.
Resultant of two vectors is always a vector, so option (B) is wrong.

12. (B)

$$\int \frac{1}{6x+2}dx = \left.\frac{\ln(6x+2)}{6}\right|_0^1 = \frac{\ln(8) - \ln(2)}{6} = \frac{2\ln 2}{6} = \frac{1}{3}(\ln 2)$$

13. (BCD)

14. (B)

Let after time *t*, car catches the scooter and the distance travelled by scooter in time *t*,

$$x = \frac{1}{2} \times (1) \times t^2 = \frac{t^2}{2} \qquad ...(\text{i})$$

The distance travelled by car in time *t*

$$x + 150 = \frac{1}{2} \times 2 \times t^2 = t^2 \qquad ...(\text{ii})$$

Solving Eqs. (i) and (ii), we get

$$t = \sqrt{300}\text{ s}$$

15. (4)

$U = 20\sqrt{3}\cos 60°\,\hat{i} + 20\sqrt{3}\sin 60°\,\hat{j}$

$V_f = 20\sqrt{3}\cos 60°\,\hat{i} + \left(20\sqrt{3}\sin 60° - gt\right)\hat{j}$

$\vec{U}.\vec{V}_f = 0$

$\left(20\sqrt{3}\right)^2 = 20\sqrt{3}\sin 60°\,gt$

$$t=\frac{20\sqrt{3}}{10\times\sqrt{3}/2}=4\,\text{sec}$$

16. (5)

If x is the length of each side of an equilateral triangle and A is its area, then $A=\frac{\sqrt{3}}{4}x^2 \Rightarrow \frac{dA}{dt}=\frac{\sqrt{3}}{4}$ $2x\frac{dx}{dt}$

Here, $x=\frac{5}{\sqrt{3}}\text{cm}$ and $\frac{dx}{dt}=2\text{ cm/sec}$

$\Rightarrow \frac{dA}{dt}=5\,\text{sq. unit per sec.}$

17. (5)

Given, $\mathbf{r}_1(t)=3t\,\hat{\mathbf{i}}+4t^2\,\hat{\mathbf{j}}$

$\therefore \frac{d\mathbf{r}_1}{dt}=3\hat{\mathbf{i}}+8t\,\hat{\mathbf{j}}$

At $t=1\text{s}$,

$\mathbf{v}_1=\frac{d\mathbf{r}_1}{dt}=3\hat{\mathbf{i}}+8\hat{\mathbf{j}}$

Again, $\mathbf{r}_2(t)=4t^2\,\hat{\mathbf{i}}+3t\,\hat{\mathbf{j}}$

$\Rightarrow \frac{d\mathbf{r}_2}{dt}=8t\,\hat{\mathbf{i}}+3\hat{\mathbf{j}}$

At $t=1\text{s}$,

$\mathbf{v}_2=\frac{d\mathbf{r}_2}{dt}=8\hat{\mathbf{i}}+3\hat{\mathbf{j}}$

Relative speed, $\mathbf{v}_{\text{rel}}=\mathbf{v}_2-\mathbf{v}_1$

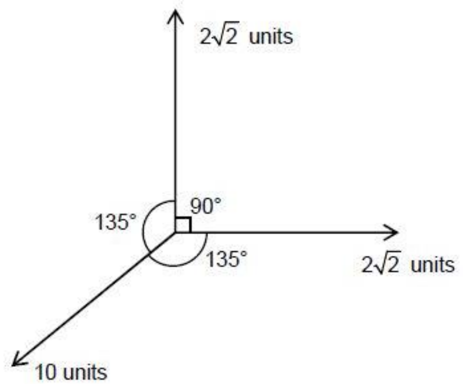
$=\left(8\hat{\mathbf{i}}+3\hat{\mathbf{j}}\right)-\left(3\hat{\mathbf{i}}+8\hat{\mathbf{j}}\right)$

$=5\hat{\mathbf{i}}-5\hat{\mathbf{j}}$

$\therefore |\mathbf{v}_{\text{rel}}|=\sqrt{(5)^2+(-5)^2}$

$=5\sqrt{2}\text{ m/s}$

18. (6)



19. (4)

$$T_1 = \frac{x}{2v_0}$$

$$\frac{T_2}{2}(v_1 + v_2) = \frac{x}{2}$$

$$T = T_1 + T_2 = \frac{x}{v_1 + v_2} + \frac{x}{2v_0} = x\left\{\frac{2v_0(v_1 + v_2)}{2v_0 + v_1 + v_2}\right\}$$

$$v_{avg} = \frac{x}{T} = \frac{2v_0(v_1 + v_2)}{2v_0 + v_1 + v_2}$$

$$\Rightarrow a + b = 2 + 2 = 4$$

PART (B) : CHEMISTRY

20. (B)

Mass of H_2O = Volume x density = 0.06 × 1

= 0.06 gm.

No. of molecules of $H_2O = \frac{0.06}{18} \times N_A$

$= 3.33 \times 10^{-3} \times 6.02 \times 10^{23}$

$= 2 \times 10^{21}$

21. (B)

$H_2 \;+\; Cl_2 \;\rightarrow\; 2HCl$

1mole 1mole 2mole

2gm 71gm 73gm

Given 5gm 5gm

Thus H_2 is in excess and Cl_2 is in limit

$\therefore 71\,gm\,Cl_2\ ^{o}\ 73\,gm\,HCl$

$5\,gm\,Cl_2 \equiv \frac{73}{71} \times 5\,gm\,HCl \equiv 5.14\,gm\,HCl.$

22. (D)

$$3BaCl_2 + 2Na_3PO_4 = 6NaCl + Ba_3(PO_4)_2$$

	$3BaCl_2$	$2Na_3PO_4$	$6NaCl$	$Ba_3(PO_4)_2$
Initially	3	2	6	1
Given mole	0.5	0.2		

Thus Na_3PO_4 is limiting reagent.

Hence Mole of $Ba_3(PO_4)_2$ formed = 0.1

23. (A)

$r_3 = r_1 \frac{3^2}{3} \quad \Rightarrow \quad r_1 = \frac{r_3}{3} = \frac{x}{3}\,cm$

$\therefore$ de-Broglie's wavelength $= \frac{2\pi x}{3}$

24. (B)

$IE = -E_1$

E_1 for $He^+ = -19.6 \times 10^{-18}$ J atom^{-1}

$$\frac{(E_1)_{He^+}}{(E_1)Li^{2+}} = \frac{\left(Z_{He^+}\right)^2}{\left(Z_{Li^{2+}}\right)^2}$$

$$\frac{-19.6\times10^{-18}}{(E_1)_{Li^{2+}}}=\frac{4}{9}$$

or $$E_1\left(Li^{2+}\right)=\frac{-19.6\times9\times10^{-18}}{4}$$
$$=-4.41\times10^{-17}\ J\ atom^{-1}$$

25. (D)

26. (B)

27. (A)

28. (B)
$120+20=140\,amu$
$2\ molecule=280\,amu$

29. (B)
$$M=\frac{1\times300+2\times200}{500}=\frac{7}{5}$$
$$\frac{7}{5}\times500=1\times(V+500)$$
$$700=V+500$$
$$V=200\,ml$$

30. (A, C)
$$r=0.53\frac{n^2}{Z}$$
for $He^+\Rightarrow Z=2, n=1,2,3.....$
$$r=0.53\times\frac{1^2}{2}=0.265$$
$$=0.53\times\frac{2^2}{2}=1.06\,Å$$
$$=0.53\times\frac{3^2}{2}=2.3856\,Å$$

31. (BCD)
$$\text{No. of moles}=\frac{\text{weight}}{G.M.W.}$$

32. (ACD)

$$\frac{4A_x}{4A_x+96}=0.572$$

$$\frac{A_x}{A_x+24}=0.572$$

$$A_x=(0.572\times 24)/(1-0.572)=32$$

33. (AC)

$$\bar{v}=RZ^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$$

$$x=R\left(\frac{1}{2^2}-\frac{1}{3^2}\right)=\frac{5R}{36}$$

$$\bar{v}_1=Rx2^2\left(1-\frac{1}{2^2}\right)=3R=\frac{36}{5}X\times 3=\frac{108x}{5}$$

Wavelength of 2nd line of lyman series of H-atom

$$\frac{1}{\lambda}=R\times 1^2\left(\frac{1}{1^2}-\frac{1}{3^2}\right)$$

$$\frac{1}{\lambda}=R\times\frac{8}{9}$$

$$\frac{1}{\lambda}=\frac{36x}{5}\times\frac{8}{9}$$

$$\lambda=\frac{5}{32x}$$

34. (5)

$$32\,\text{gm S}\Rightarrow 1\,\text{mole S}\Rightarrow 1\,\text{mole SO}_2$$

$$1\,\text{purity}=\frac{W_{SO_2}}{W_{Total}}\times 100$$

$$\Rightarrow\ \frac{64}{256}\times 100=25\%$$

$$x^2=25$$

$$x=5$$

35. (3)

$12.1\text{eV}=E_3-E_1$ for H-atom no. of spectral lines $=\dfrac{(n_2-n_1)(n_2-n_1+1)}{2}=3$

36. (9)
With principal 'n' the total number of electrons can present is given as "$2n^2$"
$\therefore$ If $n = 3$ total electrons can be $2.3^2 = 18$.
Out of them half can have $m_s = -\frac{1}{2}$

37. (1)
An orbital can accommodate maximally two electrons with opposite spin.

38. (2)
$KOH + CO_2 \rightarrow KHCO_3$
56 g 22.7 L
? 11.35 L
$= \frac{56 \times 11.35}{22.7} = 28\,g$
$= 2 \times 14\,g$

PART (C) : MATHEMATICS

39. (A)

Let α and α^2 be the roots of $x^2 - x - k = 0$. Then,

$$\alpha + \alpha^2 = 1 \text{ and } \alpha^3 = -k$$

$$\Rightarrow \quad (-k)^{1/3} + (-k)^{2/3} = 1$$

$$\Rightarrow \quad -k^{1/3} + k^{2/3} = 1$$

$$\Rightarrow \quad \left(k^{2/3} - k^{1/3}\right)^3 = 1$$

$$\Rightarrow \quad k^2 - k - 3k\left(k^{2/3} - k^{1/3}\right) = 1$$

$$\Rightarrow \quad k^2 - k - 3k(1) = 1$$

$$\Rightarrow \quad k^2 - 4k - 1 = 0$$

$$\Rightarrow \quad k = 2 \pm \sqrt{5}$$

40. (C)

$$\frac{\tan 205^\circ - \tan 115^\circ}{\tan 245^\circ + \tan 335^\circ}$$

$$= \frac{\tan 25^\circ + \cot 25^\circ}{\cot 25^\circ - \tan 25^\circ} = \frac{1 + \tan^2 25^\circ}{1 - \tan^2 25^\circ}$$

$$= \sec 50^\circ$$

41. (B)

$$\tan 40^\circ + 2\tan 10^\circ = \tan 40^\circ + 2\cot 80^\circ$$

$$= \tan 40^\circ + \frac{2\left(\cot^2 40^\circ - 1\right)}{2\cot 40^\circ}$$

$$= \frac{2\tan 40^\circ . \cot 40^\circ + 2\cot^2 40^\circ - 2}{2\cot 40^\circ}$$

$$= \cot 40^\circ$$

42. (D)

$$\cos\frac{\pi}{7} + \cos\frac{3\pi}{7} + \cos\frac{5\pi}{7}$$

$$= \frac{\cos\left(\frac{\pi}{7} + \frac{3-1}{2}.\frac{2\pi}{7}\right)}{\sin\frac{\pi}{7}}.\sin\frac{3\pi}{7}$$

$$= \frac{\cos\left(\frac{3\pi}{7}\right).\sin\left(\frac{3\pi}{7}\right)}{\sin\frac{\pi}{7}}$$

$$=\frac{\sin\frac{6\pi}{7}}{2.\sin\frac{\pi}{7}}$$

$$=\frac{\sin\frac{\pi}{7}}{2.\sin\frac{\pi}{7}}$$

$$=\frac{1}{2}$$

43. (A)

$\cos x\in[-1,1]\quad\forall\in R$ and $\sin x$ is increasing in $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

Hence, maximum value of $\sin(\cos x)=\sin 1$

44. (A)

$$\tan\frac{\pi}{3}+2\tan\frac{2\pi}{3}+4\tan\frac{4\pi}{3}+8\tan\frac{8\pi}{3}$$

$$=\tan\frac{\pi}{3}+2\tan\left(\pi-\frac{\pi}{3}\right)+4\tan\left(\pi+\frac{\pi}{3}\right)+8\tan\left(3\pi-\frac{\pi}{3}\right)$$

$$=\tan\frac{\pi}{3}-2\tan\frac{\pi}{3}+4\tan\frac{\pi}{3}-8\tan\frac{\pi}{3}$$

$$=-5\tan\frac{\pi}{3}$$

$$=-5\sqrt{3}$$

45. (C)

$$\sin 9^\circ\sin 69^\circ\sin 51^\circ=\frac{1}{4}\sin 27^\circ$$

$$\sin 21^\circ\sin 81^\circ\sin 39^\circ=\frac{1}{4}\sin 63^\circ$$

Required value $=\frac{4}{16}\frac{\sin 27^\circ\sin 63^\circ}{\sin 54^\circ}=\frac{1}{8}$

46. (B)

The roots of the equations are given by

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

(i) Let $b^2-4ac>0, b>0$

Now, if $a>0, b>0, b^2-4ac<b^2$

$\Rightarrow$ the roots are negative.

(ii) Let $b^2 - 4ac < 0$, then the roots are given by

$$x = \frac{-b \pm i\sqrt{\left(4ac - b^2\right)}}{2a}, \left(i = \sqrt{-1}\right)$$

Which are imaginary and have negative real part ($b > 0$)

$\therefore$ In each case, the roots have negative real part.

47. (A)

$$\frac{(1+\tan 11°)(1+\tan 34°)}{(1+\tan 17°)(1+\tan 28°)} = \frac{(1+\tan 11°)(1+\tan(45°-11°))}{(1+\tan 17°)(1+\tan(45°-17°))}$$

48. (C)

$a = \cos A + \cos B - \cos(A + B)$

$$a = 2.\cos\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right) - 2\cos^2\left(\frac{A+B}{2}\right) + 1$$

$$a = 2\cos\left(\frac{A+B}{2}\right)\left[\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right)\right] + 1$$

$$a = 4\sin\frac{A}{2}.\sin\frac{B}{2}.\cos\left(\frac{A+B}{2}\right) + 1$$

$\therefore$ $a = 4b + 1$

$\therefore$ $a - 4b = +1$

Hence, option (C).

49. (ABC)

$$x\cos\theta + y\sin\theta = k$$

$$x\cos\theta = k - y\sin\theta$$

$$x^2\cos^2\theta = k^2 + y^2\sin^2\theta - 2ky\sin\theta$$

$$x^2 - x^2\sin^2\theta = k^2 + y^2\sin^2\theta - 2ky\sin\theta$$

i.e. $\left(x^2 + y^2\right)\sin^2\theta - 2ky\sin\theta + k^2 - x^2 = 0$

$\therefore$ $\sin A + \sin B = \dfrac{2ky}{x^2 + y^2}$

$$\sin A.\sin B = \frac{k^2 - x^2}{x^2 + y^2}$$

$$y\sin\theta = k - x\cos\theta$$

$$y^2\left(1 - \cos^2\theta\right) = k^2 + x^2\cos^2\theta - 2kx\cos\theta$$

i.e. $\left(x^2 + y^2\right)\cos^2\theta - 2kx\cos\theta + k^2 - y^2 = 0$

$$\therefore \quad \cos A+\cos B=\frac{2kx}{x^2+y^2}$$

$$\cos A\cos B=\frac{k^2-y^2}{x^2+y^2}$$

50. (ABCD)

$$2\sin\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)=\frac{1}{\sqrt{2}}$$

$$2\cos\left(\frac{a+b}{2}\right)\cos\left(\frac{a-b}{2}\right)=\frac{\sqrt{3}}{\sqrt{2}} \qquad \Rightarrow \qquad \tan\left(\frac{a+b}{2}\right)=\frac{1}{\sqrt{3}}$$

$$\sin(a+b)=\frac{2\tan\left(\frac{a+b}{2}\right)}{1+\tan^2\left(\frac{a+b}{2}\right)}=\frac{2\times\frac{1}{\sqrt{3}}}{1+\frac{1}{3}}=\frac{\sqrt{3}}{2}$$

51. (ABC)

If roots of the equation be α, β, γ then

$$\alpha^2+\beta^2+\gamma^2=(\alpha+\beta+\gamma)^2-2(\alpha\beta+\beta\gamma+\gamma\alpha)=a^2-2b$$

$$\alpha^2\beta^2+\beta^2\gamma^2+\gamma^2\alpha^2=(\alpha\beta+\beta\gamma+\gamma\alpha)^2-2\alpha\beta\gamma(\alpha+\beta+\gamma)$$

$$=b^2-2a$$

$$\alpha^2\beta^2\gamma^2=1.$$

So, the equation whose roots are $\alpha^2, \beta^2, \gamma^2$ is

$$x^3-\left(a^2-2b\right)x^2+\left(b^2-2a\right)x-1=0$$

It is identical to $x^3-ax^2+bx-1=0$

$\therefore a^2-2b=a$ and $b^2-2a=b,$ eliminating b, we get

$$\frac{\left(a^2-a\right)^2}{4}-2a=\frac{a^2-a}{2}$$

$$\Rightarrow \quad a\left\{a(a-1)^2-8-2(a-1)\right\}=0$$

$$\Rightarrow \quad a\left(a^3-2a^2-a-6\right)=0$$

or $\quad a(a-3)\left(a^2+a+2\right)=0$

$\therefore \quad a=0$ or $a=3$ or $a^2+a+2=0$

Which give $b=0$ or $b=3$ or $b^2+b+2=0$

So, $a=b=0$ or $a=b=3$

or a, b are roots of $x^2+x+2=0$

52. (B)

Roots are lying in the interval (–1, 1) then $t(x)=4x^2-2x+4$

(i) $D\geq 0 \Rightarrow 4-4.4.a\geq 0$

$4\geq 16a$

$\frac{1}{4}\geq a$

(ii) $-1<\frac{-b}{2a}<1 \Rightarrow -1<\frac{2}{2.4}<1$

$-1<\frac{1}{4}<1$

Always true

(iii) $4f(-1)>0 \Rightarrow 4.(6+a)>0$

$a>-6$

(iv) $4.f(1)>0 \Rightarrow 4.(2+a)>0$

$a>-2$

$\therefore\ a\in\left(-2,\frac{1}{4}\right]$

Hence, option (B).

53. (4)

Let roots be $\alpha, 4\alpha$

$\therefore\ \alpha+4\alpha=\frac{-5}{3} \Rightarrow \alpha=\frac{-1}{3}$

and $\alpha.4\alpha=\frac{a}{3}$

$\Rightarrow a=3.4\left(\frac{-1}{3}\right)^2$

$\Rightarrow\ 3a=4$

54. (1)

$1\sin 40°.\sec 50°-\frac{\tan 40°}{\cot 50°}+1$

= 1 – 1 + 1

= 1

55. (0)

Let $3\sin\theta+4\cos\theta=5$

$3\cos\theta-4\sin\theta=x$

$\therefore\ 9+16=25+x^2$

$x=0$

56. (3)

$\alpha+\beta=1;\ \alpha\beta=-4$

$\frac{1}{\alpha}+\frac{1}{\beta}=\alpha\beta=\frac{\alpha+b}{\alpha\beta}-\alpha\beta$

$=\frac{1}{-4}-(-4)$

$=\frac{-1}{4}+4=\frac{15}{4}=\frac{p}{q}$

$\therefore\ p-3q=15-3(4)=3$

57. (1)